Examination Papers, 2017
[Set-I, II, III]

Time Allowed: **3 Hours**] [Maximum Marks: **90**

General Instructions:

(i) All questions are compulsory.

*(ii) The question paper consists of **31** questions divided into four sections – A, B, C and D.*

*(iii) Section A contains **4** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **11** questions of **4** marks each.*

(iv) Use of calculators is not permitted.

Set-I

SECTION-A

1. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the sun?

2. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

3. A number is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What will be the probability that square of this number is less than or equal to 1?

4. If the distance between the points $(4, k)$ and $(1, 0)$ is 5, then what can be the possible values of k?

SECTION-B

5. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

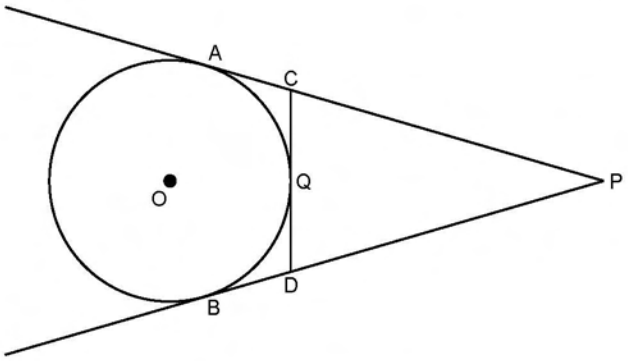
6. Find how many integers between 200 and 500 are divisible by 8.

7. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

8. Find the value of k for which the equation $x^2 + k(2x + k - 1) = 0$ has real and equal roots.

9. Draw a line segment of length 8 cm and divide it internally in the ratio 4 : 5.

10. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD.



SECTION-C

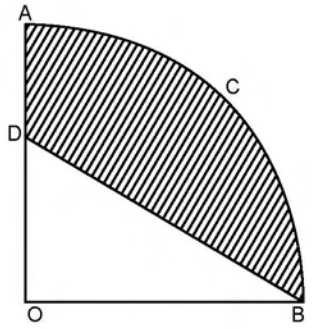
11. If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first mn term.

12. Find the sum of n terms of the series $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+ \ldots$

13. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.

14. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

15. In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region.



16. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

17. Show that ΔABC, where A(–2, 0), B(2, 0), C(0, 2) and ΔPQR where P(–4, 0), Q(4, 0), R(0, 4) are similar triangles.

18. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, –2). If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of y.

19. Two different dice are thrown together. Find the probability that the numbers obtained

(*i*) have a sum less than 7

(*ii*) have a product less than 16

(*iii*) is a doublet of odd number.

20. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

SECTION-D

21. Construct an isosceles triangle with base 8 cm and altitude 4 cm. Construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the isosceles triangle.

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

23. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$

24. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

25. If $a \neq b \neq 0$, prove that the point (a, a^2), (b, b^2) $(0, 0)$ will not be collinear.

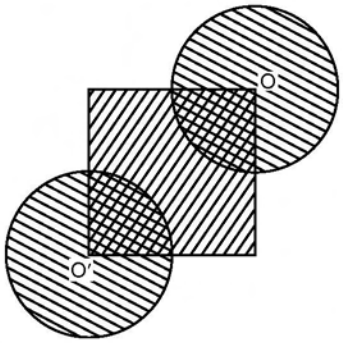
26. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

27. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25.

28. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle.

29. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

30. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where O and O′ are centres of the circles. Find the area of shaded region.



31. In a hospital used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park. Write your views on recycling of water.

Set-II [*Uncommon Questions to Set-I*]

10. Draw a line segment of length 7 cm and divide it internally in the ratio 2 : 3.

18. If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} is $\frac{1}{m}$ then show that its $(mn)^{th}$ term is 1.

19. A metallic solid sphere of radius 10.5 cm is melted and recasted into smaller solid cones, each of radius 3.5 cm and height 3 cm. How many cones will be made?

20. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

28. Draw a right triangle in which the sides (other than the hypotenuse) are of lengths 4 cm and 3 cm. Now construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

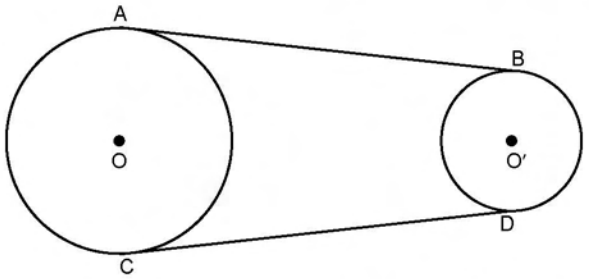
29. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first $(m + n)$ terms is zero.

30. Two points A and B are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

31. The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is $\frac{1}{27}$ of the given cone, then at what height it is cut from its base.

Set-III [*Uncommon Questions to Set-I & II*]

10. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.



18. If the p^{th} term of an A.P. is q and q^{th} term is p, prove that is n^{th} term is $(p + q - n)$.

19. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

20. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If height of the tower is 50 m, find the height of the hill.

28. Construct a triangle ABC with sides BC = 7 cm, $\angle B = 45°$ and $\angle A = 105°$. then construct a triangle whose sides are $\frac{3}{4}$ times the corresponding ΔABC.

29. If the p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms of the A.P. is $\left(\frac{pq+1}{2}\right)$.

30. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m towards the base of the tower, the angle of elevation of the top increases by 15°, find the height of the tower.

31. A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the mid-point of its height. Compare the volumes of the two parts.

Examination Papers, 2016
[Delhi Set-I, II, III]

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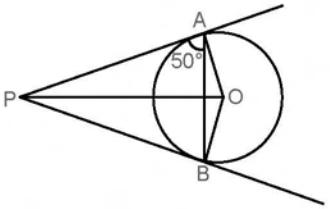
SET-I

1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$.

Sol. Given, $\angle PAB = 50°$



$\angle PAB + \angle OAB = 90°$ [$\because$ angle between radius OA and tangent PA is 90°]

$\Rightarrow 50° + \angle OAB = 90°$

$\Rightarrow \angle OAB = 90° - 50° = 40°$

Now, $PA = PB$ [$\therefore$ tangents from an external point are same]

$\Rightarrow \angle PBA = \angle PAB$

$\Rightarrow \angle PBA = 50°$

$\angle PBA + \angle OBA = 90°$ [$\therefore$ angle between radius OB and tangent PB is 180°]

$\Rightarrow 50° + \angle OBA = 90°$

$\Rightarrow \angle OBA = 90° - 50° = 40°$

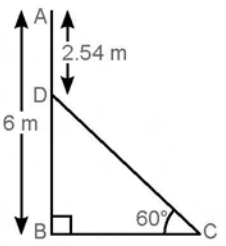
Now in ΔAOB we have

$\angle AOB + \angle ABO + \angle BAO = 180°$ [$\because$ sum of angles in triangle is 180°]

$\Rightarrow \angle AOB + 40° + 40° = 180°$

$\Rightarrow \angle AOB = 180° - 80° = 100°$

2. If Figure, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2.54 m, find the length of the ladder. (use $\sqrt{3} = 1.73$)



Sol. $BD = AB - AD$

$= 6 \text{ m} - 2.54 \text{ m} = 3.46 \text{ m}$

In ΔDBC, $\quad \frac{BD}{CD} = \sin 60°$

$\Rightarrow \quad \frac{3.46}{CD} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad CD = \frac{2 \times 3.46}{\sqrt{3}} = \frac{2 \times 3.46}{1.73} = 2 \times 2 = 4$ m

Hence, length of the ladder is 4 m.

3. **Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.**

Sol. Reversing the given A.P., we get

185, 181, 174, ..., 9, 5

Now, first term $(a) = 185$

Common difference, $(d) = 181 - 185 = -4$

We know that nth term of an A.P. is given by $a + (n - 1)d$

Ninth term $a_9 = a + (9 - 1)d$

$= 185 + 8 \times (-4) = 185 - 32 = 153$

4. **Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.**

Sol. Total possible outcomes when one card is drawn = 48

When the number on drawn card is a perfect square, total favourable cases are 4, 9, 16, 25, 36, 49, i.e. = 6

$$P(\text{perfect square number}) = \frac{\text{Number of total possible outcomes}}{\text{Number of favourable outcomes}} = \frac{6}{48} = \frac{1}{8}$$

5. **If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equations $ax^2 + 7x + b = 0$, find the values of a and b.**

Sol. Given quadratic equation is $ax^2 + 7x + b = 0$ ...(*i*)

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0 \qquad [\because x = \frac{2}{3} \text{ is the root of equation } (i)]$$

$\Rightarrow \quad \frac{4}{9}a + \frac{14}{3} + b = 0$

$\Rightarrow \quad \frac{4a + 42 + 9b}{9} = 0 \Rightarrow 4a + 9b + 42 = 0$ ...(*ii*)

$a(-3)^2 + 7(-3) + b = 0 \qquad [\because x = -3 \text{ is the root of equation } (i)]$

$\Rightarrow \quad 9a + b - 21 = 0$ ..(*iii*)

Putting the value of b from (*iii*) in (*ii*), we get

$$4a + 9[21 - 9a] + 42 = 0$$

$$\Rightarrow \quad 4a + 189 - 81a + 42 = 0$$

$$\Rightarrow \quad 231 - 77a = 0$$

$$\Rightarrow \quad 77a = 231$$

$$\Rightarrow \quad a = 3$$

Putting $a = 3$ in (*iii*), we have

$$27 + b = 21$$

$$\Rightarrow \quad b = -6$$

$$a = 3, b = -6$$

6. Find the ratio in which *y*-axis divides the line segment joining the points A(5, –6) and B(–1, –4). Also find the coordinates of the point of division.

Sol. Let the point on y-axis be P$(0, y)$ and AP : PB $= k : 1$.

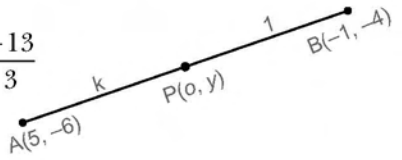
$\therefore$ Co-ordinates of P given by: $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Then, taking x-axis of A, B; $\frac{5 \times 1 + k(-1)}{k+1} = 0 \Rightarrow \frac{5-k}{k+1} = 0 \Rightarrow k = 5$

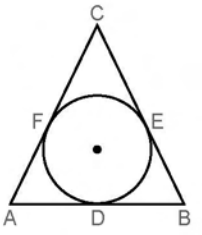
Hence the required ratio is 5 : 1

Now, taking y-axis, $y = \frac{(-4)(5) + (1)(-6)}{5+1} = \frac{-13}{3}$

Hence point on y-axis is $\left(0, \frac{-13}{3}\right)$



7. In given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Sol. Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $AD = AF = x$ [$\because$ Tangent drawn from external point to circle are equal]

$\therefore$ $DB = BE = 12 - x$ and $CF = CE = 10 - x$

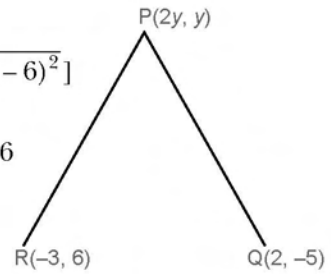
$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$

$\Rightarrow \quad x = 7$

$\therefore$ AD = 7 cm, BE = 5 cm and CF = 3 cm

8. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, –5) and R(–3, 6), find the coordinates of P.

Sol. Let the required point be $(2y, y)$. Let Q(2, – 5) and R(–3, 6) are given points.



Now, PQ = PR $\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

[$\because$ using Distance formula, $\sqrt{(x-2)^2+(y+5)^2} = \sqrt{(x+3)^2+(y-6)^2}$]

Squaring both sides we get

$4y^2 + 4 - 8y + y^2 + 10y + 25 = 4y^2 + 9 + 12y + y^2 - 12y + 36$

$\Rightarrow \quad 2y + 29 = 45$

$\Rightarrow \quad 2y = 45 - 29 = 16$

$\Rightarrow \quad y = 8$

$\Rightarrow \quad 2y = 16$

Hence coordinates of P are (16, 8)

9. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero?

Sol. Let the number of terms taken for sum to be zero be n.

Then, sum of n terms $(S_n) = 0$ (Given)

First term $(a) = 18$

Common difference $(d) = -2$

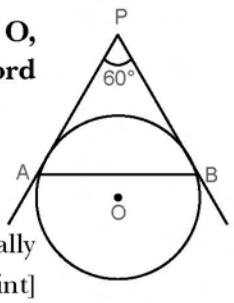
Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)] \quad \Rightarrow \quad 0 = 38 - 2n$

$\Rightarrow \quad n = 19$

$\therefore$ Hence, sum of 19 terms is 0.

10. If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle$APB = 60°. Find the length of chord AB.



Sol. In ΔAPB we have AP = BP

$\Rightarrow \quad \angle PAB = \angle PBA$

[$\because$ Tangents from an external point are equally inclined to segment joining centre to point]

Let $\angle PAB = x$,

then in ΔAPB, $x + x + 60° = 180°$ [$\because$ sum of angles of Δ is 180°]

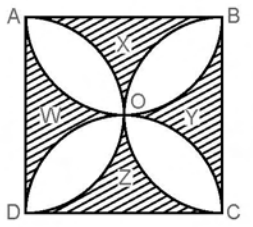
$2x = 180° - 60° = 120°$

$x = 60°$

As all three angles of ΔAPB are 60°. So ΔAPB is an equilateral triangle.

Hence AP = BP = AB = 5 cm

11. In figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Area of the square ABCD = $14 \times 14 = 196 \text{ cm}^2$

Area of semicircle AOB = $\frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$ $\left(\because r = \frac{14}{2}\right)$

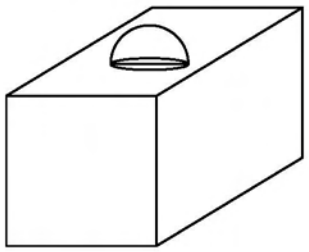
Similarly, area of semicircle DOC = 77 cm^2

Hence, the area of shaded region (Part W and Part Y) = Area of square – Area of two semicircles AOB and COD

$= 196 - 154 = 42 \text{ cm}^2$

Therefore, area of four shaded parts (i.e. X, Y, W, Z) = $(2 \times 42) \text{ cm}^2 = 84 \text{ cm}^2$

12. In Figure, a decorative block, made up of two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{use } \pi = \frac{22}{7}\right)$



Sol. Side of the cube = 6 cm

Total surface area of cube = $6 \times (\text{side})^2 = 6 \times (6)^2 = 216 \text{ cm}^2$

Area covered on the face of cube by circular part of hemisphere = $\pi r^2 = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \text{ cm}^2$

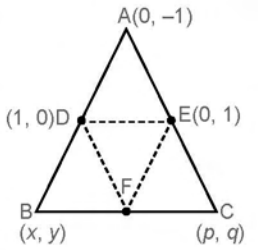
Curved surface area of hemisphere = $2 \times \pi \times r^2 = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \text{ cm}^2$

So, Total surface area of the block = Surface area of cube – Area of circular face of hemisphere + Curved surface area of hemisphere

$= 216 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$

$= 216 + \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} = 216 + 9.625 = 225.625 \text{ cm}^2$

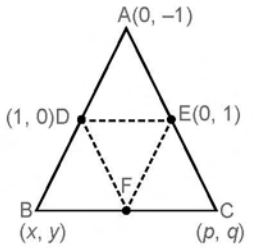
13. In figure ABC is a triangle coordinates of whose vertex A are (0, –1). D and E respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0, 1) respectively. If F is the mid-point of BC, find the areas of ΔABC and ΔDEF.



Sol. Let coordinates of B are (x, y). Then using mid point formula we have

$$\frac{x+0}{2} = 1 \quad \Rightarrow \quad x = 2$$

$$\frac{y-1}{2} = 0 \quad \Rightarrow \quad y = 1$$



Coordinates of B are (2,1)

Let coordinates of C are (p, q)

Similarly coordinates of C we have

$$\frac{p+0}{2} = 0 \quad \Rightarrow \quad p = 0$$

$$\frac{q-1}{2} = 1 \quad \Rightarrow \quad q = 3$$

Coordinates of C are (0, 3)

Area of ΔABC

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = \frac{1}{2}[0(1-3) + 2(3+1) + 0(-1-1)]$$

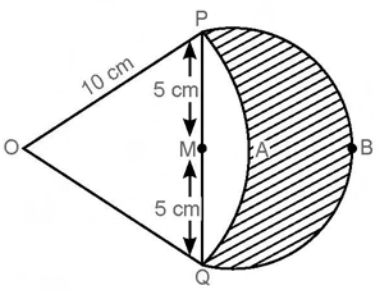
$$= \frac{1}{2} \times 8 = 4 \text{ sq. units}$$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ i.e. (1, 2) [$\because$ Using mid-point formula]

$$\text{Area of } \Delta DEF = \frac{1}{2}[1(1-2) + 0(2-0) + 1(0-1)] = \frac{1}{2}[-1 + 0 - 1]$$

$$= \frac{1}{2} \times (-2) = [-1] = 1 \text{ sq. units} \quad [\because \text{Area cannot be negative}]$$

14. In figure, two arcs PAQ and PBQ are shown. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semicircle drawn on PQ as diameter with centre M. If OP = PQ = 10 cm, show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm².



