

EXERCISE 1F

1. Write the rationalising factor of the denominator in $\frac{1}{\sqrt{2} + \sqrt{3}}$. [2014]

2. Rationalise the denominator of each of the following.

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{\sqrt{5}}{2\sqrt{3}}$

(iii) $\frac{1}{2 + \sqrt{3}}$

(iv) $\frac{1}{\sqrt{5} - 2}$

(v) $\frac{1}{5 + 3\sqrt{2}}$

(vi) $\frac{1}{\sqrt{7} - \sqrt{6}}$ [2010]

(vii) $\frac{4}{\sqrt{11} - \sqrt{7}}$ [2010]

(viii) $\frac{1 + \sqrt{2}}{2 - \sqrt{2}}$ [2014]

(ix) $\frac{3 - 2\sqrt{2}}{3 + 2\sqrt{2}}$

3. It being given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$, find the value to three places of decimals, of each of the following.

(i) $\frac{2}{\sqrt{5}}$

(ii) $\frac{2 - \sqrt{3}}{\sqrt{3}}$

(iii) $\frac{\sqrt{10} - \sqrt{5}}{\sqrt{2}}$ [2010]

4. Find rational numbers a and b such that

(i) $\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = a + b\sqrt{2}$ [2012]

(ii) $\frac{2 - \sqrt{5}}{2 + \sqrt{5}} = a\sqrt{5} + b$ [2014]

(iii) $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = a + b\sqrt{6}$ [2010, '11]

(iv) $\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = a + b\sqrt{3}$ [2014]

5. It being given that $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{6} = 2.449$ and $\sqrt{10} = 3.162$, find to three places of decimal, the value of each of the following.

(i) $\frac{1}{\sqrt{6} + \sqrt{5}}$ [2014]

(ii) $\frac{6}{\sqrt{5} + \sqrt{3}}$ [2011]

(iii) $\frac{1}{4\sqrt{3} - 3\sqrt{5}}$ [CBSE Sample Paper]

$$(iv) \frac{3+\sqrt{5}}{3-\sqrt{5}} \quad [2010] \quad (v) \frac{1+2\sqrt{3}}{2-\sqrt{3}}$$

$$(vi) \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \quad [2012]$$

6. Simplify by rationalising the denominator.

$$(i) \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$(ii) \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \quad [2014]$$

7. Simplify

$$(i) \frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$(ii) \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}} \quad [2011]$$

$$(iii) \frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \quad [2014]$$

$$(iv) \frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}} \quad [2010, '11]$$

8. Prove that

$$(i) \frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1 \quad [2011]$$

$$(ii) \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} \\ + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$$

9. Find the values of a and b if

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}. \quad [2014]$$

10. Simplify $\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}} + \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$. [2015]

11. If $x = 3 + 2\sqrt{2}$, check whether $x + \frac{1}{x}$ is rational or irrational. [2010]

12. If $x = 2 - \sqrt{3}$, find the value of $\left(x - \frac{1}{x}\right)^3$.

13. If $x = 9 - 4\sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$. [2011]

14. If $x = \frac{5-\sqrt{21}}{2}$, find the value of $x + \frac{1}{x}$. [2014]

15. If $a = 3 - 2\sqrt{2}$, find the value of $a^2 - \frac{1}{a^2}$. [2010]

16. If $x = \sqrt{13} + 2\sqrt{3}$, find the value of $x - \frac{1}{x}$.

17. If $x = 2 + \sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$.

[2015]

18. If $x = \frac{5 - \sqrt{3}}{5 + \sqrt{3}}$ and $y = \frac{5 + \sqrt{3}}{5 - \sqrt{3}}$, show that $x - y = -\frac{10\sqrt{3}}{11}$.

[2015]

19. If $a = \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$ and $b = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$, show that $3a^2 + 4ab - 3b^2 = 4 + \frac{56}{3}\sqrt{10}$.

20. If $a = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ and $b = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$, find the value of $a^2 + b^2 - 5ab$.

[2011]

21. If $p = \frac{3 - \sqrt{5}}{3 + \sqrt{5}}$ and $q = \frac{3 + \sqrt{5}}{3 - \sqrt{5}}$, find the value of $p^2 + q^2$.

[2010]

22. Rationalise the denominator of each of the following.

(i) $\frac{1}{\sqrt{7} + \sqrt{6} - \sqrt{13}}$ [2011]

(ii) $\frac{3}{\sqrt{3} + \sqrt{5} - \sqrt{2}}$ [2015]

(iii) $\frac{4}{2 + \sqrt{3} + \sqrt{7}}$

[2015]

23. Given, $\sqrt{2} = 1.414$ and $\sqrt{6} = 2.449$, find the value of $\frac{1}{\sqrt{3} - \sqrt{2} - 1}$ correct to 3 places of decimal.

