

The University of Zambia
Department of Mathematics and Statistics
MAT 3300 Real Analysis
Test 1

Duration: Two (2) Hours

Instructions:

- Answer all questions
 - Show all necessary work to earn full marks
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1. (a) Determine whether the following sets are closed in \mathbb{R} :

i. $A = \{x \in \mathbb{Q} : 0 \leq x \leq 1\}$,

ii. $B = \{\frac{m}{n} : 0 \leq m \leq 1, 0 < n \leq 1\}$,

iii. $C = C_1 \cap C_2$, where C_1 is compact and C_2 is connected.

(b) Determine whether the following statements are true or false:

i. The Cartesian product of two closed subsets of \mathbb{R} is closed in \mathbb{R}^2 ,

ii. If A is a closed subset of \mathbb{R} , then $A' = \{\frac{1}{a} : a \in A\}$ is closed.

iii. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function and A is closed in \mathbb{R} , then $f(A)$ is closed in \mathbb{R} .

2. (a) Show directly that the following sets are not compact in \mathbb{R} :

i. $A = (-2, 2)$

ii. $B = [0, \infty)$

(b) Prove the following:

i. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and A is compact in \mathbb{R} , then $f(A)$ is compact in \mathbb{R} .

ii. If A and B are compact subsets of \mathbb{R} , then $A \times B$ is a compact subset of \mathbb{R}^2 .

3. (a) Show that the following sets are disconnected:

i. The Cantor set C

ii. The set of all natural numbers \mathbb{N}

(b) Show that the following sets are intervals in \mathbb{R} and therefore connected:

i. $(-2, 2)$

ii. $(0, \infty)$

(c) Prove the following:

- i. Every countable subset of \mathbb{R} is disconnected,
- ii. A subset B of a connected set A in \mathbb{R} may be disconnected,
- iii. The intersection of any two connected sets in \mathbb{R} is connected.