Sol. OP = OQ = 10 cm (radii of circle)

PQ = 10 cm (given)

So, ΔOPQ is an equilateral triangle

$\therefore \quad \angle POQ = 60°$

$$\text{Area of segment PAQM} = \text{Area of sector OPAQ} - \text{Area of } \Delta OPQ$$

$$= \frac{60}{360} \times \pi \times 10 \times 10 - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{cm}^2$$

$$\text{Area of semicircle} = \frac{1}{2}\times\pi\times5\times5 = \frac{25}{2}\pi \text{ cm}^2$$

$$\text{Area of the shaded region} = \frac{25}{2}\pi - \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) = \frac{25\pi}{2} - \frac{50\pi}{3} + 25\sqrt{3}$$

$$= \frac{75\pi - 100\pi}{6} + 25\sqrt{3} = 25\sqrt{3} - \frac{25\pi}{6}$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2$$

15. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.

Sol. **Given:** $S_7 = 49$, where $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \frac{7}{2}[2a + (7-1)d] = 49$$

$$\Rightarrow 2a + 6d = 14 \Rightarrow a + 3d = 7 \quad ...(i)$$

Similarly, $S_{17} = 289$

$$\frac{17}{2}[2a + (17-1)d] = 289$$

$$\Rightarrow 2a + 16d = 34 \Rightarrow a + 8d = 17 \quad ...(ii)$$

Solving (i) and (ii), we get

$$a = 1 \text{ and } d = 2$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2 + 2n - 2] = n \times n = n^2$$

16. Solve for x: $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, \quad x \neq 3, -\frac{3}{2}$

Sol. $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0 \Rightarrow \frac{2x(2x+3) + (x-3) + (3x+9)}{(x-3)(2x+3)} = 0$

$$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0 \Rightarrow 4x^2 + 10x + 6 = 0$$

$$\Rightarrow 2x^2 + 5x + 3 = 0 \Rightarrow 2x^2 + 2x + 3x + 3 = 0$$

$$\Rightarrow 2x(x+1) + 3(x+1) = 0 \Rightarrow (x+1)(2x+3) = 0$$

$$\Rightarrow x + 1 = 0 \text{ or } 2x + 3 = 0$$

$$\Rightarrow x = -1 \text{ or } x = \frac{-3}{2} \quad \left(\because \text{ given, } x \neq \frac{-3}{2}\right)$$

$\because$ It is given that $x \neq \frac{-3}{2}$.

Hence, solution of the given equation is $x = -1$.

17. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Sol. Radius of the well = 2 m, height of the well = 21 m

Volume of the earth dug out $= \pi r^2 h$

$= \pi \times 2 \times 2 \times 21 \text{ m}^3 = 264 \text{ m}^3$

$\therefore$ Radius of embankment = Radius of well + width of embankment

$= 2 + 3 = 5$ m

Volume of embankment $= (\pi r_1^{\,2} h - \pi r_2^{\,2} h)\text{m}^3$

$= [\pi \times (5)^2 \times h - \pi(2)^2 h]\text{m}^3$

$= (\pi \times 25 \times h - \pi - 4h)\text{m}^3$

$= \left(\frac{22}{7} \times 21 \times h\right) \text{m}^3 = 66\,h \text{ m}^3$

As per condition,

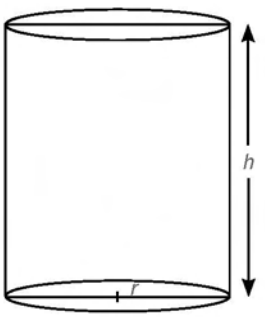
$\Rightarrow$ Volume of earth dug out = Volume of embankment

$\Rightarrow \quad 264 \text{ m}^3 = 66h \text{ m}^3$

$\Rightarrow$ Height of embankment, $h = \frac{264}{66} = 4$ m

18. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left(\textbf{use } \pi = \frac{\mathbf{22}}{\mathbf{7}}\right)$

Sol. Here $r + h = 37$ [Given, where $r \rightarrow$ radius, $h \rightarrow$ height]



Total surface area of cylinder $= 2\pi r(h + r) = 2\pi rh + 2\pi r^2$

$\Rightarrow \quad 2\pi r(h + r) = 1628$

$\Rightarrow \quad 2\pi r \times 37 = 1628$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 37 = 1628$

$\Rightarrow \quad r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7$ cm

Given, $r + h = 37$

$7 + h = 37$

$\Rightarrow \quad h = 37 - 7 = 30$ cm

Hence, volume of cylinder $= \pi r^2 h$

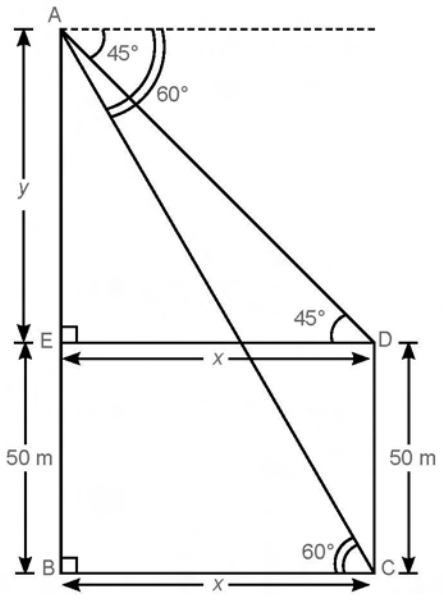
$= \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$

19. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3} = 1.73$)

Sol. Let CD is the building of height 50 m and AB be the tower.

Let horizontal distance between the tower and building is BC is x metre.

$\because$ BCDE is a rectangle



So, $ED = BC$ and $BE = CD$

Also, $ED = x$ and $BE = 50$ m

Let $AE = y$

Now, in ΔAED, $\frac{y}{x} = \tan 45° \Rightarrow \frac{y}{x} = 1$

$\Rightarrow$ $y = x$ ...(*i*)

Now, in ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow$ $\frac{AE + EB}{BC} = \sqrt{3} \Rightarrow \frac{y + 50}{x} = \sqrt{3}$

$\Rightarrow$ $x + 50 = \sqrt{3}x$ [$\because$ $y = x$, using(*i*)]

$\Rightarrow$ $\sqrt{3}x - x = 50$

$\Rightarrow$ $(\sqrt{3} - 1)x = 50$

$\Rightarrow$ $x = \frac{50}{\sqrt{3} - 1}$

$\Rightarrow$ $x = \frac{50(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$= \frac{50(\sqrt{3} + 1)}{3 - 1} = \frac{50(\sqrt{3} + 1)}{2}$

$\Rightarrow$ $x = 25(\sqrt{3} + 1) = 25(1.73 + 1) = 25 \times 2.73 = 68.25$ m

$\therefore$ Height of the tower $= 50 + y = 50 + 68.25$ ($\because$ $x = y$)

$= 118.25$ m

Horizontal distance between the tower and the building $= x = 68.25$ m.

20. In a single throw of a pair of different dice, what is the probability of getting (*i*) a prime number on each dice? (*ii*) a total of 9 or 11?

Sol. (*i*) Total possible cases when two dice are thrown together $= 6 \times 6 = 36$

Favourable cases when both numbers are prime are (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5), i.e. 9 outcomes

P(a prime number on each dice) $= \frac{\text{Favourable cases}}{\text{Total cases}} = \frac{9}{36} = \frac{1}{4}$

(*ii*) Favourable cases when sum of numbers are 9 or 11 are (3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5), i.e. 6 outcomes

P(a total of 9 or 11) $= \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{6}{36} = \frac{1}{6}$

21. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane.

What value is depicted in this question?

Sol. Let the usual speed of plane be x km/hr.

Time taken to cover 1500 km with usual speed $= \frac{1500}{x}$ hours.

When the speed of plane is increased, then new speed $= (x + 250)$ km/hr.

Time taken to cover 1500 km with the new speed $(x + 250)$ km/hr $= \frac{1500}{x+250}$

According to question, $\frac{1500}{x} = \frac{1500}{x+250} + \frac{1}{2} \Rightarrow \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$$\Rightarrow \frac{1500x + 1500 \times 250 - 1500x}{x(x+250)} = \frac{1}{2} \Rightarrow \frac{1500 \times 250}{x^2 + 250x} = \frac{1}{2}$$

$$\Rightarrow x^2 + 250x = 750000 \Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow x^2 + 1000x - 750x - 750000 = 0 \Rightarrow x(x + 1000) - 750(x + 1000) = 0$$

$$\Rightarrow (x + 1000)(x - 750) = 0 \Rightarrow x + 1000 = 0 \text{ or } x - 750 = 0$$

$$\Rightarrow x = -1000 \text{ or } x = 750$$

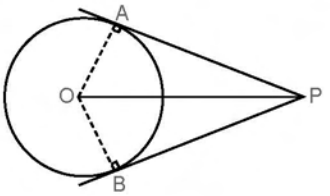
$$\Rightarrow x = 750 \quad (\because \text{Speed cannot be negative})$$

Hence, the usual speed of plane is 750 km/hr.

In this question, pilot's caring behaviour towards passengers is shown as well as other side pilot is cautious and alert for his duty to reach destination point at scheduled time.

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. **Given:** A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.



To Prove: PA = PB

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.

$\angle OAP = \angle OBP = 90°$ ...(*i*)

[Radius is perpendicular to the tangent at the point of contact]

OA = OB (radii) ...(*ii*)

OP is common ...(*iii*)

$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*) and (*iii*)]

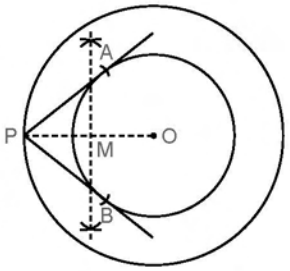
Hence, AP = BP (CPCT)

23. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length.

Sol. Now after measuring, PA and PB comes out to be 4 cm.

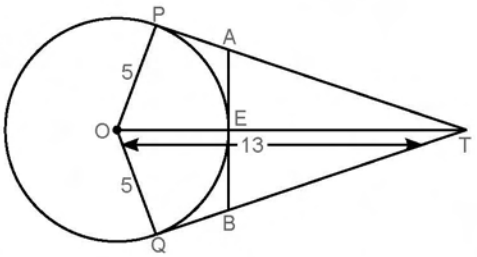
Steps of construction of tangents:

1. Take point O. Draw 2 concentric circles of radii 3 cm and 5 cm respectively.
2. Locate point P on the circumference of larger circle.
3. Join OP and bisect it. Let M be mid-point of OP.
4. Taking M as centre and MP as radius, draw an arc intersecting smaller circle at A and B.



Join PA and PB. Thus, PA, PB are required tangents.

24. In given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Sol. In ΔOPT, $OP^2 + PT^2 = OT^2$ [$\because$ Pythagoras theorem]

$$PT = \sqrt{OT^2 - OP^2}$$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and $TE = OT - OE = 13 - 5 = 8$ cm

Let $PA = AE = x$ [tangent from outer point A]

In ΔTEA, $TE^2 + EA^2 = TA^2$ [$\because$ Pythagoras theorem]

$$(8)^2 + (x)^2 = (12 - x)^2$$

$$64 + x^2 = (12 - x)^2$$

$$\Rightarrow \quad 64 + x^2 = 144 + x^2 - 24x$$

$$\Rightarrow \quad 80 = 24x$$

$$\Rightarrow \quad x = 3.3 \text{ cm}$$

Thus $AB = 2 \times 3.3$ cm $= 6.6$ cm [$\because$ AE = EB, as AB is tangent to circle at E]

25. Find x in terms of a, b and c: $\dfrac{a}{x-a} + \dfrac{b}{x-b} = \dfrac{2c}{x-c}$, $x \neq a, b, c$

Sol.
$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c} \Rightarrow \frac{a(x-b) + b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$$

$$\Rightarrow \quad \frac{(ax + bx - 2ab)}{(x^2 - ax - bx + ab)} = \frac{2c}{x-c}$$

$$\Rightarrow \quad (ax + bx - 2ab)(x - c) = 2c(x^2 - ax - bx + ab)$$

$$\Rightarrow \quad ax^2 + bx^2 - 2abx - acx - bcx + 2abc = 2cx^2 - 2acx - 2bcx + 2abc$$

$\Rightarrow \quad (a + b - 2c)x^2 + (ac + bc - 2ab)x = 0$

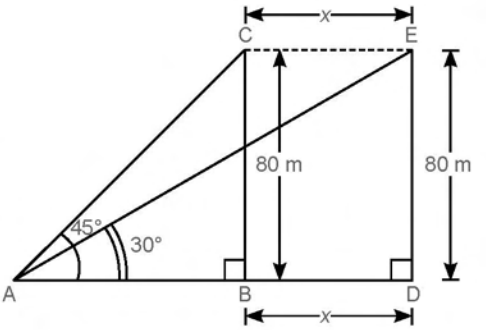
$\Rightarrow \quad x\{(a + b - 2c)x + (ac + bc - 2ab)\} = 0$

$\Rightarrow \quad x = 0 \quad \text{or} \quad x = \dfrac{2ab - ac - bc}{a + b - 2c}$

26. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$)

Sol. Let BC is 80 m high tree.

After 2 seconds, position of bird is E.



Let $CE = x$

In ΔCBA, $\dfrac{BC}{AB} = \tan 45°$

$\Rightarrow \quad \dfrac{80}{AB} = 1$

$\Rightarrow \quad AB = 80 \text{ m}$

In ΔEDA, $\dfrac{ED}{AD} = \tan 30°$

$\Rightarrow \quad \dfrac{80}{AB + BD} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \dfrac{80}{80 + x} = \dfrac{1}{\sqrt{3}} \Rightarrow 80\sqrt{3} = 80 + x \qquad [\because AB = 80 \text{ m}]$

$\Rightarrow \quad x = 80\sqrt{3} - 80 \Rightarrow x = 80(\sqrt{3} - 1)$

$\Rightarrow \quad x = 80(1.732 - 1) \Rightarrow x = 80 \times 0.732$

$\Rightarrow \quad x = 58.56 \text{ m}$

$\Rightarrow \quad BD = x = 58.56 \text{ m}$

So, the speed of flying of the bird $= \dfrac{\text{distance (BD)}}{\text{Time}}$

$= \dfrac{58.56}{2} = 29.28 \text{ m/s}$

27. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

Sol. Let total time be $(n - 1)$ minutes in which the police catch the thief.

Since thief ran 1 minute before police start running.

$\because$ Time taken by thief before he was caught $= (n - 1 + 1) = n$ minute

Then total distance covered by thief $= (100 \times n)$ metres

Total distance covered by policeman in $(n-1)$ minute

$$= 100 + 110 + 120 + \ldots + (n-1) \text{ terms}$$

$$= \frac{(n-1)}{2}[2000 + (n-2)10] \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$

According to question,

Total distance covered by thief in 'n' minute

= total distance covered by policeman in $(n-1)$ minute

$$100n = \frac{(n-1)}{2}[200 + (n-2)10]$$

$$\Rightarrow \quad 200n = (n-1)[200 + 10n - 20] \Rightarrow 200n = (n-1)(10n + 180)$$

$$\Rightarrow \quad 200n = 10n^2 + 180n - 10n - 180 \Rightarrow 10n^2 - 30n - 180 = 0$$

$$\Rightarrow \quad n^2 - 3n - 18 = 0 \Rightarrow n^2 - 6n + 3n - 18 = 0$$

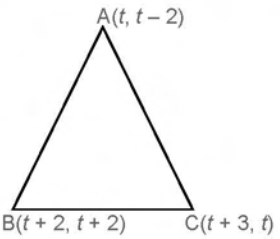
$$\Rightarrow \quad n(n-6) + 3(n-6) = 0 \Rightarrow (n-6)(n+3) = 0$$

$$\Rightarrow \quad n = 6 \text{ or } n = -3 \text{ (rejected)}$$

Hence, time taken by policeman to catch the thief is $(6-1)$, i.e. 5 minutes.

28. Prove that the area of a triangle with vertices $(t, t-2)$, $(t+2, t+2)$ and $(t+3, t)$ is independent of t.

Sol. Given vertices of triangle are $\{t, t-2\}$, $\{t+2, t+2\}$, $\{t+3, t\}$



Let (x_1, y_1), (x_2, y_2), (x_3, y_3) are vertices of the triangle.

Area of the triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

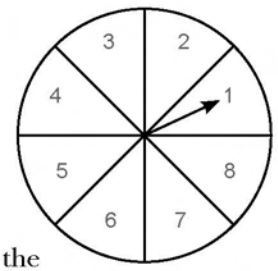
$= \frac{1}{2}[t(t+2-t) + (t+2)\{t-t+2\} + (t+3)\{t-2-t-2\}]$

$= \frac{1}{2}[2t + 2t + 4 - 4t - 12]$

$= \frac{1}{2} \times (-8) = 4$ sq units, since area can't be negative.

Hence, area is independent of t.

29. A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1, 2, 3, ..., 8 which are equally likely outcomes. What is the probability that the arrow will point at (*i*) an odd number (*ii*) a number greater than 3 (*iii*) a number less than 9?



Sol. (*i*) Total possible outcomes when the arrow points at one of the numbers are 8.

Favourable outcomes when the required number is odd are 1, 3, 5, 7, i.e. 4 outcomes.

$$\therefore \text{P(an odd number)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{4}{8} = \frac{1}{2}$$

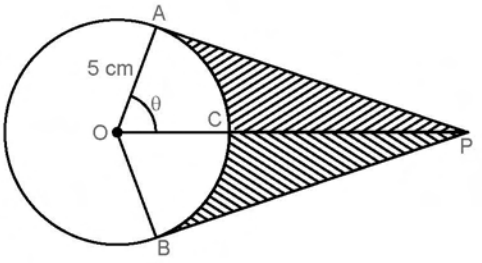
(*ii*) Favourable outcomes when the required number is more than 3 are 4, 5, 6, 7, 8, i.e. 5 outcomes.

$$\text{P(a number is more than 3)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcome}} = \frac{5}{8}$$

(*iii*) Favourable outcomes when the required number is less than 9 are 1, 2, 3, 4, 5, 6, 7, 8, i.e. 8 outcomes.