[2014]

24. If $x = \frac{1}{2 - \sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.

25. Evaluate $\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}}$, it being given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

HINT
$$\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}} = \frac{15}{\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}}$$

$$= \frac{15}{3\sqrt{10} - 3\sqrt{5}} = \frac{5}{\sqrt{10} - \sqrt{5}}$$

ANSWERS (EXERCISE 1F)

1. $\sqrt{2} - \sqrt{3}$

2. (i) $\frac{\sqrt{7}}{7}$

(ii) $\frac{\sqrt{15}}{6}$

(iii) $2 - \sqrt{3}$

(iv) $\sqrt{5} + 2$

(v) $\frac{5 - 3\sqrt{2}}{7}$

(vi) $\sqrt{7} + \sqrt{6}$

(vii) $\sqrt{11} + \sqrt{7}$

(viii) $\frac{4 + 3\sqrt{2}}{2}$

(ix) $17 - 12\sqrt{2}$

EXERCISE 1G

1. Simplify

(i) $2^{\frac{2}{3}} \times 2^{\frac{1}{3}}$ [2015]

(ii) $2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$ [2014]

(iii) $7^{\frac{5}{6}} \times 7^{\frac{2}{3}}$

(iv) $(1296)^{\frac{1}{4}} \times (1296)^{\frac{1}{2}}$

2. Simplify

(i) $\frac{6^{1/4}}{6^{1/5}}$

(ii) $\frac{8^{1/2}}{8^{2/3}}$

(iii) $\frac{5^{6/7}}{5^{2/3}}$

3. Simplify

(i) $3^{\frac{1}{4}} \times 5^{\frac{1}{4}}$

(ii) $2^{\frac{5}{8}} \times 3^{\frac{5}{8}}$

(iii) $6^{\frac{1}{2}} \times 7^{\frac{1}{2}}$

4. Simplify

(i) $(3^4)^{\frac{1}{4}}$

(ii) $(3^{1/3})^4$

(iii) $\left(\frac{1}{3^4}\right)^{\frac{1}{2}}$

5. Evaluate

(i) $(125)^{\frac{1}{3}}$

(ii) $(64)^{\frac{1}{6}}$

(iii) $(25)^{\frac{3}{2}}$

(iv) $(81)^{\frac{3}{4}}$

(v) $(64)^{-\frac{1}{2}}$

(vi) $(8)^{-\frac{1}{3}}$

6. If $a = 2$, $b = 3$, find the values of

(i) $(a^b + b^a)^{-1}$

(ii) $(a^a + b^b)^{-1}$ [2014]

7. Simplify

(i) $\left(\frac{81}{49}\right)^{-\frac{3}{2}}$ [2011]

(ii) $(14641)^{0.25}$ [2015]

(iii) $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$ [2011]

(iv) $\left(\frac{7776}{243}\right)^{-\frac{3}{5}}$ [2014]

8. Evaluate

$$(i) \frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$$

[2015]

$$(ii) \left(\frac{64}{125}\right)^{-\frac{2}{3}} + \left(\frac{256}{625}\right)^{-\frac{1}{4}} + \left(\frac{3}{7}\right)^0$$

$$(iii) \left(\frac{81}{16}\right)^{-\frac{3}{4}} \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3} \right]$$

[2010]

$$(iv) \frac{(25)^{\frac{5}{2}} \times (729)^{\frac{1}{3}}}{(125)^{\frac{2}{3}} \times (27)^{\frac{2}{3}} \times 8^{\frac{4}{3}}}$$

9. Evaluate

$$(i) (1^3 + 2^3 + 3^3)^{\frac{1}{2}} \quad [2015]$$

$$(ii) \left[5 \left(8^{\frac{1}{3}} + 27^{\frac{1}{3}} \right)^3 \right]^{\frac{1}{4}}$$

[2011, '15]

$$(iii) \frac{2^0 + 7^0}{5^0} \quad [2015]$$

$$(iv) \left[(16)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$

[2014]

10. Prove that

$$(i) \left[8^{-\frac{2}{3}} \times 2^{\frac{1}{2}} \times 25^{-\frac{5}{4}} \right] \div \left[32^{-\frac{2}{5}} \times 125^{-\frac{5}{6}} \right] = \sqrt{2}$$

[2015]

$$(ii) \left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}} = \frac{65}{16}$$

[2011]

$$(iii) \left[7 \left\{ (81)^{\frac{1}{4}} + (256)^{\frac{1}{4}} \right\}^4 \right] = 16807$$

[2015]