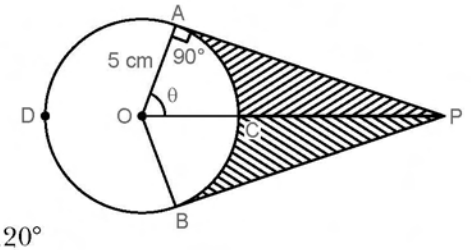
$$\text{P(number is less than 9)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{8}{8} = 1$$

30. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. (use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Sol. **Given:** $AO = 5$ cm and $OP = 10$ cm

In right ΔAOP,



$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 60°$

$$\angle AOB = \theta' = 2 \times 60 = 120°$$

$$\text{Length of ADB} = \frac{360° - \theta'}{360°} \times 2\pi r \qquad (\because \theta' = 120°)$$

$$= \frac{240}{360} \times 2 \times 3.14 \times 5$$

$$= \frac{2}{3} \times 10 \times 3.14 = 20.93 \text{ cm}$$

Hence, length of belt in contact = 20.93 cm

Now, in right ΔOAP, we have

$$\tan\theta = \frac{AP}{AO}$$

$$\tan 60° = \frac{AP}{5} \quad \Rightarrow \quad AP = 5\sqrt{3} \text{ cm}$$

$$\text{Area of } (\Delta OAP + \Delta OBP) = \frac{1}{2} \times AO \times AP + \frac{1}{2} \times OB \times PB$$

$$= \frac{1}{2} \times 5 \times 5\sqrt{3} + \frac{1}{2} \times 5 \times 5\sqrt{3} \quad (\because AP = BP \text{ and } OA = OB)$$

$$= 25\sqrt{3} \text{ cm}^2 = 25 \times 1.73 \text{ cm}^2$$

$$= 43.25 \text{ cm}^2$$

$$\text{Area of sector OACB} = \frac{\theta'}{360} \times \pi r^2$$
$$= \frac{120}{360} \times 3.14 \times 5 \times 5$$
$$= \frac{1}{3} \times 3.14 \times 25 = 26.16 \text{ cm}^2$$
$$\text{Shaded Area} = \text{Area of } (\Delta OAP + \Delta OBP) - \text{Area of OACB}$$
$$= 43.25 - 26.16 = 17.09 \text{ cm}^2$$

31. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (use $\pi = 3.14$)

Sol. Let R, r and V be the upper radius, lower radius and volume of the frustum respectively then

R = 20 cm, r = 12 cm and V = 12308.8 cm^3

$$\text{Volume of frustum of cone} = \frac{1}{3} \times \pi[(R^2 + r^2 + Rr)]h$$
$$12308.8 = \frac{1}{3} \times 3.14\,[400 + 144 + 240]h$$
$$h = \frac{12308.8 \times 3}{3.14 \times 784} \Rightarrow h = 15 \text{ cm}$$

Now, l (slant height) $= \sqrt{(20-12)^2 + 15^2} = \sqrt{64+225} = \sqrt{289} = 17$ cm^2

Total area of metal sheet used = Curved Surface Area of cone + Area of base
$$= \pi(R + r)l + \pi r^2$$
$$= \pi(20 + 12) \times 17 + \pi \times 12 \times 12$$
[$\because$ lower radius, r = 12 cm]
$$= \pi \times 32 \times 17 + 144\pi = 688\pi$$
$$= 688 \times 3.14 = 2160.32 \text{ cm}^2$$

SET-II [UNCOMMON QUESTIONS TO SET-I]

10. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero?

Sol. In the given A.P.,

Here, first term (a) = 27

Common difference (d) = – 3

Sum of n terms (S_n) = 0

Therefore, $$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$0 = \frac{n}{2}[2 \times 27 + (n-1)(-3)]$$
$$\Rightarrow \quad 54 - 3n + 3 = 0$$
$$\Rightarrow \quad 3n = 57 \Rightarrow n = 19$$

Thus, the sum of 19 terms of given A.P. is zero.

18. Solve for x: $\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}$; $x \neq 1, -2, 2$

Sol.

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}$$

$$\Rightarrow \quad \frac{x+1}{x-1}+\frac{x-2}{x+2}+\frac{2x+3}{x-2}=4$$

$$\Rightarrow \frac{(x+1)(x+2)(x-2)+(x-2)^2(x-1)+(2x+3)(x-1)(x+2)}{(x-1)(x+2)(x-2)}=4$$

$$\Rightarrow (x+1)(x^2-4)+(x-1)(x^2+4-4x)+(2x+3)(x^2+x-2)=4(x-1)(x^2-4)$$

$$\Rightarrow x^3-4x+x^2-4+x^3+4x-4x^2-x^2-4+4x+2x^3+2x^2-4x+3x^2+3x-6=4(x^3-4x-x^2+4)$$

$$\Rightarrow x^3+x^2-4x-4+x^3-5x^2+8x-4+2x^3+5x^2-x-6=4(x^3-x^2-4x+4)$$

$$\Rightarrow \quad 4x^3+x^2+3x-14=4x^3-4x^2-16x+16$$

$$\Rightarrow \quad 5x^2+19x-30=0$$

$$\Rightarrow \quad 5x^2+25x-6x-30=0$$

$$\Rightarrow \quad 5x(x+5)-6(x+5)=0$$

$$\Rightarrow \quad (x+5)(5x-6)=0$$

$$\Rightarrow \quad x+5=0 \text{ or } 5x-6=0$$

$$\Rightarrow \quad x=-5 \text{ or } x=\frac{6}{5}$$

Thus, solutions of given equation are $x=-5$ and $x=\frac{6}{5}$

19. Two different dice are thrown together. Find the probability of:

(*i*) getting a number greater than 3 on each die.

(*ii*) getting a total of 6 or 7 of the numbers on two dice.

Sol. (*i*) When two dice are thrown together total possible outcomes $= 6 \times 6 = 36$

Favourable outcomes when both dice have number more than 3 are (4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6), i.e. 9 outcomes.

$$\text{P(a number greater than 3 on each due)} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{9}{36} = \frac{1}{4}$$

(*ii*) Favourable outcomes when sum of the numbers appearing on the dice is 6 or 7 are, i.e. (1, 5), (1, 6), (2, 4), (2, 5), (3, 3), (3, 4), (4, 2), (4, 3), (5, 1), (5, 2), (6, 1), outcomes.

$$\text{P(a total of 6 or 7)} = \frac{11}{36}$$

20. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm^2. Find the volume of the cone. (use $\pi = 3.14$)

Sol. Radius of cone $(r) = 3$ cm

Curved surface area $= \pi rl = 47.1$ cm^2

$$\therefore \quad l = \frac{47.1}{\pi r} = \frac{47.1}{3.14 \times 3} = 5 \text{ cm, where } l = \text{slant height of cone}$$

We know that, $h = \sqrt{l^2 - r^2}$

$= \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4 = 3.14 \times 3 \times 4 = 37.68 \text{ cm}^3$

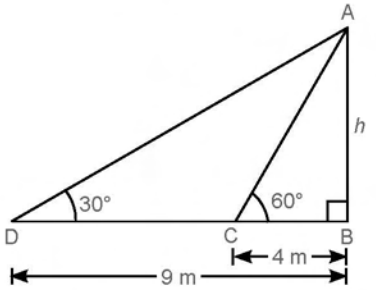
28. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are 60° and 30° respectively. Find the height of the tower.

Sol. Let AB be the tower of height 'h'.

In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \frac{h}{4} = \sqrt{3}$

$\Rightarrow h = 4\sqrt{3}$

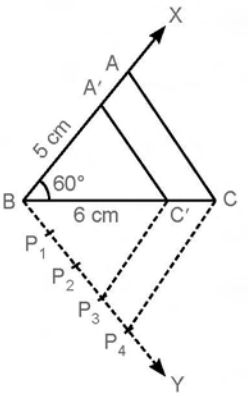


29. Construct a triangle ABC in which BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

Sol. **Steps of construction:**

1. Draw ΔABC with side BC = 6 cm, AB = 5 cm, $\angle ABC = 60°$.
2. Draw ray BX making an acute angle with BC on opposite side of vertex A.
3. Locate 4 points P_1, P_2, P_3, P_4 on line segment BY.
4. Join P_4C and draw a line through P_3, parallel to P_4C intersecting BC at C′.
5. Draw a line through C′ parallel to AC intersecting AB at A′.

$\Delta A'BC'$ is the required triangle.



30. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

Sol. Here, Perimeter of triangle, $a + b + c = 60$ [Given]

$a + b + 25 = 60$

$a + b = 35$ cm

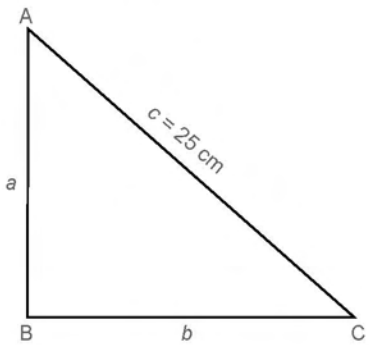
Using Pythagoras theorem,

$a^2 + b^2 = (25)^2 = 625$

Now, $(a + b)^2 = a^2 + b^2 + 2ab$

$(35)^2 = 625 + 2ab$

$1225 - 625 = 2ab$



$$\Rightarrow \quad 2ab = 600$$

$$\Rightarrow \quad ab = 300$$

$$\text{Area } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 300 = 150 \text{ cm}^2$$

SET-III [UNCOMMON QUESTIONS TO SET-I & II]

10. How many terms of the A.P. 65, 60, 55, ... be taken so that their sum is zero?

Sol. In the given A.P.,

$$\text{First term } (a) = 65$$

$$\text{Common difference } (d) = 60 - 65 = -5$$

$$\text{Sum of } n \text{ terms } (S_n) = 0$$

Therefore, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$0 = \frac{n}{2}[2 \times 65 + (n-1)(-5)]$$

$$0 = 130 - 5n + 5$$

$$\Rightarrow \quad -5n = -135 \quad \Rightarrow \quad n = 27$$

$\therefore$ Hence, sum of 27 terms is zero.

18. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but 'Kewal' another shopkeeper will not buy shirts with major defects. A shirt is taken out of the box at random. What is the probability that

(*i*) Ramesh will buy the selected shirt?

(*ii*) 'Kewal' will buy the selected shirt?

Sol. (*i*) When one shirt is taken out, then number of total possible outcomes = 100

Ramesh will purchase when shirt is good,

$\therefore$ Favourable outcomes = number of good shirts = 88

$$\text{P(Ramesh buys shirt)} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{88}{100} = \frac{22}{25}$$

(*ii*) Kewal will buy shirt if a shirt is not having major defect.

Number of favourable outcomes = Number of shirts without major defect = 96

$$\text{P(Kewal buys a shirt)} = \frac{96}{100} = \frac{24}{25}$$

19. Solve the following quadratic equation for *x*:

$$\boldsymbol{x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0}$$

Sol.

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

$$\Rightarrow \quad x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+\left(\frac{a}{a+b}\right)\left(\frac{a+b}{a}\right)=0$$

$$\Rightarrow \quad x^2+\left(\frac{a}{a+b}\right)x+\left(\frac{a+b}{a}\right)x+\left(\frac{a}{a+b}\right)\cdot\left(\frac{a+b}{a}\right)=0$$

$$\Rightarrow \quad x\left[x+\frac{a}{a+b}\right]+\left(\frac{a+b}{a}\right)\left[x+\frac{a}{a+b}\right]=0$$

$$\Rightarrow \quad \left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right)=0$$

$$\Rightarrow \quad x+\frac{a}{a+b}=0 \quad \text{or} \quad x+\frac{a+b}{a}=0$$

$$\Rightarrow \quad x=-\left(\frac{a}{a+b}\right) \text{ or } x=-\left(\frac{a+b}{a}\right)$$

20. A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy. $\left(\text{use } \pi = \frac{22}{7}\right)$

Sol. Here, given that $h = 15.5 - 3.5 = 12$ cm

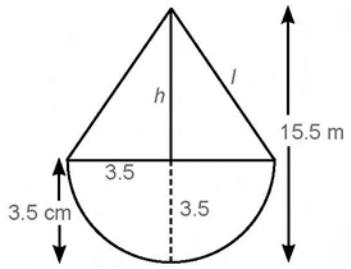
Also, slant height of cone, $l = \sqrt{h^2+r^2}$

$= \sqrt{(12)^2+(3.5)^2}$ cm

$= \sqrt{144+12.25}$ cm

$= \sqrt{156.25}$ cm

$= 12.5$ cm



$\therefore$ Curved Surface Area of cone $= \pi rl = \frac{22}{7} \times 3.5 \times 12.5$

$= 137.5 \text{ cm}^2$

$\therefore$ Surface area of hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times (3.5)^2 = 77 \text{ cm}^2$

Hence, Total Surface Area of toy = Surface area of hemisphere + Curved Surface Area of cone

$= 77 + 137.5 = 214.5 \text{ cm}^2$

28. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers.

Sol. Let the three numbers in A.P. are $a - d, a, a + d$

Then $a - d + a + a + d = 12$ [$\because$ Given that, $S_3 = 12$]

$\Rightarrow \quad 3a = 12 \Rightarrow a = 4$

Also, $(a-d)^3 + a^3 + (a+d)^3 = 288$ [$\because$ Sum of their cubes = 288]

$\Rightarrow \quad (4-d)^3 + (4)^3 + (4+d)^3 = 288$

$\Rightarrow 64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

$\Rightarrow \quad 24d^2 + 192 = 288 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

For $d = 2$, the numbers will be 2, 4, 6. For $d = -2$, numbers will be 6, 4, 2.

Hence, required numbers are 2, 4, 6.

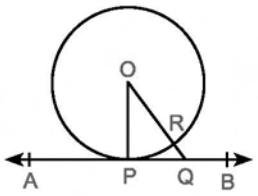
29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Sol. **Given:** A circle C(O, r) and a tangent AB at a point P.

To prove: OP $\perp$ AB

Construction: Take any point Q other than P on the tangent AB.

Join OQ, intersecting circle at R.



Proof: We have, $OP = OR$ [Radii]

$OQ = OR + RQ$

$\therefore$ $OQ > OR \Rightarrow OQ > OP$ [$\because$ OR = OP = radius]

Thus, OP < OQ, i.e. OP is shorter than any other segment joining O to any point of AB.

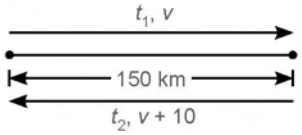
But among all line segments, joining point O to point on AB, shortest one is perpendicular from O on AB.

Hence, $OP \perp AB$

30. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

Sol. Let t_1 and t_2 be the time taken in going and returning respectively.

Also, v and $v + 10$ be the speed in going and returning respectively.



As, $\text{Velocity} = \dfrac{\text{Distance}}{\text{Time}}$

$\Rightarrow$ $\text{Time} = \dfrac{\text{Distance}}{\text{Velocity}}$

$\therefore$ $t_1 = \dfrac{150}{v}$ and $t_2 = \dfrac{150}{v+10}$

According to question, $t_1 - t_2 = 2\dfrac{1}{2}$

$$\frac{150}{v} - \frac{150}{v+10} = \frac{5}{2} \Rightarrow 150\left[\frac{1}{v} - \frac{1}{v+10}\right] = \frac{5}{2}$$

$$\frac{1}{v} - \frac{1}{v+10} = \frac{1}{60}$$

$$60[v + 10 - v] = v(v + 10)$$

$$v^2 + 10v - 600 = 0$$

$$v^2 + 30v - 20v - 600 = 0$$

$$v(v + 30) - 20(v + 30) = 0$$

$$v = 20 \quad \text{or} \quad v = -30 \text{ (rejected)}$$

Hence, velocity in going 20 km/hour and in returning 30 km/hour.

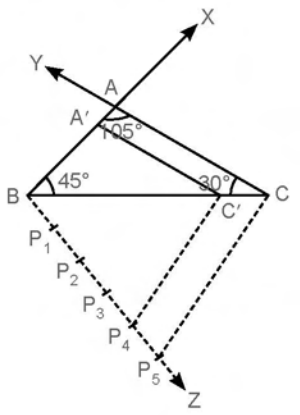
31. **Draw a triangle ABC with BC = 7 cm, $\angle B = 45°$ and $\angle A = 105°$. Then construct a triangle whose sides are $\frac{4}{5}$ times the corresponding sides of ΔABC.**

Sol. Given, $\angle B = 45°, \angle A = 105°$

$\therefore$ Sum of all interior angles in $\Delta = 180°$

$\therefore$ $\angle A + \angle B + \angle C = 180°$

$\angle C = 30°$



Steps of construction:

1. Draw ΔABC with side BC = 7 cm, $\angle B = 45°$, $\angle C = 30°$.
2. Draw a ray BX making an acute angle with BC on opposite side of vertex A.
3. Locate 5 points P_1, P_2, P_3, P_4, P_5 on BZ.
4. Join P_5C. Draw line through P_4 parallel to P_5C intersecting BC at C′.
5. Through C′, draw line parallel to AC intersecting AB at A′.

$\Delta A'BC'$ is the required triangle.

Examination Papers, 2016
[All India Set-I, II, III]

General Instructions:

(i) All questions are compulsory.

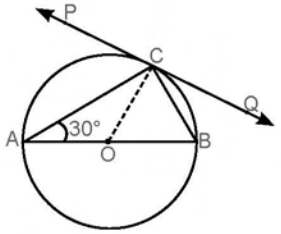
*(ii) The question paper consists of **31** questions divided into four sections – A, B, C and D.*

*(iii) Section A contains **4** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **11** questions of **4** marks each.*

(iv) Use of calculators is not permitted.

SET-I

1. In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$.



Sol. **Construction:** Join AO.

Given: PQ is tangent. AB is diameter $\angle CAB = 30°$.

To Find: $\angle PCA$

Solution: In ΔAOC, $AO = CO$ ($\because$ Equal radii)

$\angle CAO = \angle OCA$ ($\because$ Angles opposite to equal sides are equal)

or $\angle CAB = \angle OCA$

But, $\angle CAB = 30°$ So, $\angle OCA = 30°$ *(i)*

Since, $OC \perp PQ$ ($\because$ Tangent is perpendicular to radius at point of contact)

$\Rightarrow$ $\angle PCO = 90° \Rightarrow \angle OCA + \angle PCA = 90° \Rightarrow 30° + \angle PCA = 90°$

$\therefore$ $\angle PCA = 60°$

2. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P.?

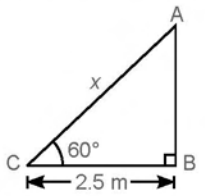
Sol. Given that $k + 9$, $2k - 1$ and $2k + 7$ are in A.P.

Then $(2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$

$\Rightarrow$ $k - 10 = 8 \Rightarrow k = 18$

3. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Sol. Let AC be the ladder of length x.



In ΔABC, $\frac{BC}{x} = \cos 60°$

$\Rightarrow$ $\frac{2.5}{x} = \frac{1}{2} \Rightarrow x = 2 \times 2.5 = 5$ m

Thus, length of the ladder is 5 m.

4. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

Sol. Number of toal possible outcomes when one card is drawn = 52

Number of favourable outcomes when card is neither red nor queen = 28

$$\text{Required probability} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{28}{52} = \frac{7}{13}$$

5. If – 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Sol. $\because$ – 5 is the root of the quadratic equation $2x^2 + px - 15 = 0$

$\Rightarrow$ $2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow$ $50 - 5p - 15 = 0 \Rightarrow 35 - 5p = 0$

$\Rightarrow$ $5p = 35 \Rightarrow p = 7$

Now, given that equation $p(x^2 + x) + k = 0$ has equal roots

i.e. $7(x^2 + x) + k = 0$ has equal roots

i.e. $7x^2 + 7x + k = 0$ has equal roots

$\Rightarrow$ $7^2 - 4 \times 7 \times k = 0$ [$\because$ For equal roots, D = 0, i.e. $b^2 - 4ac = 0$]

$\Rightarrow$ $7(7 - 4k) = 0$

$\Rightarrow$ $k = \frac{7}{4}$

6. Let P and Q be the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4) such that P is nearer to A. Find the coordinates of P and Q.

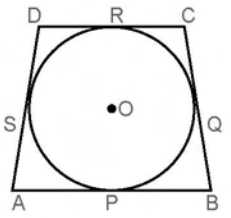
Sol. Let A(2, – 2), B(– 7, 4) be given points. Let P(x, y), Q(x', y') are point of trisection.

P divides AB in the ratio 1 : 2

Coordinates of P are $\left(\frac{2\times2+1(-7)}{1+2}, \frac{(-2)(2)+1(4)}{1+2}\right)$ or (–1, 0)

Q is mid point of PB. So using mid point formula coordinates of Q are $\left(\frac{-1-7}{2}, \frac{0+4}{2}\right)$ or (–4, 2)

7. In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.



Sol. We know that tangents drawn to a circle from an outer points are equal.

So, $AP = AS,\ BP = BQ,\ CR = CQ$ and $DR = DS$.

Now, consider

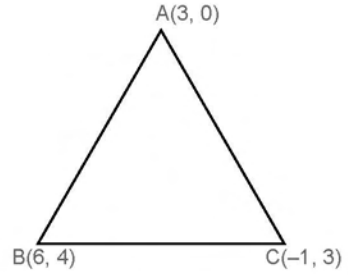
$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow \quad AB + CD = AD + BC$$

Hence proved.

8. Prove that the points (3, 0), (6, 4) and (–1, 3) are the vertices of a right angled isosceles triangle.

Sol. Let the triangle be ΔABC as shown in figure. Distances are:



Using distance formula,

$AB = \sqrt{(3-6)^2+(0-4)^2} = 5$

$BC = \sqrt{(6+1)^2+(4-3)^2} = 5\sqrt{2}$

$CA = \sqrt{(-1-3)^2+(3-0)^2} = 5$

Here, AB = AC $\Rightarrow$ ΔABC is isosceles triangle

Consider, $AB^2 + AC^2 = (5)^2 + (5)^2 = 25 + 25 = 50$

$\Rightarrow$ and, $BC^2 = \left(5\sqrt{2}\right)^2 = 50$

$\therefore$ Here, $AB^2 + AC^2 = BC^2$

$\Rightarrow$ ΔABC is a right angled triangle.

[$\because$ In right Δ, using Pythagoras theorem $(H)^2 = (P)^2 + (B)^2$]

where H = hypotenuse, B = base, P = perpendiculars

9. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

Sol. Let a be first term and d be the common difference of the A.P. Then

$$a_n = a + (n-1)d$$

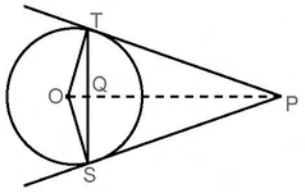
$$a_4 = a + (4-1)d$$

$0 = a + 3d \Rightarrow a = -3d$ [$\because$ Given, $a_4 = 0$]

Now $a_{25} = a + (25-1)d = a + 24d = -3d + 24d = 21d = 3 \times 7d$

Hence, $a_{25} = 3 \times a_{11}$ [$\because$ Since $a_{11} = a + (11-1)d = -3d + 10d = 7d$]

10. In given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If PO = $2r$, show that $\angle OTS = \angle OST = 30°$.



Sol. Let $\angle TOP = \theta$

In right triangle OTP we have

$\therefore \quad \cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} = \cos 60° \Rightarrow \theta = 60°$

Hence $\angle TOS = 2 \times 60 = 120°$ [$\because \angle TOP = \angle POS$ as angles opposite to equal tangent are equal]

In ΔOTS, we have $OT = OS$ [$\because$ Equal radii]

$\Rightarrow \quad \angle OTS = \angle OST$ [$\because$ Angle opposite to equal sides are equal]

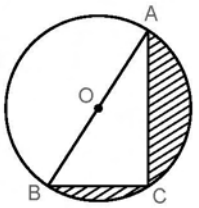
In ΔOTS,

$\angle OTS + \angle OST + \angle TOS = 180°$

$2\angle OST = 60°$

$\therefore \quad \angle OST = \angle OTS = 30°$ Hence proved.

11. In figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take $\pi = 3.14$)



Sol. Here, $BC^2 = AB^2 - AC^2 = 169 - 144 = 25$

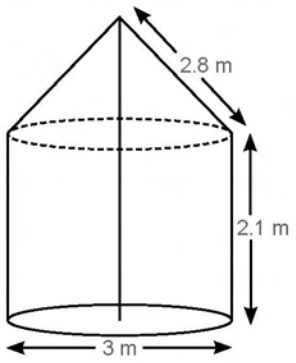
$\therefore \quad BC = 5$

Area of shaded region = Area of semicircle – Area of right triangle ABC

$= \frac{1}{2} \times \pi r^2 - \frac{1}{2} AC \times BC$

$= \frac{1}{2} \times 3.14 \left(\frac{13}{2}\right)^2 - \frac{1}{2} \times 12 \times 5 = 66.33 - 30 = 36.33 \text{ cm}^2$

12. In figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹ 500/sq. metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Area canvas needed = curved surface area of cylinder + curved surface area of cone

$$= 2\pi rh + \pi rl \quad [\text{where } r = \text{radius}, h = \text{height}, l = \text{slant height}]$$

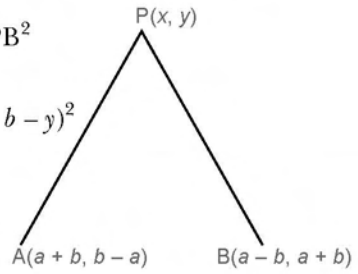
$$= 2 \times \frac{22}{7} \times 1.5 \times 2.1 + \frac{22}{7} \times 1.5 \times 2.8$$

$$= \frac{22}{7}[6.3 + 4.2] = \frac{22}{7} \times 10.5 = 33 \text{ m}^2$$

Cost of canvas = $33 \times 500 = ₹ 16500$

13. If the point P(x, y) is equidistant from the points A($a + b$, $b - a$) and B($a - b$, $a + b$). Prove that $bx = ay$.

Sol. Given, $PA = PB \Rightarrow PA^2 = PB^2$



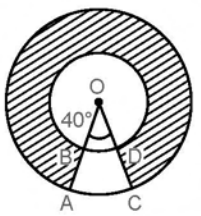
Applying distance formula,

$$\Rightarrow (a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$$

$$\Rightarrow (a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay$$
$$= (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$$

$\Rightarrow 4ay = 4bx \Rightarrow ay = bx$ or $bx = ay$ Hence proved.

14. In figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Area of shaded region $= \frac{360° - \theta}{360°} \times \pi(R^2 - r^2)$

$$= \frac{320°}{360°} \times \pi[(14)^2 - (7)^2]$$

$$= \frac{8}{9} \times \frac{22}{7}(196 - 49) = \frac{8}{9} \times \frac{22}{7} \times 147$$

$$= \frac{1232}{3} = 410.67 \text{ cm}^2$$

15. If the ratio of the sum of first n terms of two A.P.'s is $(7n + 1) : (4n + 27)$, find the ratio of their mth terms.

Sol. Let S_n and S'_n be the sum of n terms of two A.P.'s. Let a, a' and d, d' be first terms and common difference of two A.P.'s. Then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']}$$

$$= \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27} \qquad \ldots(i)$$

Since $\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)a'}$ [$\because$ Let t_m, t_m be m^{th} terms of two A.P.'s]

So replacing $\frac{n-1}{2}$ by $m-1$, i.e. $n = 2m-1$ in (i)

$$\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)a'} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Thus, the ratio of their m^{th} terms is $14m - 6 : 8m + 23$.

16. Solve for x: $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$, $x \neq 1, 2, 3$

Sol. $$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$$

$$\Rightarrow \quad \frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3} \quad \Rightarrow \quad \frac{(2x-4)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow \quad 3 = (x-1)(x-3) \quad \Rightarrow \quad x^2 - 4x + 3 = 3$$

$$\Rightarrow \quad x^2 - 4x = 0 \quad \Rightarrow \quad x(x-4) = 0 \quad \Rightarrow \quad x = 0 \text{ or } x = 4$$

17. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Volume of water in conical vessel

$$= \frac{1}{3}\pi r^2 h \qquad \text{[where } r = \text{radius, } h = \text{height]}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24 \text{ cm}^3$$

Let height of water in cylindrical vessel be h then volume of water in cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 10 \times 10 \times h \text{ cm}^3$$

A.T.Q.

Volume of water in conical vessel = Volume of water in cylindrical vessel

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24 = \frac{22}{7} \times 10 \times 10 \times h$$

$$\Rightarrow \quad h = \frac{5 \times 5 \times 24}{3 \times 10 \times 10} = 2 \text{ cm}$$

$\therefore$ Height to which water rises in cylindrical vessel = 2 cm

18. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylidrical vessel.

Sol. We know that, Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \times (6)^3 = \frac{4}{3}\pi \times 216 \text{ cm}^3$$

Let radius of cylindrical vessel be r cm then, volume of cylinder is:

$$V = \pi r^2 h = \pi r^2 \times \frac{32}{9} \text{ cm}^3$$

According to question,

$$\text{volume of sphere} = \text{volume of cylinder}$$

$$\frac{4}{3}\pi \times 216 = \pi \times r^2 \times \frac{32}{9}$$

$$r^2 = \frac{4 \times 216 \times 9}{3 \times 32} = 27 \times 3 = 81$$

$$r^2 = 81$$

$$r = 9 \text{ cm}$$

$\therefore$ Diameter of cylindrical vessel $= 2r = 18$ cm

19. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Sol. Let AB be the water level, DA be the height of ship = 10 m.

Let BC be the hill of height h from water level.

Let AB $= x$

In ΔDEB,

$$\frac{BE}{DE} = \tan 30° \Rightarrow \frac{10}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = 10\sqrt{3} \text{ m} \qquad \ldots(i)$$

Now, in ΔCED, $\frac{CE}{x} = \tan 60°$

$$\Rightarrow \quad \frac{h-10}{10\sqrt{3}} = \sqrt{3} \qquad \text{[From } (i)\text{]}$$

$$\Rightarrow \quad h - 10 = 30$$

$$\Rightarrow \quad h = 40 \text{ m}$$

So, distance of hill from ship $= 10\sqrt{3}$ m and the height of the hill = 40 m.

20. Three different coins are tossed together. Find the probability of getting

(*i*) exactly two heads (*ii*) at least two heads (*iii*) at least two tails.

Sol. Possible outcomes when three coins are tossed

HHH, HHT, HTT, TTT, THH, TTH, HTH, THT

(*i*) Number of exactly two heads are HHT, HTH and THH.

P(exactly two heads) $= \frac{3}{8}$

(*ii*) In case of at least two heads, outcomes are HHT, HTH, THH and HHH.

$$\text{P(at least two heads)} = \frac{4}{8} = \frac{1}{2}$$

(*iii*) In case of at least two tails, outcomes are TTH, THT, HTT and TTT.

$$\text{P(at least two tails)} = \frac{4}{8} = \frac{1}{2}$$

21. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq.m, find the amount shared by each school to set up the tents. What value is generated by the above problem?

$\left(\textbf{Use } \pi = \frac{22}{7}\right)$

Sol. Slant height of conical part $= \sqrt{r^2 + h^2} = \sqrt{(2.8)^2 + (2.1)^2} = \sqrt{7.84 + 4.41}$

$= \sqrt{12.25} = 3.5$ m

Area of tents = Curved surface area of cylindrical part + Curved surface area of conical part

$= 2\pi rh + \pi rl$

$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5$

$= 3 \times \frac{22}{7} \times 2.8 \times 3.5 = 92.4 \text{ m}^2$

$\therefore$ Canvas required for 1500 tents $= 1500 \times 92.4 = 138600 \text{ m}^2$

Cost of 1500 tents $= (1500 \times 92.4) \times 120 =$ ₹ 16632000

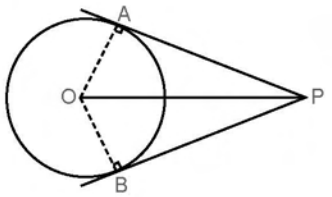
[$\because$ Making tents costs ₹ 120 per sq. m]

Share of each school $= \frac{1663200}{50} =$ ₹ 332640

School authorities are concerned about safety of children and their families.

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. **Given:** A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.



To Prove: PA = PB

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.

$\angle OAP = \angle OBP = 90°$ ...(*i*)

[Radius is perpendicular to the tangent at the point of contact]

$OA = OB$ (radii) ...(*ii*)

OP is common ...(*iii*)

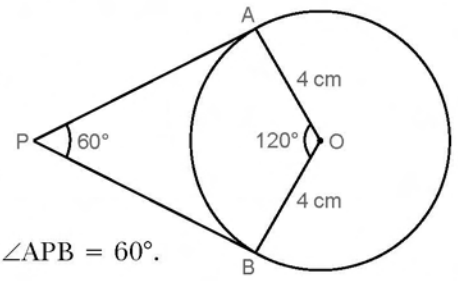
$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*) and (*iii*)]

Hence, $AP = BP$ (CPCT)

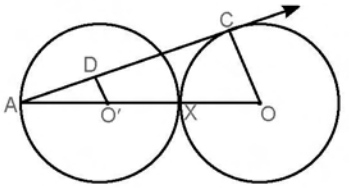
23. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Sol. Steps of construction:

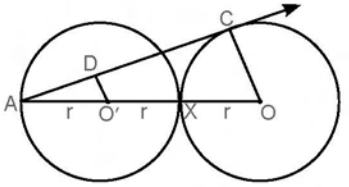
1. Draw a circle of radius 4 cm with centre O.
2. Take point A on circle. Join OA.
3. Draw line AP perpendicular to radius OA.
4. Draw $\angle AOB = 120°$ at O.
5. Join A and B at P, to get 2 tangents. Here $\angle APB = 60°$.



24. In given figure, two equal circles, with centres O and O′, touch each other at X. OO′ produced meets the circle with centre O′ at A. AC is tangent to the circle with centre O at the point C. O′D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Sol. AC is tangent to the circle with centre O.