11. Simplify $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponential form of x .

[2011]

12. Simplify the product $\sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32}$.

13. Simplify

$$(i) \left(\frac{15^{1/3}}{9^{1/4}}\right)^{-6} \quad [2010]$$

$$(ii) \left(\frac{12^{1/5}}{27^{1/5}}\right)^{5/2} \quad [2010]$$

$$(iii) \left(\frac{15^{1/4}}{3^{1/2}}\right)^{-2} \quad [2011]$$

14. Find the value of x in each of the following.

$$(i) \sqrt[5]{5x+2} = 2 \quad [2014]$$

$$(ii) \sqrt[3]{3x-2} = 4 \quad [2014]$$

$$(iii) \left(\frac{3}{4}\right)^3 \left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x} \quad [2010]$$

$$(iv) 5^{x-3} \times 3^{2x-8} = 225 \quad [2010]$$

$$(v) \frac{3^{3x} \cdot 3^{2x}}{3^x} = \sqrt[4]{3^{20}} \quad [2015]$$

15. Prove that

$$(i) \sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x} = 1.$$

$$(ii) \left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \cdot \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \cdot \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}} = 1$$

[2015]

$$(iii) \frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$$

[2010]

$$(iv) \frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4} = 1$$

16. If x is a positive real number and exponents are rational numbers, simplify

[2011]

$$\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c}.$$

17. If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$, prove that $m - n = 1$.

18. Write the following in ascending order of magnitude.

[2015]

$$\sqrt[6]{6}, \sqrt[3]{7}, \sqrt[4]{8}.$$

ANSWERS (EXERCISE 1G)

$$1. (i) 2 \quad (ii) 2^{\frac{13}{15}} \quad (iii) 7^{\frac{3}{2}} \quad (iv) 216 \quad 2. (i) 6^{\frac{1}{20}} \quad (ii) 8^{-\frac{1}{6}} \quad (iii) 5^{\frac{4}{21}}$$

$$3. (i) (15)^{\frac{1}{4}} \quad (ii) (6)^{\frac{5}{8}} \quad (iii) (42)^{\frac{1}{2}} \quad 4. (i) 3 \quad (ii) 3^{\frac{4}{3}} \quad (iii) \frac{1}{9}$$

$$5. (i) 5 \quad (ii) 2 \quad (iii) 125 \quad (iv) 27 \quad (v) \frac{1}{8} \quad (vi) \frac{1}{2}$$

$$6. (i) \frac{1}{17} \quad (ii) \frac{1}{13}$$

$$7. (i) \frac{343}{729} \quad (ii) 11 \quad (iii) \frac{81}{16} \quad (iv) \frac{1}{8}$$

$$8. (i) 214 \quad (ii) \frac{61}{16} \quad (iii) 1 \quad (iv) \frac{125}{16}$$

$$9. (i) 6 \quad (ii) 5 \quad (iii) 2 \quad (iv) 2$$

$$11. x^{\frac{1}{6}} \quad 12. 2 \quad 13. (i) \frac{27}{225} \quad (ii) \frac{2}{3} \quad (iii) \frac{3}{(15)^{1/2}}$$

$$14. (i) x = 6 \quad (ii) x = 22 \quad (iii) x = 5 \quad (iv) x = 5 \quad (v) x = \frac{5}{4}$$

$$16. 1 \quad 18. \sqrt[6]{6} < \sqrt[4]{8} < \sqrt[3]{7}$$

3. (i) 0.894 (ii) 0.155 (iii) 0.655
4. (i) $a = 3, b = -2$ (ii) $a = 4, b = -9$ (iii) $a = 5, b = 2$ (iv) $a = 11, b = -6$
5. (i) 0.213 (ii) 1.512 (iii) 4.545 (iv) 6.854
(v) 16.660 (vi) 4.441
6. (i) $\frac{114 - 41\sqrt{6}}{30}$ (ii) $\frac{4\sqrt{30} + 9}{21}$
7. (i) $\frac{42}{11}$ (ii) $2\sqrt{2}$ (iii) $16 - \sqrt{3}$ (iv) 0
9. $a = 1, b = 1$ 10. 24 11. Rational 12. $-24\sqrt{3}$ 13. 322
14. 5 15. $-24\sqrt{2}$ 16. $4\sqrt{3}$ 17. 52 20. 93 21. 47
22. (i) $\frac{7\sqrt{6} + 6\sqrt{7} + \sqrt{546}}{84}$ (ii) $\frac{2\sqrt{3} - 3\sqrt{2} - \sqrt{30}}{4}$ (iii) $\frac{2\sqrt{3} - \sqrt{21} + 3}{3}$
23. -1.466 24. 3 25. 5.398