In $\Delta ADO'$ and ΔACO, $\angle ADO' = \angle ACO$ (each 90°)

$\angle DAO = \angle CAO$ (common)

$\therefore$ By AA criterion, $\frac{AO'}{AO} = \frac{DO'}{CO}$ [$\because$ corresponding parts of similar triangle]

$AO = AO' + O'X + XO = r + r + r = 3r$

$\frac{DO'}{CO} = \frac{r}{3r}$ [$\therefore$ $AO = AO' + O'X + XO = 3AO$]

$\frac{DO'}{CO} = \frac{1}{3}$

25. Solve for x: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$, $x \neq -1, -2, -4$

Sol. $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4} \Rightarrow \frac{x+2+2x+2}{(x+1)(x+2)} = \frac{4}{x+4}$

$\Rightarrow$ $(3x + 4)(x + 4) = 4(x^2 + 3x + 2) \Rightarrow 3x^2 + 16x + 16 = 4x^2 + 12x + 8$

$\Rightarrow$ $x^2 - 4x - 8 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16+32}}{2} \Rightarrow x = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

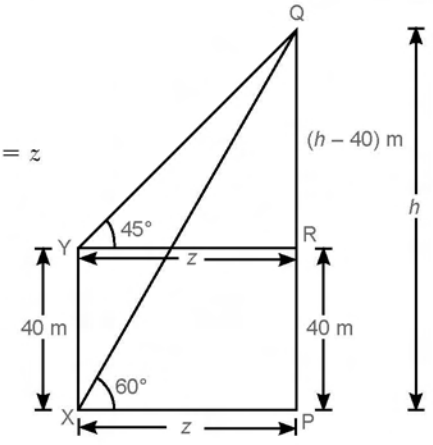
26. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3} = 1.73$)

Sol. Let height of PQ be h.

Let z be the distance between X and P.

$\because$ XPRY is a rectangle.

$\therefore$ RP = XY = 40 m and PX = YR = z



In ΔQPX, $\frac{PQ}{PX} = \tan 60° \Rightarrow \frac{h}{z} = \sqrt{3}$

$\Rightarrow \frac{h}{\sqrt{3}} = z$ ...(*i*)

In ΔQRY, $\frac{QR}{YR} = \tan 45°$

$\Rightarrow \frac{h-40}{z} = 1 \Rightarrow h - 40 = z$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\frac{h}{\sqrt{3}} = h - 40 \Rightarrow h = h\sqrt{3} - 40\sqrt{3} \Rightarrow h\sqrt{3} - h = 40\sqrt{3}$$

$$\Rightarrow h(\sqrt{3} - 1) = 40\sqrt{3}$$

$$\Rightarrow h = \frac{40\sqrt{3}}{(\sqrt{3}-1)} = \frac{40\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{40(3+\sqrt{3})}{2}$$

$\Rightarrow h = 20(3 + 1.73) = 20 \times 4.73 = 94.6$ m ...(*iii*)

So, height of the tower PQ = 94.6 m

and the distance PX = 94.6 – 40 = 54.6 m [From (*ii*) and (*iii*)]

27. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceding the house numbered X is equal to sum of the numbers of houses following X.

Sol. The A.P. of numbers of houses preceding house numbered x is: $1 + 2 + 3 + ... + (x - 1)$

$\therefore$ Sum, $S_n = \frac{n}{2}[2a + (n-1)d]$, where $a \rightarrow$ first term

$d \rightarrow$ common difference

$$= \frac{(x-1)}{2}[2 \times 1 + (x - 1 - 1) \times 1]$$

$$= \frac{(x-1)}{2} \times [2 + x - 2] = \frac{x(x-1)}{2}$$

Now, A.P. of total number of houses following x is: $(x + 1) + (x + 2) + ... + 49$

$$n = 49 - (x + 1) + 1 = 49 - x$$

$\therefore$ Sum of these numbers, $S_n = \frac{n}{2}[a + l]$, where l is last term

$$= \frac{(49 - x)}{2}[x + 1 + 49] = \frac{(49 - x)}{2}(x + 50)$$

According to question,

$$\frac{x(x-1)}{2} = \frac{(49 - x)(x + 50)}{2}$$

$\Rightarrow$ $x^2 - x = 49x + 2450 - x^2 - 50x$

$\Rightarrow$ $2x^2 = 2450$

$\Rightarrow$ $x^2 = 1225 \Rightarrow x = 35$

Justification:

Now, A.P. of numbers before house numbered $x = 1 + 2 + ... + 34$

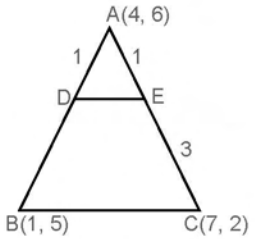
$\therefore$ $S_{34} = \frac{34}{2}[a + l] = \frac{34}{2} \times [1 + 34] = 17 \times 35 = 595$

Now, A.P. of numbers following house numbered $x = 36 + 37 ... + 49$

$\therefore$ $S' = \frac{14}{2}[36 + 49] = 7 \times 85 = 595$

Hence, for value of $x = 35$, the sum of numbers of houses preceding house numbered x is equal to sum of numbers of houses following x.

28. In fig., the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.



Sol. Given: $\frac{AD}{AB} = \frac{1}{3}$

$3AD = AB$

$\therefore$ $3AD = AD + DB$

$2AD = DB$

$\frac{AD}{DB} = \frac{1}{2}$

Similarly, $\frac{AE}{EC} = \frac{1}{2}$

Calculated using section formula

Coordinates of D are $\left(\frac{1(1) + 2(4)}{1 + 2}, \frac{1(5) + 2(6)}{1 + 2}\right)$ i.e. $\left(3, \frac{17}{3}\right)$

Coordinates of E are $\left(\frac{1(7) + 2(4)}{1 + 2}, \frac{1(2) + z(6)}{1 + 2}\right)$ i.e. $\left(5, \frac{14}{3}\right)$

$$\text{Area of } \Delta ADE = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\left[4\left(\frac{17}{3}-\frac{14}{3}\right)+3\left(\frac{14}{3}-6\right)+6\left(6-\frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4+3\left(\frac{-4}{3}\right)+5\left(\frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\left[4-4+\frac{5}{3}\right] = \frac{5}{6} \text{ sq. units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$$

$$= \frac{1}{2}[4 \times 3 + (-4) + 7 \times 1]$$

$$= \frac{1}{2}[12 - 4 + 7]$$

$$= \frac{1}{2} \times 15 = \frac{15}{2} \text{ sq. units}$$

Hence, $$\frac{\text{Area } (\Delta ADE)}{\text{Area } (\Delta ABC)} = \frac{5/6}{15/12} = \frac{5}{6} \div \frac{15}{12} = \frac{5}{6} \times \frac{12}{15} = \frac{2}{3}$$

29. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

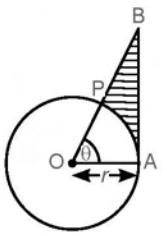
Sol. x can be any one of 1, 2, 3 or 4 and y can be any one of 1, 4, 9 or 16.

Total number of cases of xy = 16

Number of cases when product is less than 16 are 1×1, 1×4, 1×9, 2×1, 2×4, 3×1, 3×4, 4×1, i.e. 8 cases.

$$\text{Required probability} = \frac{\text{Number of favourable cases}}{\text{Total number of cases}} = \frac{8}{16} = \frac{1}{2}$$

30. In figure, is shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$.



Sol. $$\text{Length are } \overset{\frown}{AP} = \frac{\theta}{360} \times 2\pi r = \frac{\pi r\theta}{180}$$

Now $$\frac{AB}{r} = \tan\theta \Rightarrow AB = r\tan\theta$$

$$\frac{OB}{r} = \sec\theta \Rightarrow OB = r\sec\theta$$

$$PB = OB - r = r\sec\theta - r$$

Perimeter of shaded region = AB + PB + $\overset{\frown}{AP}$

$$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180}$$

$$= r\left[\tan\theta + \sec\theta - 1 + \frac{\pi\theta}{180}\right]$$

31. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let the speed of the stream be x km/h

Upstream case: Speed of boat = $(24 - x)$ km/h

Time taken for going 32 km upstream = $\frac{32}{24-x}$ hours

Downstream case: Speed of boat = $(24 + x)$ km/h

Time taken for going 32 km downstream = $\frac{32}{24+x}$ hours

According to question,

$$\frac{32}{24-x} - \frac{32}{24+x} = 1 \Rightarrow 32\left[\frac{24+x-24+x}{(24-x)(24+x)}\right] = 1 \qquad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Velocity}}\right]$$

$$\Rightarrow \quad 32\left(\frac{2x}{576-x^2}\right) = 1 \Rightarrow 64x = 576 - x^2$$

$$\Rightarrow \quad x^2 + 64x - 576 = 0 \Rightarrow (x + 72)(x - 8) = 0$$

$$\Rightarrow \quad x = 8, \quad x = -72$$

$$\Rightarrow \quad x = 8 \qquad \text{(as speed cannot be negative)}$$

Hence, speed of the stream is 8 km/hr.

SET-II [UNCOMMON QUESTIONS TO SET-I]

10. Solve for x: $\sqrt{2x+9} + x = 13$.

Sol. $\sqrt{2x+9} + x = 13$

$$\Rightarrow \quad \sqrt{2x+9} = (13 - x)$$

Squaring both sides, we get

$$\Rightarrow \quad 2x + 9 = (13 - x)^2$$

$$\Rightarrow \quad 2x + 9 = 169 - 26x + x^2$$

$$\Rightarrow \quad x^2 - 28x + 160 = 0$$

$$\Rightarrow \quad (x - 20)(x - 8) = 0$$

$$\Rightarrow \quad x = 20 \text{ or } x = 8$$

$$\Rightarrow \quad x = 8 \qquad [\text{as } x = 20 \text{ does not satisfy the equation}]$$

18. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

Sol. Let the required numbers in A.P. are $a - d$, a, $a + d$ respectively.

Now, $a - d + a + a + d = 15$ [$\because$ Sum of digits = 15]

$\Rightarrow$ $3a = 15 \Rightarrow a = 5$

According to question, number is

$100\,(a - d) + 10a + a + d$, i.e. $111a - 99d$

Number on reversing the digits is

$100\,(a + d) + 10a + a - d$, i.e. $111a + 99d$

Now, as per given condition in question,

$$(111a - 99d) - (111a + 99d) = 594$$
$$-198d = 594$$
$$d = -3$$

$\therefore$ Digits of number are $[5 - (-3), 5, (5 + (-3)] = 8, 5, 2$

$\therefore$ Required number is $111 \times (5) - 99\,(-3) = 555 + 297 = 852$

19. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

Sol. $(a - b)x^2 + (b - c)x + (c - a) = 0$

For equal roots, discriminant, D = 0

$\Rightarrow$ $(b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow$ $b^2 - 2bc + c^2 - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow$ $b^2 - 2bc + c^2 - 4ac + 4a^2 + 4bc - 4ab = 0$

$\Rightarrow$ $4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$

$\Rightarrow$ $(2a - b - c)^2 = 0$

$\Rightarrow$ $2a - b - c = 0$

$\Rightarrow$ $2a = b + c$

20. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is:

(*i*) a black King. (*ii*) a card of red colour. (*iii*) a card of black colour.

Sol. Removed red colour cards = $3 \times 2 = 6$

Remaining cards = $52 - 6 = 46$

(*i*) Number of black kings = 2

P(a black king) = $\frac{2}{46} = \frac{1}{23}$

(*ii*) Number of red colour cards = 26

Remaining red colour cards = 26 – 6 = 20

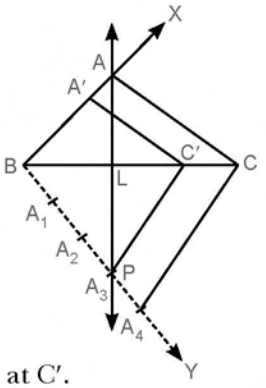
P(a card of red colour) = $\frac{20}{46} = \frac{10}{23}$

(*iii*) Number of black cards = 26

P(a black colour card) = $\frac{26}{46} = \frac{13}{23}$

28. Draw an isosceles ΔABC in which BC = 5.5 cm and altitude AL = 3 cm. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Sol. Steps of construction:



1. Draw BC = 5.5 cm.
2. Construct AP the perpendicular bisector of BC meeting BC at L.
3. Along LP cut off LA = 3 cm.
4. Join BA and CA. Then ΔABC so obtained is the required ΔABC.
5. Draw an acute angle CBY and cut 4 equal lengths as $BA_1 = A_1A_2 = A_2A_3 = A_3A_4$ and join CA_4.
6. Now draw a line through A_3 parallel to CA_4 intersecting BC at C′.
7. Draw a line through C′ and parallel to AC intersecting AB at A′. BA′C′ is the required triangle.

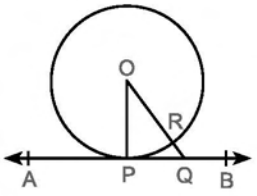
29. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact.

Sol. Given: A circle C(O, *r*) and a tangent AB at a point P.

To prove: OP ⊥ AB

Construction: Take any point Q other than P on the tangent AB.

Join OQ, intersecting circle at R.



Proof: We have, OP = OR [Radii]

$OQ = OR + RQ$

$\therefore \quad OQ > OR \Rightarrow OQ > OP$ [$\because$ OR = OP = radius]

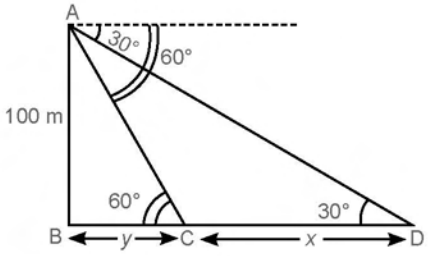
Thus, OP < OQ, i.e. OP is shorter than any other segment joining O to any point of AB. But among all line segments, joining point O to point on AB, shortest one is perpendicular from O on AB.

Hence, OP ⊥ AB

30. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it , changes from 30° to 60°. Find the distance travelled by the ship during the period of observation.

(Use $\sqrt{3} = 1.73$)

Sol. Let AB be the tower of height 100 m.



Let BC = y and CD = x.

In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \frac{100}{y} = \sqrt{3} \Rightarrow y = \frac{100}{\sqrt{3}}$ ...(*i*)

In ΔABD, $\frac{AB}{BD} = \tan 30° \Rightarrow \frac{100}{y+x} = \frac{1}{\sqrt{3}}$

$\Rightarrow x + y = 100\sqrt{3} \Rightarrow x = 100\sqrt{3} - y$

$\Rightarrow x = 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{300-100}{\sqrt{3}} = \frac{200}{\sqrt{3}}$ ...[(From (*i*)]

$\Rightarrow x = \frac{200\sqrt{3}}{3} = \frac{200 \times 1.73}{3} = 115.33\text{m}$

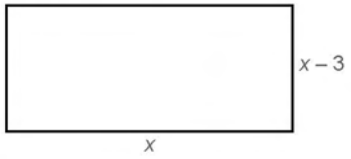
The distance travelled by the ship is 115.33 m.

31. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

Sol. Let length of rectangular park be x m and breadth be $(x - 3)$ m

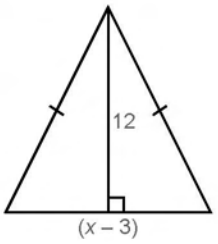
Base of isosceles $\Delta = (x - 3)$ m

Altitude of $\Delta = 12$ m



According to question,

Area of rectangular park = Area of Δ + 4



$\Rightarrow x(x-3) = \frac{1}{2}(x-3) \times 12 + 4$

$\Rightarrow x(x-3) = 6(x-3) + 4$

$\Rightarrow x^2 - 3x = 6x - 18 + 4$

$\Rightarrow x^2 - 9x + 14 = 0$

$\Rightarrow (x-7)(x-2) = 0$

$\Rightarrow x = 7$ or $x = 2$

But $x = 2$ is rejected otherwise breadth will be –ve which is not possible.

$\therefore$ Length of rectangular park is 7 m and breadth is 4 m.

SET-III [UNCOMMON QUESTIONS TO SET-I & II]

10. Solve for x: $\sqrt{6x + 7} - (2x - 7) = 0$.

Sol. $\sqrt{6x+7} - (2x - 7) = 0$

$$\Rightarrow \quad \sqrt{6x+7} = 2x - 7$$

Squaring both sides, we get

$$\Rightarrow \quad 6x + 7 = (2x - 7)^2$$

$$\Rightarrow \quad 6x + 7 = 4x^2 - 28x + 49$$

$$\Rightarrow \quad 4x^2 - 34x + 42 = 0$$

$$\Rightarrow \quad 2x^2 - 17x + 21 = 0$$

$$\Rightarrow \quad 2x^2 - 14x - 3x + 21 = 0$$

$$\Rightarrow \quad 2x(x - 7) - 3(x - 7) = 0$$

$$\Rightarrow \quad (2x - 3)(x - 7) = 0$$

$$\Rightarrow \quad x = 7 \text{ or } x = \frac{3}{2}$$

$$\Rightarrow \quad x = 7 \qquad [\text{as } x = \frac{3}{2} \text{ does not satisfy the equation}]$$

18. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card:

(*i*) is divisible by 9 and is a perfect square.

(*ii*) is a prime number greater than 80.

Sol. Total possible cases = 100

(*i*) Favourable cases when number is a perfect square and is divisible by 9 are 9, 36 and 81.

So, number of favourable cases = 3

$$\text{Required probability} = \frac{\text{Number of favourable cases}}{\text{Total possible cases}} = \frac{3}{100}$$

(*ii*) Favourable cases the prime numbers greater than 80 are 83, 89 and 97

So, number of favourable cases = 3

$$\text{Required probability} = \frac{\text{Number of favourable cases}}{\text{Total possible cases}} = \frac{3}{100}$$

19. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

Sol. Let three consecutive natural numbers are $x - 1$, x and $x + 1$.

According to question,

$$x^2 - [(x + 1)^2 - (x - 1)^2] = 60$$

$$\Rightarrow x^2 - [(x + 1 - x + 1)(x + 1 + x - 1)] = 60$$
$$\Rightarrow x^2 - 4x - 60 = 0$$
$$\Rightarrow x^2 - 10x + 6x - 60 = 0$$
$$\Rightarrow x(x - 10) + 6(x - 10) = 0$$
$$\Rightarrow (x + 6)(x - 10) = 0$$
$$\Rightarrow x = 10 \qquad (x = -6, \text{ rejected})$$

Hence, the numbers are 9, 10 and 11.

20. The sums of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and their common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Sol. Here, sum of n terms of AP is $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_1 = \frac{n}{2}[2 + (n-1)1] = \frac{n(n+1)}{2}$ [$\because$ where $a = 1, d = 1$]

$S_2 = \frac{n}{2}[2 + (n-1)2] = \frac{n}{2}(2n) = n^2$ [$\because$ where $a = 1, d = 2$]

$S_3 = \frac{n}{2}[2 + (n-1)3]$

$= \frac{n}{2}[2 + 3n - 3] = \frac{n}{2}[3n - 1]$

Now, consider $S_1 + S_3 = \frac{n^2 + n + 3n^2 - n}{2}$

$= \frac{4n^2}{2} = 2n^2 = 2S_2$

28. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Sol. Let one pipe takes x minutes to fill the tank.

Then, another pipe takes $x + 5$ to fill the tank.

According to question,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100} \qquad \left(\because 11\frac{1}{9} = \frac{100}{9}\right)$$
$$100[x + 5 + x] = 9[x^2 + 5x]$$
$$200x + 500 = 9x^2 + 45x$$
$$9x^2 - 155x - 500 = 0$$
$$9x^2 - 180x + 25x - 500 = 0$$
$$9x(x - 20) + 25(x - 20) = 0$$
$$(9x + 25)(x - 20) = 0$$

$$x = 20 \quad \text{or} \quad x = -\frac{25}{9} \text{ (Rejected)}$$

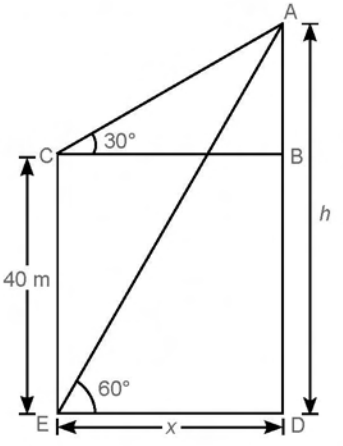
Hence, one pipe takes 20 minutes and another takes 25 minutes to fill the tank.

29. From a point on the ground, the angle of elevation of the top of a tower is observed to be 60°. From a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 30°. Find the height of the tower and its horizontal distance from the point of observation.

Sol. Let h be the height of the tower and x be the horizontal distance from the point of observation.

$\because$ BDEC is a rectangle,

$\therefore$ CB = ED = x and CE = BD = 40 m



In ΔABC, $\tan 30° = \frac{AB}{BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{x} \Rightarrow x = AB\sqrt{3} \qquad ...(i)$$

Now, in ΔAED, $\tan 60° = \frac{AD}{DE}$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{DE} \Rightarrow x = \frac{h}{\sqrt{3}} \qquad ...(ii)$$

From equation (i) and (ii), we get

$$AB\sqrt{3} = \frac{h}{\sqrt{3}} \quad [\because AB + 40 = h \Rightarrow AB = h - 40]$$

$$\sqrt{3}(h - 40) = \frac{h}{\sqrt{3}}$$

$$3(h - 40) = h \Rightarrow 3h - 120 = h$$

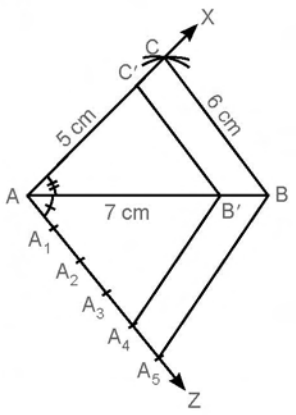
$$2h = 120 \Rightarrow h = 60 \text{ m}$$

From (ii), $x = \frac{h}{\sqrt{3}} \Rightarrow x = \frac{60}{\sqrt{3}} \Rightarrow x = \frac{60\sqrt{3}}{3}$

$$\Rightarrow \quad x = 20\sqrt{3} \Rightarrow x = 34.641 \text{ m}$$

30. Draw a triangle with sides 5 cm, 6 cm and 7 cm. Then draw another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of first triangle.

Sol. Steps of construction:



1. Draw a line segment AB of length 7 cm.

   Then using A as centre and distance 5 cm draw an arc C. Also draw an arc using B as centre and with distance 6 cm, which intersect earlier drawn arc at C. Join AC and BC.
2. Draw an acute angle BAZ and cut AZ as $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$ and join BA_5.
3. Through A_4 draw a line parallel BA_5 intersecting AB at B′.
4. Through B′ draw a line parallel to BC intersecting AC at C′.

$\Delta AB'C'$ is the required triangle.

31. **A number x is selected at random from the numbers 1, 4, 9, 16 and another number y is selected at random from the numbers 1, 2, 3, 4. Find the probability that the value of xy is more than 16.**

Sol. x can be 1, 4, 9 or 16 and y can be 1, 2, 3 or 4.

Total number of cases of xy are 16.

Number of cases when xy is more than 16 are (9×2), (9×3), (9×4), (16×2), (16×3), (16×4), i.e. 6 cases.

$$\text{P(value of } xy \text{ more than 16)} = \frac{6}{16} = \frac{3}{8}$$

Examination Papers, 2016
[Foreign Set-I, II, III]

General Instructions:

(i) All questions are compulsory.

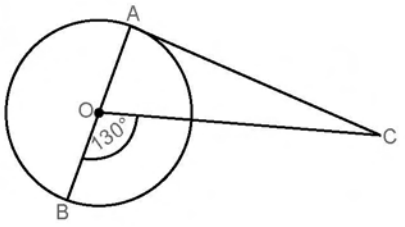
*(ii) The question paper consists of **31** questions divided into four sections – A, B, C and D.*

*(iii) Section A contains **4** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **11** questions of **4** marks each.*

(iv) Use of calculators is not permitted.

SET-I

1. In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠BOC = 130°, then find ∠ACO.



Sol. $\angle AOC + \angle BOC = 180°$

[$\because$ Linear Pair Axiom]

$\angle AOC + 130° = 180°$

$\angle AOC = 180° - 130°$

$\angle AOC = 50°$

Now, $\angle OAC = 90°$ [angle between radius OA and tangent AC is 90°]

Now, in ΔAOC,

$\angle OAC + \angle AOC + \angle ACO = 180°$ [$\because$ sum of angles in triangle is 180°]

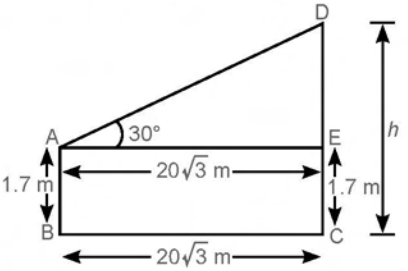
$90° + 50° + \angle ACO = 180°$

$\angle ACO = 180° - 140°$

$\angle ACO = 40°$

2. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.

Sol. Let CD be the tower of height h.



In ΔDEA, $\frac{DE}{AE} = \tan 30°$

$\Rightarrow \frac{h - 1.7}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow h - 1.7 = 20$

$\Rightarrow h = 20 + 1.7 = 21.7$ m

So, height of tower is 21.7 m.

3. **For what value of k will the consecutive terms $2k + 1, 3k + 3$ and $5k - 1$ form an A.P.?**

Sol. Given that $2k + 1$, $3k + 3$ and $5k - 1$ are in A.P.

So, $(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)$

$\Rightarrow$ $k + 2 = 2k - 4$

$\Rightarrow$ $2k - k = 2 + 4 \Rightarrow k = 6$

4. **20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7.**

Sol. When one ticket is drawn, total possible cases are 20.

Favourable cases when the number is a multiple of 3 or 7 are 3, 6, 9, 12, 15, 18, 7, 14, i.e. 8 cases.

$$\text{Required probability} = \frac{\text{Number of favourable cases}}{\text{Total number of possible cases}} = \frac{8}{20} = \frac{2}{5}$$

5. **A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.**

Sol. Let ones digit of number $= x$

Let tens digit of number $= y$

$\therefore$ Number will be $= 10y + x$

According to question,

$\Rightarrow$ $10y + x = 4(x + y)$

$\Rightarrow$ $10y + x = 4x + 4y$

$3x - 6y = 0$

$x - 2y = 0 \Rightarrow x = 2y$ ...(*i*)

and $10y + x = 3xy$ ...(*ii*)

Putting $x = 2y$ from (*i*) in (*ii*), we get,

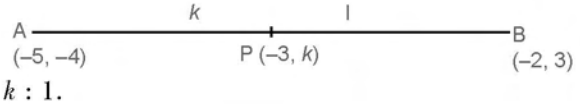
$10y + 2y = 3(2y)y \Rightarrow 12y = 6y^2 \Rightarrow y = 2$

and $x = 2y \Rightarrow x = 4$ ($\because y = 2$)

The required number $= 10(2) + 4 = 20 + 4 = 24$.

6. **Find the ratio in which the point $(-3, k)$ divides the line-segment joining the points $(-5, -4)$ and $(-2, 3)$. Also find the value of k.**

Sol.



Let P divides AB in $k : 1$.

Then $-3 = \dfrac{k\times(-2)+1(-5)}{k+1}$ [Using section formula, $(x, y) = \left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$]

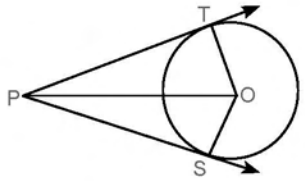
$\Rightarrow \quad -3k - 3 = -2k - 5$

$\Rightarrow \quad -k = -2$

$\Rightarrow \quad k = 2$

Hence the required ratio is 2 : 1

7. In given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120°$, Prove that OP = 2PS.



Sol. Let PT = x = PS [$\because$ Tangent drawn from external point to circle are equal]

$\angle SPT = 120°$ [Given]

In ΔOTP and ΔOSP, $\angle OTP = \angle OSP$

[$\because$ each equal to 90°, since tangent perpendicular r radius]

$OT = OS$ [$\because$ Equal radii]

$OP = OP$ [common]

$\Rightarrow \quad \Delta OSP \cong \Delta OTP$ [$\because$ By SAS congruence rule]

$\therefore \quad \angle TPO = \angle SPO$ [$\because$ By CPCT]

$\Rightarrow \quad \angle TPO = \dfrac{1}{2}\angle SPT = \dfrac{1}{2} \times 120 = 60°$

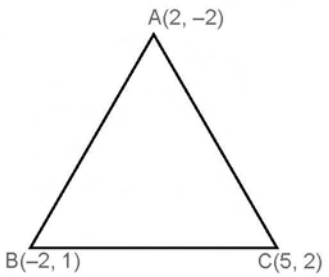
In ΔOTP, $\dfrac{OP}{x} = \text{Sec } 60°$

$\Rightarrow \quad \dfrac{OP}{x} = 2 \Rightarrow OP = 2x \Rightarrow OP = 2PS$

Hence proved.

8. Prove that the points (2, –2), (–2, 1) and (5, 2) are the vertices of a right angled triangle. Also find the area of this triangle.

Sol. Let A(2, –2), B(–2, 1) and C(5, 2) be the given points. So,



Using Distance formula

$AB^2 = (2 + 2)^2 + (-2 - 1)^2 = 16 + 9 = 25$

$\therefore \quad AB = 5$

$BC^2 = (-2 - 5)^2 + (1 - 2)^2 = 49 + 1 = 50$

$BC = 5\sqrt{2}$

$AC^2 = (5 - 2)^2 + (2 + 2)^2 = 9 + 16 = 25$

$AC = 5$

$\therefore$ $BC^2 = AC^2 + AB^2$, so ΔABC is a right angled triangle in which BC is hypotenuse.

$\therefore$ $$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \text{ sq. units}$$

9. If the ratio of sum of the first m and n terms of an A.P. is $m^2 : n^2$, show that the ratio of its mth and nth terms is $(2m - 1) : (2n - 1)$.

Sol. Let S_m and S_n be the sum of first m and n terms of the A.P. Let first term and common difference of an A.P. is a and d respectively. Then

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}$$

$\Rightarrow$ $$\frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n}$$

$\Rightarrow$ $$2an + (mn - n)d = 2am + (mn - m)d$$

$\Rightarrow$ $$2a(n - m) = (mn - m - mn + n)d$$

$\Rightarrow$ $$2a(n - m) = (n - m)d$$

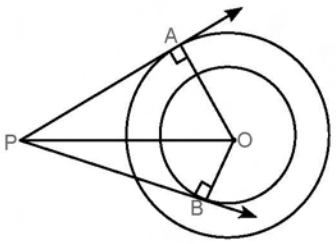
$\Rightarrow$ $$d = 2a$$

Consider, $$\frac{a_m}{a_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1)(2a)}{a + (n-1)(2a)} = \frac{a + 2am - 2a}{a + 2an - 2a}$$

$$= \frac{2am - a}{2an - a} = \frac{2m - 1}{2n - 1}$$

Hence, ratio of m^{th} and n^{th} term is $2m - 1 : 2n - 1$.

10. In given figure, two concentric circles are of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8cm, find the length of BP.



Sol. $OA = 6$ cm [$\because$ Given radius]

$OB = 4$ cm [$\because$ Given radius]

$AP = 8$ cm

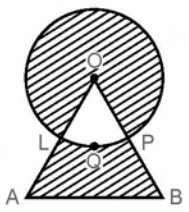
In ΔOAP, $OP^2 = OA^2 + AP^2 = 36 + 64 = 100$ [$\because$ Pythagoras theorem]

$\Rightarrow$ $OP = 10$ cm

In ΔOBP, $BP^2 = OP^2 - OB^2 = 100 - 16 = 84$ [$\because$ Pythagoras theorem]

$BP = 2\sqrt{21}$ cm

11. Find the area of shaded region in figure, where a circle of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Sol. $$\text{area of } \Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times (12)^2$$

$= 36\sqrt{3} = 36 \times 1.73$

$= 62.28 \text{ cm}^2$

area of circle with centre O $= \pi r^2 = 3.14 \times (6)^2$

$= 3.14 \times 36 = 113.04 \text{ cm}^2$

area of sector (OLQP) $= \pi r^2 \times \frac{\theta}{360°} = 3.14 \times (6)^2 \times \frac{60°}{360°}$

$= 3.14 \times 36 \times \frac{1}{6} = 18.84 \text{ cm}^2$

area of shaded region = area of ΔOAB + area of circle – 2 area of sector OLQP

$= (62.28 + 113.04 - 2 \times 18.84) \text{ cm}^2$

$= 137.64 \text{ cm}^2$

12. A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty? $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Radius of hemispherical tank = 1.5 m

Volume of water in hemispherical tank $= \frac{2}{3} \times \pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (1.5)^3 = \frac{99}{14} \text{ m}^3$

Rate of water taken out in 1 second $= \frac{25}{7}$ litre/second

Let time taken to empty to empty half the tank be 't' sec.

A.T.Q.,

Rate of flow of water $\times$ t sec $= \frac{1}{2} \times$ volume of water in the hemispherical tank

$\therefore \quad \frac{25}{7} \times \frac{1}{1000} \times t = \frac{1}{2} \times \frac{99}{14}$

$\therefore \quad t = 990 \text{ s}$

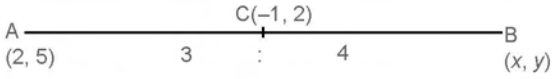
$\therefore$ Time taken to empty half the tank = 16 min. 30 sec.

13. If the point C(–1, 2) divides internally the line-segment joining the points A(2, 5) and B(x, y) in the ratio 3 : 4, find the value of $x^2 + y^2$.

Sol. Using section formula,

$$-1 = \frac{3 \times x + 4 \times 2}{3 + 4}$$

$$-1 = \frac{3x + 8}{7}$$



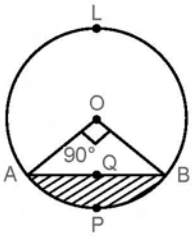
$3x + 8 = -7 \quad \Rightarrow \quad 3x = -15 \quad \Rightarrow \quad x = -5$

Similarly, $2 = \dfrac{3 \times y + 4 \times 5}{3 + 4}$

$14 = 3y + 20 \Rightarrow 3y = -6 \Rightarrow y = -2$

Hence, $x^2 + y^2 = (-5)^2 + (-2)^2 = 25 + 4 = 29$

14. In figure, AB is a chord of a circle, with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence, find the area of major segment ALBQA. (Use $\pi = 3.14$)



Sol. Area of minor segment APBQ $= \dfrac{\theta}{360^\circ} \times \pi r^2 - r^2 \sin 45^\circ \cos 45^\circ$

$= \dfrac{3.14 \times 100}{4} - 100 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}}$ $\left(\because \dfrac{90^\circ}{360^\circ} = \dfrac{1}{4}\right)$

$= (78.5 - 50)\ \text{cm}^2 = 28.5\ \text{cm}^2$

Area of major segment ALBQA $= \pi r^2$ – area of minor segment

$= 3.14 \times (10)^2 - 28.5$

$= (314 - 28.5)\ \text{cm}^2 = 285.5\ \text{cm}^2$

15. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1st and 4th) to the product of means (2nd and 3rd) is 5 : 6.