LAWS OF EXPONENTS

Let $a > 0, b > 0$ be real numbers and let m and n be rational numbers. Then, we have

- (i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$
- (iii) $(a^m)^n = a^{mn}$ (iv) $a^m \times b^m = (ab)^m$
- (v) $(ab)^m = a^m b^m$ (vi) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- (vii) $a^{-n} = \frac{1}{a^n}$ (viii) $a^0 = 1$

(ii) $(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5})$ and $(3\sqrt{3} - \sqrt{2} + \sqrt{5})$

(iii) $(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11})$ and $(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11})$

2. Multiply

(i) $3\sqrt{5}$ by $2\sqrt{5}$

(ii) $6\sqrt{15}$ by $4\sqrt{3}$

(iii) $2\sqrt{6}$ by $3\sqrt{3}$

(iv) $3\sqrt{8}$ by $3\sqrt{2}$

(v) $\sqrt{10}$ by $\sqrt{40}$

(vi) $3\sqrt{28}$ by $2\sqrt{7}$

3. Divide

(i) $16\sqrt{6}$ by $4\sqrt{2}$

(ii) $12\sqrt{15}$ by $4\sqrt{3}$

(iii) $18\sqrt{21}$ by $6\sqrt{7}$

4. Simplify

(i) $(3 - \sqrt{11})(3 + \sqrt{11})$ [2014]

(ii) $(-3 + \sqrt{5})(-3 - \sqrt{5})$

[2014]

(iii) $(3 - \sqrt{3})^2$

(iv) $(\sqrt{5} - \sqrt{3})^2$

(v) $(5 + \sqrt{7})(2 + \sqrt{5})$

(vi) $(\sqrt{5} - \sqrt{2})(\sqrt{2} - \sqrt{3})$

5. Simplify $(3 + \sqrt{3})(2 + \sqrt{2})^2$.

[2010]

6. Examine whether the following numbers are rational or irrational:

(i) $(5 - \sqrt{5})(5 + \sqrt{5})$

(ii) $(\sqrt{3} + 2)^2$

(iii) $\frac{2\sqrt{13}}{3\sqrt{52} - 4\sqrt{117}}$

(iv) $\sqrt{8} + 4\sqrt{32} - 6\sqrt{2}$

7. On her birthday Reema distributed chocolates in an orphanage. The total number of chocolates she distributed is given by $(5 + \sqrt{11})(5 - \sqrt{11})$.

[2014]

(i) Find the number of chocolates distributed by her.

(ii) Write the moral values depicted here by Reema.

8. Simplify

(i) $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

[2010]

(ii) $\frac{2\sqrt{30}}{\sqrt{6}} - \frac{3\sqrt{140}}{\sqrt{28}} + \frac{\sqrt{55}}{\sqrt{99}}$

(iii) $\sqrt{72} + \sqrt{800} - \sqrt{18}$

[2014]

ANSWERS (EXERCISE 1D)

1. (i) $3(\sqrt{3} - \sqrt{2})$

(ii) $\sqrt{2} + 8\sqrt{3} - 6\sqrt{5}$

(iii) $\sqrt{7} + \sqrt{2} + 5\sqrt{11}$

2. (i) 30

(ii) $72\sqrt{5}$

(iii) $18\sqrt{2}$

(iv) 36

(v) 20

(vi) 84

4. (i) -2

(ii) 4

3. (i) $4\sqrt{3}$

(ii) $3\sqrt{5}$

(iii) $3\sqrt{3}$

(v) $10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(iii) $12 - 6\sqrt{3}$

(iv) $8 - 2\sqrt{15}$

5. $18 + 12\sqrt{2} + 6\sqrt{3} + 4\sqrt{6}$

(vi) $\sqrt{10} - \sqrt{15} - 2 + \sqrt{6}$

Clearly, point P on the number line represents $2.3\bar{2}$ up to 4 decimal places.

EXERCISE 1E

1. Represent $\sqrt{5}$ on the number line. [2010, '11, '14]
2. Locate $\sqrt{3}$ on the number line. [2012, '15]
3. Locate $\sqrt{10}$ on the number line. [2010]
4. Locate $\sqrt{8}$ on the number line. [2011]
5. Represent $\sqrt{4.7}$ geometrically on the number line.
6. Represent $\sqrt{10.5}$ on the number line. [2014]
7. Represent $\sqrt{7.28}$ geometrically on the number line.
8. Represent $(1 + \sqrt{9.5})$ on the number line. [2015]
9. Visualize the representation of 3.765 on the number line using successive magnification. [2015]
10. Visualize the representation of $4.\overline{67}$ on the number line up to 4 decimal places. [2011]