Sol. Let the four parts of the A.P. are $a - 3d, a - d, a + d, a + 3d$

Now, $a - 3d + a - d + a + d + a + 3d = 56$ [$\because$ Sum = 56]

$\Rightarrow \quad 4a = 56 \Rightarrow a = 14$

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6} \qquad [\because \text{ Putting } a = 14]$$

$$\Rightarrow \quad \frac{196-9d^2}{196-d^2} = \frac{5}{6}$$

$$\Rightarrow \quad 1176 - 54d^2 = 980 - 5d^2$$

$$\Rightarrow \quad 49d^2 = 196 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

Thus, 4 parts are $a - 3d, a - d, a + d, a + 3d$, i.e. 8, 12, 16, 20.

16. Solve the given quadratic equation for x: $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

Sol. $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Roots of above quadratic equation are given by quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-9(a+b)] \pm \sqrt{[-9(a+b)]^2 - 4(9)(2a^2+5ab+2b^2)}}{2(9)}$$

$$x = \frac{9(a+b) \pm 3\sqrt{9a^2+9b^2+18ab-8a^2-8b^2-20ab}}{18}$$

$$x = \frac{9(a+b) \pm 3\sqrt{a^2+b^2-2ab}}{18}$$

$$x = \frac{9(a+b) \pm 3\sqrt{(a-b)^2}}{18}$$

$$x = \frac{9(a+b) \pm 3(a-b)}{18}$$

$\therefore$ $$x = \frac{9a+9b+3a-3b}{18} \text{ and } x = \frac{9a+9b-3a+3b}{18}$$

$$x = \frac{12a+6b}{18} \text{ and } x = \frac{6a+12b}{18}$$

$\therefore$ $$x = \frac{2a+b}{3} \text{ and } x = \frac{a+2b}{3}$$

17. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cone.

Sol. Volume of ice cream cylinder $= \pi r^2 h$

$= \pi \times (6)^2 \times 15 \text{ cm}^3$

Volume of 1 ice cream cone $= \frac{1}{3} \times \pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3} \times \pi \times r^2(4r) + \frac{2}{3}\pi r^3$ [$\because$ Height $= 2 \times$ diameter]

$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3 \text{ cm}^3$

Volume of ice cream 10 such cones $= 10 \times 2\pi r^3 = 20\pi r^3 \text{ cm}^3$

According to Question,

Volume of 10 ice-cream cones = Volume of cylinder

$$20\pi r^3 = \pi \times 36 \times 15$$

$$r^3 = \frac{36 \times 15}{20}$$

$\Rightarrow$ $r^3 = 27 \Rightarrow r = 3$ cm

Diameter of conical ice cream cup = 6 cm

18. A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends are as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹ 35 per litre. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Volume of milk in container $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$$= \frac{1}{3} \times \frac{22}{7} \times 21\,[400 + 64 + 160] = \frac{22 \times 624}{1000} \text{ litres}$$

$$\left[\because 1\text{cm}^3 = \frac{1}{1000} l\right]$$

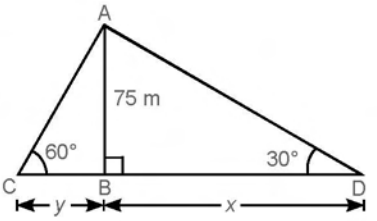
$\therefore$ Cost of 1 litre milk = ₹ 35

$$\text{Cost of milk in container} = \frac{35 \times 22 \times 624}{1000} = ₹\ 480.48$$

19. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men (Use $\sqrt{3}$ = 1.73)

Sol. Let C and D be the positions of two men.

Let CB = y and BD = x



In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow$ $\frac{75}{y} = \sqrt{3}$

$\Rightarrow$ $y = \frac{75}{\sqrt{3}} = \frac{75\sqrt{3}}{3} = 15\sqrt{3}$ m

$= 15 \times 1.73 = 25.95$ m

Now, in ΔABD, $\tan 30° = \frac{75}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow x = 75\sqrt{3} \Rightarrow 75 \times 1.73 = 129.75$ m

Hence, distance between two men is $x + y = 129.75 + 25.95 = 155.7$ m

20. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh will win the game if all the tosses show the same result, (i.e. either all three heads or all three tails) and loses the game otherwise. Find the probability that Ramesh will lose the game.

Sol. When coins are tossed three times, then total outcomes are

HHH, HHT, HTT, TTT, TTH, THH, HTH, THT

Total possible cases = 8

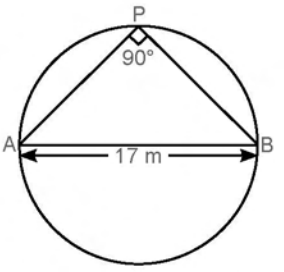
Favourable cases to win the game are HHH or TTT, i.e. two cases.

$$\text{P(Ramesh will win the game)} = \frac{\text{Number of favourable cases}}{\text{Total possible cases}} = \frac{2}{8} = \frac{1}{4}$$

$$\text{Required probability} = \text{P(Ramesh will loose the game)} = 1 - \frac{1}{4} = \frac{3}{4}$$

21. A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected.

Sol. Let P be the position of the pole.



$$\angle APB = 90° \text{(angle in a semi circle)}$$

By Pythagoras Theorem, $AB^2 = AP^2 + PB^2$

$$17^2 = AP^2 + PB^2 \quad ...(i)$$

$$AP - PB = 7 \quad ...(ii)$$

Squaring both sides, we get

$$(AP - PB)^2 = 49$$

$$AP^2 + PB^2 - 2AP \,.\, PB = 49 \quad ...(iii)$$

From (i) and (iii), we have

$$17^2 - 2AP \,.\, PB = 49$$

$$\Rightarrow \quad 2AP \,.\, PB = 289 - 49 \quad \Rightarrow \quad AP \,.\, PB = \frac{240}{2}$$

$$\Rightarrow \quad AP \,.\, PB = 120 \quad ...(iv)$$

From (ii) and (iv), we have

$$120 = PB(7 + PB)$$

Let $PB = x$

$$120 = x(7 + x) \quad \Rightarrow \quad x^2 + 7x - 120 = 0$$

$$\Rightarrow \quad x^2 + 15x - 8x - 120 = 0 \quad \Rightarrow \quad x(x + 15) - 8(x + 15) = 0$$

$$\Rightarrow \quad (x - 8)(x + 15) = 0$$

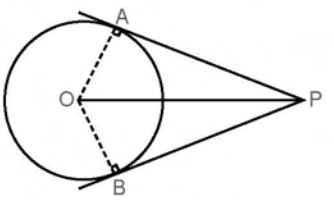
$$x = 8 \quad \text{or} \quad x = -15 \text{ (Rejected)}$$

$$PB = x = 8 \text{ m}$$

$$\therefore \quad AP = 7 + PB = 7 + 8 = 15 \text{ m}$$

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. **Given:** A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.



To Prove: PA = PB

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.

$$\angle OAP = \angle OBP = 90° \quad ...(i)$$

[Radius is perpendicular to the tangent at the point of contact]

$OA = OB$ (radii) ...(*ii*)

OP is common ...(*iii*)

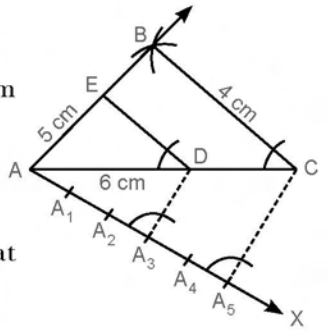
$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*) and (*iii*)]

Hence, $AP = BP$ (CPCT)

23. Draw a ΔABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

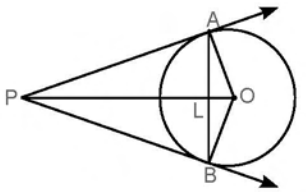
Sol. Steps of construction:

1. Draw a line segment AC = 6 cm.
2. Draw an arc with A as centre and radius equal to 5 cm.
3. Draw an arc with C as centre and radius equal to 4 cm intersecting the previous drawn arc at B.
4. Join AB and CB, then ΔABC is required triangle.
5. Below AC make an acute angle CAX.
6. Along AX mark of 5 points A_1, A_2, A_3, A_4, A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
7. Join A_5C.
8. From A_3 draw $A_3D \parallel A_3C$ meeting AC at D.
9. From D, draw $ED \parallel BC$ meeting AB at E.



Then we have ΔEDA which is the required triangle.

24. In given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA.



Sol. Let $PL = x$

As OP is perpendicular bisector of AB. Then

$AL = BL = 8$ cm

In ΔALO, $OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 36 \Rightarrow OL = 6$ cm

$AP^2 = OP^2 - OA^2$ [$\because$ Pythagoras theorem]

In ΔOAP, $AP^2 = (x + 6)^2 - 10^2$

$AP^2 = AL^2 + PL^2$ [$\because$ Pythagoras theorem]

In ΔALP, $AP^2 = x^2 + 64$

Now, $(x + 6)^2 - 10^2 = x^2 + 64$

$x^2 + 12x + 36 - 100 = x^2 + 64$

$\Rightarrow$ $12x = 128 \Rightarrow x = \frac{128}{12} = \frac{32}{3}$ cm

From ΔALP,

$$AP^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1024}{9} + 64 = \frac{1024+576}{9} \text{ cm}$$

$$AP^2 = \frac{1600}{9} \text{ cm}$$

$$AP = \frac{40}{3} \text{ cm} = 13.3 \text{ cm}$$

25. Find the positive value(s) of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have real roots.

Sol. It is given that quadratic equation, $x^2 + kx + 64 = 0$ has real roots.

For real roots $D \geq 0$

$\therefore$ $k^2 - 4(1)(64) \geq 0 \Rightarrow k^2 - 256 \geq 0$

$\Rightarrow$ $k^2 \geq 256 \Rightarrow k \geq \pm 16$

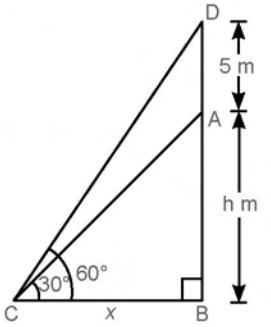
Again, it is given that quadratic equation, $x^2 - 8x + k = 0$ has real roots.

$\therefore$ $(-8)^2 - 4(1)(k) \geq 0 \Rightarrow 64 - 4k \geq 0$

$\Rightarrow$ $64 \geq 4k \Rightarrow k \leq 16$

26. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of the top and bottom of the flagstaff are 60° and 30° respectively. Find the height of the tower and the distance of the point from the tower. (take = $\sqrt{3}$ = 1.732)

Sol. Let AB is the tower of height h and DA is the flagstaff of height 5 m and BC = x.



In ΔABC, $\frac{AB}{BC} = \tan 30°$

$\Rightarrow$ $\frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $x = h\sqrt{3}$ ...(*i*)

Now, in ΔDBC, $\frac{DB}{BC} = \tan 60°$

$\frac{5+h}{x} = \sqrt{3}$

$\Rightarrow$ $\frac{5+h}{\sqrt{3}} = x$ ..(*ii*)

From (*i*) and (*ii*), we get

$$h\sqrt{3} = \frac{h+5}{\sqrt{3}} \Rightarrow 3h = h + 5$$

$\Rightarrow$ $2h = 5$

$\Rightarrow$ $h = \frac{5}{2} = 2.5$ m

Height of tower is 2.5 m.

Distance of point C from tower = $2.5 \times \sqrt{3} = 2.5 \times 1.732 = 4.33$ m [From (*i*)]

27. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?

What value is reflected in this question?

Sol. The amounts saved form an A.P. 450, 470, 490, ... in which

first term (a) = ₹ 450

Common difference (d) = ₹ 20

Total terms (n) = 12 (number of months)

Then, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{12} = \frac{12}{2}[2 \times 450 + (12-1)(20)] = 6[900 + 220] = 6 \times 1120 = 6720$$

Now, $6720 > 6500$

$\therefore$ Reshma will be able to send her daughter to school as she has saved more than ₹ 6500.

Now, Reshma is very much concerned about her daughter's education. She is aware and dedicated towards her daughter's education.

28. The coordinates of the points A, B and C are (6, 3), (–3, 5) and (4, –2) respectively. P(x, y) is any point in the plane. Show that $\frac{\text{ar}(\Delta PBC)}{\text{ar}(\Delta ABC)} = \left|\frac{x+y-2}{7}\right|$.

Sol. Taking points P, B, C. Firstly,

$$\text{area}(\Delta PBC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[x(7) - 3(-2 - y) + 4(y - 5)]$$

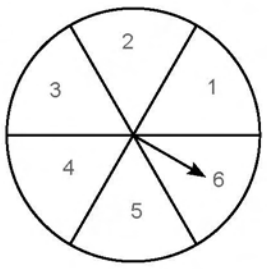
$$= \frac{1}{2}[7x + 7y - 14] \text{ sq. units}$$

Now, $\text{area}(\Delta ABC) = \frac{1}{2}[6 \times 7 - 3(-5) + 4(3 - 5)]$

$$= \frac{1}{2}[42 + 15 - 8] = \frac{1}{2} \times 49 \text{ sq. units}$$

Hence, $\left|\frac{\text{area}(\Delta PBC)}{\text{area}(\Delta ABC)}\right| = \left|\frac{7x+7y-14}{49}\right| = \left|\frac{x+y-2}{7}\right|$

29. In figure a disc is shown on which a player spins an arrow twice. The fraction $\frac{a}{b}$ is formed, where 'a' is the number of sector on which arrow stops on the first and 'b' is the number of the sector in which the arrow stops on second spin. On each spin, each sector has equal chance of selection by the arrow. Find the probability that $\frac{a}{b} > 1$.



Sol. For $\frac{a}{b} > 1$, when $a = 1$, b can not take any value

$a = 2$, b can take 1 value

$a = 3$, b can take 2 value, i.e. 1 and 2

$a = 4$, b can take 3 value, i.e. 1, 2, 3

$a = 5$, b can take 4 value, i.e. 1, 2, 3, 4

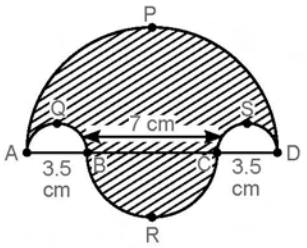
$a = 6$, b can take 5 value, i.e. 1, 2, 3, 4, 5

Total possible outcomes $= 6 \times 6 = 36$

Favourable outcomes $= 1 + 2 + 3 + 4 + 5 = 15$

$$P\left(\frac{a}{b} > 1\right) = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{15}{36} = \frac{5}{12}$$

30. Find the area of the shaded region in figure, where $\overset{\frown}{APD}$, $\overset{\frown}{AQB}$, $\overset{\frown}{BRC}$ and $\overset{\frown}{CSD}$ are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. $\left[\text{Use } \pi = \frac{22}{7}\right]$



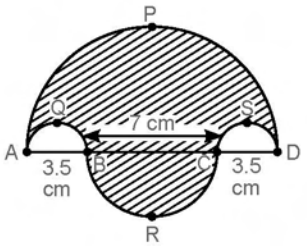
Sol. Area of shaded region

= Area of semicircle APD + Area of semicircle BRC – 2 × Area of semicircle AQB

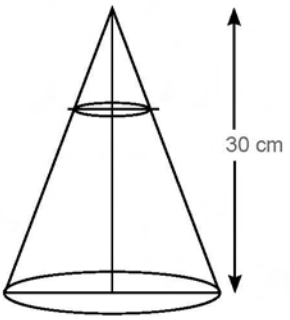
$$= \frac{1}{2}\pi(7)^2 + \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 - 2 \times \frac{1}{2}\pi\left(\frac{7}{4}\right)^2 = \frac{\pi}{2}\left[49 + \frac{49}{4} - \frac{49}{8}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{392 + 98 - 49}{8}\right) = \frac{11}{7} \times \frac{441}{8}$$

$$= \frac{693}{8} \text{ cm}^2 = 86.625 \text{ cm}^2$$



31. In figure, a right circular cone of height 30 cm is shown. A small cone is cut off from the top by a plane parallel to the base. If the volume of the small cone is $\frac{1}{27}$ of the volume of given cone, find at what height above the base is the section made.



Sol. Given height of cone = 30 cm

Let small cone is cut off at height h from top.

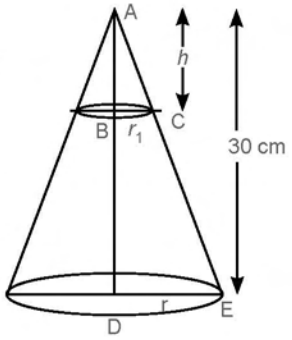
Let radius of big cone be $r_2 = r$ and small cone be r_1

In ΔADE and ΔABC we have

$$\frac{r_1}{r} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30r_1}{r}$$

Now, $$\frac{\text{Volume of small cone}}{\text{Volume of large cone}} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{1}{27} \quad \text{[Given]}$$



$$\Rightarrow \quad \frac{r_1^2 h}{r_2^2 \times 30} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{r_1^2 \times 30 \times r_1}{r^2 \times 30 \times r} = \frac{1}{27}$$

$$\frac{r_1^3}{r^3} = \frac{1}{27} \Rightarrow \frac{r_1}{r} = \frac{1}{3}$$

$$\Rightarrow \quad h = 30 \times \frac{r_1}{r} = 30 \times \frac{1}{3} = 10$$

$$\therefore \quad h = 10 \text{ cm}$$

Thus, at a height 20 cm above base, a small cone is cut.

SET-II [UNCOMMON QUESTIONS TO SET-I]

10. Solve for x: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$.

Sol. $$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0 \Rightarrow \sqrt{3}x^2 + \sqrt{2}x - 3\sqrt{2}x - 2\sqrt{3} = 0$$

$$\Rightarrow x(\sqrt{3}x + \sqrt{2}) - \sqrt{6}(\sqrt{3}x + \sqrt{2}) = 0 \Rightarrow (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$$

$$\therefore \quad x = \frac{-\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{6}}{3} \text{ and } x = \sqrt{6}$$

18. A 7 m long fagstaff is fixed on the top of a tower standing on the horizontal plane. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$)

Sol. Let AB is the tower of height h and DA is the flagstaff of height 7 m and BC is x.

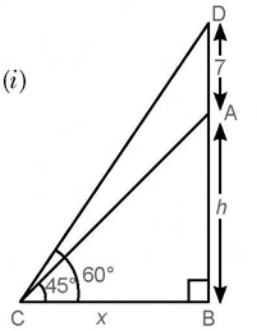
In ΔABC, $$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \quad \frac{h}{x} = 1 \Rightarrow h = x \qquad \ldots(i)$$

Now, in ΔDBC, $$\frac{DB}{BC} = \tan 60° \Rightarrow \frac{h+7}{x} = \sqrt{3}$$

$$\Rightarrow \quad h + 7 = \sqrt{3}h \qquad [\because h = x, \text{ using } (i)]$$

$$\Rightarrow \quad (\sqrt{3} - 1)h = 7$$



$$\Rightarrow \quad h = \frac{7}{\sqrt{3}-1} = \frac{7(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{7\times(1.73+1)}{2} = 9.5 \text{ m}$$

So, height of the tower is 9.5 m.

19. The pth, qth and rth terms of an A.P. are a, b and c respectively. Show that $a(q-r) + b(r-p) + c(p-q) = 0$

Sol. Let A and d be the first term and common difference of the given A.P., then

$$a_p = A + (p-1)d = a \quad ...(i)$$

$$a_q = A + (q-1)d = b \quad ...(ii)$$

$$a_r = A + (r-1)d = c \quad ...(iii)$$

Now, subtracting (i) and (ii), we get

$$(p-q)d = a - b$$

$$p - q = \frac{a}{d} - \frac{b}{d}$$

Multiplying by 'c' both sides,

$$c(p-q) = \frac{ca}{d} - \frac{cb}{d} \quad ...(iv)$$

Now, $(ii) - (iii)$, we get

$$(q-r)d = b - c$$

$$q - r = \frac{b}{d} - \frac{c}{d}$$

Multiplying by 'a' both sides,

$$a(q-r) = \frac{ab}{d} - \frac{ac}{d} \quad ...(v)$$

Now, $(iii) - (i)$, we get

$$(r-p)d = c - a$$

$$(r-p) = \frac{c}{d} - \frac{a}{d}$$

Multiplying by 'b' both sides,

$$(r-p)b = \frac{bc}{d} - \frac{ba}{d} \quad ...(vi)$$

Adding (iv), (v) and (vi), we get

$$a(q-r) + b(r-p) + c(p-q) = \frac{ab}{d} - \frac{ac}{d} + \frac{bc}{d} - \frac{ba}{d} + \frac{ca}{d} - \frac{cb}{d} = 0$$

20. Solve for x: $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}$, $x \neq 0, \frac{3}{2}, 2$

Sol.

$$\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}$$

$$\Rightarrow \quad \frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2} \quad \Rightarrow \quad \frac{(4x-3)}{2x^2-3x} = \frac{1}{x-2}$$

$$\Rightarrow \quad (x-2)(4x-3) = 2x^2 - 3x \Rightarrow 4x^2 - 3x - 8x + 6 = 2x^2 - 3x$$

$\Rightarrow \quad 4x^2 - 3x - 8x + 6 - 2x^2 + 3x = 0 \qquad \Rightarrow \quad 2x^2 - 8x + 6 = 0$

$\Rightarrow \quad x^2 - 4x + 3 = 0 \qquad \Rightarrow \quad x^2 - 3x - x + 3 = 0$

$\Rightarrow \quad x(x-3) - 1(x-3) = 0 \qquad \Rightarrow \quad (x-3)(x-1) = 0$

$\Rightarrow \quad x = 3, x = 1$

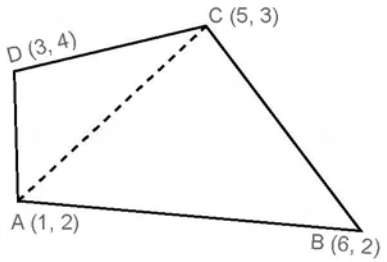
28. Find the area of the quadrilateral ABCD, the coordinates of whose vertices are A(1, 2), B(6, 2), C(5, 3) and D(3, 4).

Sol. Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(2-3) + 6(3-2) + 5(2-2)]$

$= \frac{1}{2}[-1 + 6 + 0]$

$= \frac{5}{2}$ sq. units



Now, Area of $(\Delta ACD) = \frac{1}{2}[1(3-4) + 5(4-2) + 3(2-3)]$

$= \frac{1}{2}[-1 + 10 - 3]$

$= \frac{1}{2} \times 6 = 3$ sq. units

Hence, Area (quadrilateral ABCD) $= \frac{5}{2} + 3 = \frac{11}{2}$ sq. units

29. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal.

Sol. It is given that quadratic equation, $x^2 + 2px + mn = 0$ has real and equal roots.

$\therefore \quad (2p)^2 - 4(mn)(1) = 0 \qquad [\because$ For real and equal roots $b^2 - 4ac = 0]$

$\Rightarrow \quad 4p^2 - 4mn = 0$

$\Rightarrow \quad p^2 - mn = 0$

$p^2 = mn \qquad ...(i)$

Now, for quadratic equation, $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$

$D = b^2 - 4ac$

$D = [-2(m + n)]^2 - 4(1)(m^2 + n^2 + 2p^2)$

$D = 4(m + n)^2 - 4(m^2 + n^2 + 2mn) \qquad$ [From (i)]

$D = 4(m + n)^2 - 4(m + n)^2$

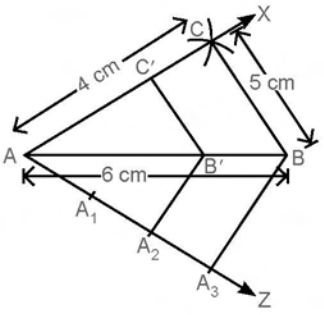
$D = 0$

$\therefore$ Roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$, also has equal roots.

30. **Draw a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of given (first) triangle.**

Sol. **Steps of construction:**

1. Draw a line segment AB of length 6 cm.
   Then using A as centre at a distance of 4 cm, draw an arc C. Also draw an arc using B as centre at a distance of 5 cm, which intersect earlier drawn arc at C. Join AC and BC.
2. Draw an acute angle BAZ and cut AZ as $AA_1 = A_1A_2 = A_2A_3$ and join BA_3.
3. Through A_2 draw a line parallel to BA_3 intersecting AB at B′.
4. Through B′ draw a line parallel to BC intersecting AC at C′.
   ΔAB′C′ is the required triangle.

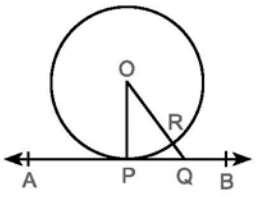


31. **Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.**

Sol. **Given:** A circle C(O, r) and a tangent AB at a point P.

To prove: OP ⊥ AB

Construction: Take any point Q other than P on the tangent AB.
Join OQ, intersecting circle at R.



Proof: We have, OP = OR [Radii]

$OQ = OR + RQ$

$\therefore \quad OQ > OR \Rightarrow OQ > OP$ [$\because$ OR = OP = radius]

Thus, OP < OQ, i.e. OP is shorter than any other segment joining O to any point of AB.

But among all line segments, joining point O to point on AB, shortest one is perpendicular from O on AB.

Hence, OP ⊥ AB

SET-III [UNCOMMON QUESTIONS TO SET-I & II]

10. **Solve for x: $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$, $x \neq 3, -5$.**

Sol. $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}; x \neq 3, -5$

$\Rightarrow \quad \frac{(x+5)-(x-3)}{(x-3)(x+5)} = \frac{1}{6} \quad \Rightarrow \quad \frac{x+5-x+3}{x^2+2x-15} = \frac{1}{6}$

$\Rightarrow \quad 8 \times 6 = x^2 + 2x - 15 \quad \Rightarrow \quad x^2 + 2x - 15 = 48$

$\Rightarrow \quad x^2 + 2x - 63 = 0 \quad \Rightarrow \quad (x + 9)(x - 7) = 0$

$\Rightarrow \quad x = 7 \text{ or } x = -9$

18. The sums of first n terms of three A.Ps' are S_1, S_2 and S_3. The first term of each is 5 and their common differences are 2, 4 and 6 respectively. Prove that $S_1 + S_3 = 2S_2$.

Sol. Here $a = 5$ and $d_1 = 2$, $d_2 = 4$ and $d_3 = 6$. Let sum of 'n' terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Now, $S_1 = \frac{n}{2}[2 \times 5 + (n-1)2]$

$= \frac{n}{2}[10 + 2n - 2] = \frac{(2n+8)n}{2} = n(n+4)$

$S_2 = \frac{n}{2}[2 \times 5 + (n-1)4]$

$= \frac{n}{2}[10 + 4n - 4] = \frac{n(4n+6)}{2} = n(2n+3) = 2n^2 + 3n$

$S_3 = \frac{n}{2}[2 \times 5 + (n-1)6]$

$= \frac{n}{2}[10 + 6n - 6] = \frac{n}{2}[6n + 4] = n(3n+2)$

Consider $S_1 + S_3 = n^2 + 4n + 3n^2 + 2n = 4n^2 + 6n = 2(2n^2 + 3n) = 2S_2$

19. Solve for x (in terms of a and b): $\frac{a}{x-b} + \frac{b}{x-a} = 2, x \neq a, b$

Sol. $\frac{a}{x-b} + \frac{b}{x-a} = 2; x \neq a, b$

$\Rightarrow \frac{a(x-a)+b(x-b)}{(x-a)(x-b)} = 2$

$\Rightarrow ax - a^2 + bx - b^2 = 2(x^2 - ax - bx + ab)$

$\Rightarrow ax + bx - a^2 - b^2 = 2x^2 - 2ax - 2bx + 2ab$

$\Rightarrow 2x^2 - 3ax - 3bx + a^2 + b^2 + 2ab = 0$

$\Rightarrow 2x^2 - 3(a+b)x + (a+b)^2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3(a+b) \pm \sqrt{9(a+b)^2 - 4 \times 2(a+b)^2}}{2 \cdot 2}$

$= \frac{3(a+b) \pm \sqrt{(a+b)^2}}{4} = \frac{3(a+b) \pm (a+b)}{4}$

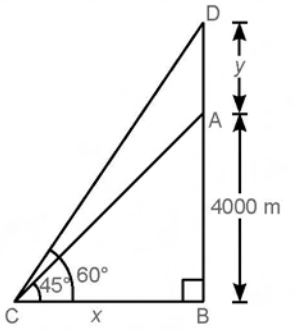
$\Rightarrow x = \frac{3(a+b)+(a+b)}{4}$ or $x = \frac{3(a+b)-(a+b)}{4}$

$\Rightarrow x = \frac{4(a+b)}{4}$ or $x = \frac{2(a+b)}{4}$

$\Rightarrow x = a + b$ or $x = \frac{a+b}{2}$

20. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Take $\sqrt{3} = 1.73$)

Sol. Let y is the vertical distance between the aeroplanes.



In ΔABC, $\frac{AB}{BC} = \tan 45° \Rightarrow \frac{4000}{x} = 1$

$\Rightarrow x = 4000$ m ...(*i*)

Now, in ΔDBC, $\frac{DB}{BC} = \tan 60°$

$\Rightarrow \frac{y+4000}{x} = \sqrt{3}$

$\Rightarrow \frac{y+4000}{4000} = \sqrt{3}$ [From (*i*)]

$\Rightarrow y + 4000 = 4000\sqrt{3} \Rightarrow y = 4000(\sqrt{3} - 1)$

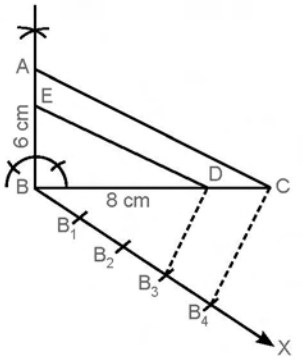
$y = 4000(1.73 - 1)$

$\Rightarrow y = 4000 \times 0.73 \Rightarrow y = 2920$ m

So, distance between the aeroplanes is 2920 m.

28. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the sides of first triangle.

Sol. Steps of construction:



1. Draw line BC = 8 cm then at B draw a line making angle of 90°.
2. Cut a length of 6 cm and name it A. Join AC. ΔABC is the right triangle.
3. Below BC make an acute angle $\angle CBX$.
4. Along BX mark off 4 points B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
5. Join B_4C.
6. From B_3 draw $B_3D \parallel B_4C$ meeting BC at D.
7. From D draw $ED \parallel AC$ meeting BA at E.

Now we have ΔEBD which is the required triangle whose sides are $\frac{3}{4}$ th of the corresponding sides of ΔABC.

29. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let x be the time taken by larger diameter tap.

$\therefore x + 8$ be the time taken by smaller diameter tap.

According to question,

$$\frac{1}{x}+\frac{1}{x+8}=\frac{10}{96} \qquad \left(\because 9 \text{ hrs } 36 \text{ min}=\frac{96}{10}\text{ hrs}\right)$$

$$\Rightarrow \quad \frac{x+8+x}{x(x+8)}=\frac{10}{96}$$

$$\Rightarrow \quad 96(2x+8)=10(x^2+8x)$$

$$\Rightarrow \quad 10x^2-112x-768=0$$

$$\Rightarrow \quad 5x^2-56x-384=0$$

$$\Rightarrow \quad x=\frac{56\pm\sqrt{(56)^2-4\times5\times(-384)}}{2\times5}$$

$$\Rightarrow \quad x=\frac{56+104}{10} \text{ or } \frac{56-104}{10}$$

$$\Rightarrow \quad x=16 \text{ or } x=-4.8 \qquad \text{(Rejected)}$$

Hence, time taken by larger and smaller taps are 16 hrs and 24 hrs respectively.

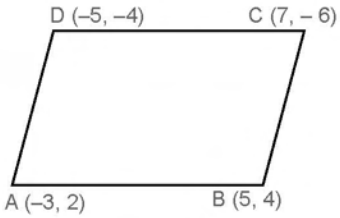
30. Find the area of a quadrilateral ABCD, the coordinates of whose vertices are A(–3, 2), B(5, 4), C(7, –6) and D(–5, –4).

Sol. $\text{Area of } \Delta ABD=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$=\frac{1}{2}[-3(8)+5(-6)+(-5)(2-4)]$$

$$=\frac{1}{2}[-24-30+10]$$

$$=\frac{1}{2}\times(-44)=(-22)=22 \text{ sq. units}$$



Since area can't be negative

$$\text{area of } \Delta BCD=\frac{1}{2}[5(-2)+7(-8)-5(10)]$$

$$=\frac{1}{2}[-10-56-50]$$

$$=\frac{1}{2}(-116)=(-58)=58 \text{ sq. units}$$

Since area cannot be negative.

Area of quadrilateral ABCD = Area (ΔABD) + area (ΔBCD)

$$=22+58=80 \text{ sq. units.}$$

31. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Sol. Let the fraction be $\frac{x}{2x+1}$

According to question,

$$\frac{x}{2x+1}+\frac{2x+1}{x}=2\frac{16}{21}$$

Consider $\frac{x}{2x+1} = y$, then above equation becomes,

$$y + \frac{1}{y} = 2\frac{16}{21} \Rightarrow \frac{y^2+1}{y} = \frac{58}{21} \Rightarrow 21y^2 + 21 = 58y$$

$$\Rightarrow \quad 21y^2 - 58y + 21 = 0 \Rightarrow y = \frac{58 \pm \sqrt{58^2 - 4\times 21\times 21}}{2\times 21}$$

$$\Rightarrow \quad y = \frac{58 \pm \sqrt{3364 - 1764}}{42} \Rightarrow y = \frac{58 \pm \sqrt{1600}}{42} \Rightarrow y = \frac{58 \pm 40}{42}$$

$$\Rightarrow \quad y = \frac{58+40}{42} \quad \text{or} \quad y = \frac{58-40}{42}$$

$\Rightarrow \quad y = \frac{98}{42} \Rightarrow y = \frac{7}{3}$	$\Rightarrow \quad y = \frac{18}{42} \Rightarrow y = \frac{3}{7}$
$\Rightarrow \quad \frac{x}{2x+1} = \frac{7}{3}$	$\Rightarrow \quad \frac{x}{2x+1} = \frac{3}{7}$
$\Rightarrow \quad 3x = 14x + 7$	$\Rightarrow \quad 7x = 6x + 3$
$\Rightarrow \quad -11x = 7$	$\Rightarrow \quad x = 3$
$\Rightarrow \quad x = \frac{-7}{11}$ (Rejected)	$\Rightarrow \quad \frac{x}{2x+1} = \frac{3}{2\times 3+1} = \frac{3}{7}$

$\therefore$ The required fraction is $\frac{3}{7}$.