MODEL QUESTION PAPER-1

For Reduced Syllabus 2020-21
MATHEMATICS :SECOND PUC
Subject code: 35

Time: 3 hours 15 minute **Max. Marks: 100**

Instructions:

i. The question paper has five parts namely A, B, C, D and E. Answer all the parts.
ii. Use the graph sheet for the question on linear programming in **PART – E**.

PART A

Answer ALL the questions **$10 \times 1 = 10$**

1. Define an empty relation.

2. Write the domain of the function $y = \sec^{-1} x$.

3. If a matrix has 5 elements, what are the possible orders it can have?

4. Find the values of x for which $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

5. If $y = \tan\sqrt{x}$, find $\frac{dy}{dx}$.

6. Find $\int (2x^2 + e^x)\, dx$.

7. Define a negative vector.

8. If a line makes angles 90°, 135° and 45° with the *x, y* and *z*-axis respectively, find its direction cosines.

9. Define Optimal solution in a linear programming problem.

10. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate $P(A|B)$.

PART B

Answer any TEN questions: **$10 \times 2 = 20$**

11. Let $*$ be a binary operation on Q defined by $a * b = \frac{ab}{2}, \forall a, b \in Q$. Show that $*$ is associative.

12. Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$.

13. Find the area of the triangle whose vertices are $(-2,-3), (3,2)$ and $(-1,-8)$ using determinants.

14. Find $\frac{dy}{dx}$, if $y = \cos(\log x + e^x),\ x > 0$

15. Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$.

16. If $y = x^3 + \tan x$, then find $\frac{d^2y}{dx^2}$.

17. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

18. Find $\int_0^{\pi} (\sin^2\frac{x}{2} - \cos^2\frac{x}{2})dx$.

19. Find $\int x \sec^2 x\, dx$.

20. Find the order and degree of the differential equation, $y''' + 2y'' + y' = 0$.

21. Find the projection of the vector $\vec{a} = \hat{i} + 3j + 7k$ on the vector $\vec{b} = 7\hat{i} - j + 8k$.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-j+3k$ and $\vec{b}=2\hat{i}-7j+k$.

23. Find the equation of the plane with intercepts 2, 3 and 4 on *X*, *Y* and *Z* axes respectively.

24. Assume that each child born in a family is equally likely to be a boy or girl. If a family has two children, what is the conditional probability that both are girls, given that the youngest is a girl.

PART C

Answer any TEN questions: **10 × 3=30**

25. Show that the relation R defined in the set A of all triangles as $R=\{(T_1,T_2):T_1 \text{ is similar to} T_2\}$, is equivalence relation.

26. For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that

(i) $A+A'$ is a symmetric matrix (ii) $A-A'$ is a skew- symmetric matrix.

27. If $x=2at^2,\ y=at^4$, then find $\frac{dy}{dx}$.

28. Find $\frac{dy}{dx}$, if $x^y=y^x$.

29. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is (a) strictly increasing (b) strictly decreasing.

30. Evaluate: $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cdot\cos^2 x}dx$.

31. Find $\int \frac{(x-3)e^x}{(x-1)^3}dx$.

32. Evaluate: $\int \frac{dx}{(x+1)(x+2)}$.

33. Find the area of the region bounded by $x^2=4y,\ y=2, y=4$ and the *y*-axis in the first quadrant.

34. Solve, $\frac{dy}{dx}=e^{x+y}$.

35. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3,\ |\vec{b}|=4,\ |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

36. Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear.

37. Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$, $x+y+z-2=0$ and the point (2, 2, 1).

38. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART D

Answer any SIX questions: **6 × 5=30**

39. Check the injectivity and surjectivity of the function $f:R\to R$ defined by $f(x)=3-4x$. Is it a bijective function?

40. If $A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$ then compute $A+B$ and $B-C$

Also, verify that $A+(B-C)=(A+B)-C$.

41. Solve the system of equations by matrix method:
$2x+3y+3z=5,\ x-2y+z=-4,\ 3x-y-2z=3.$

42. If $y=\left(\tan^{-1}x\right)^2$, show that $(1+x^2)^2y_2+2x(1+x^2)y_1=2.$

43. The length x of a rectangle is decreasing at the rate of $3\ cm/minute$ and the width y is Increasing at the rate of $2\ cm/minute$. When $x=10\ cm$ and $y=6\ cm$, find the rates of change of (i) the perimeter (ii) the area of the rectangle.

44. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\frac{1}{\sqrt{9-25x^2}}$.

45. Using the method of integration, find the area enclosed by the circle $x^2+y^2=a^2$.

46. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\ (x\neq 0)$.

47. Derive the equation of the line in space passing through a given point and parallel to a given vector both in vector and Cartesian form.

48. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

PART E

Answer any ONE question: **$1\times 10=10$**

49. (a) Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$. **6**

(b) Find the value of k if $f(x)=\begin{cases} kx+1, & if\ \ x\leq\pi \\ \cos x, & if\ \ x>\pi \end{cases}$ is continuous at $x=\pi$. **4**

50.(a) Miximise $z=4x+y$ subject to constraints $x+y\leq 50,\ 3x+y\leq 90,\ x\geq 0,\ y\geq 0$ by graphical method. **6**

(b) If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, satisfies the equation $A^2-5A+7I=O$, then find the inverse of A using this equation, where I is the identity matrix of order 2. **4**

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MODEL QUESTION PAPER-2

For Reduced Syllabus 2020-21

MATHEMATICS :SECOND PUC

Subject code: 35

Time: 3 hours 15 minute **Max. Marks: 100**

Instructions:

i. The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii. Use the graph sheet for the question on linear programming in **PART – E**.

PART – A

I. Answer all the questions **10 x 1 = 10**

1. Examine whether the operation $* : Z^+ \to Z^+$ defined by $a*b = |a-b|$, where Z^+ is the set of all positive integers, is a binary operation or not.
2. Find the domain of $\sin^{-1} x$.
3. Construct a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$.
4. If A is a square matrix and $\text{adj}(A) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then find $|A|$.
5. Differentiate $\cos\sqrt{x}$ with respect to x.
6. Evaluate: $\int \sqrt{ax+b}\, dx$.
7. Find the vector components of the vector with initial point (2,1) and terminal point (-5,7).
8. Find the distance of the plane $3x-4y+12z-3=0$ from the origin.
9. Define the objective function in a linear programming problem.
10. If F is an event of a sample space S of an experiment then find $P(S|F)$.

PART – B

II. Answer any Ten questions **10 x 2 = 20**

11. On R $*$ is defined by $a*b = \frac{a+b}{2}$, verify whether $*$ is associative.
12. Evaluate: $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$.
13. Find the equation of the line passing through (1, 2) and (3, 6) using determinants.
14. Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$.
15. Differentiate $\cos^{-1}(\sin x)$ with respect to x.
16. If $y = x^{\sin x}$, x>0. Find $\frac{dy}{dx}$.
17. Find the local maximum value of the function $g(x) = x^2 - 3x$.
18. Evaluate: $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}$ dx.
19. Evaluate: $\int \log_e x \ dx$.
20. Find order and degree of the differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$.
21. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.
22. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the positive direction of the axes.
23. Find the Cartesian equation of the line that passes through the points

(3, -2, -5) and (3, -2, 6).

24. Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black

PART – C

III. Answer any TEN questions **10 x 3 = 30**

25. Show that the relation R in the set of all integers Z defined by $R=\{(a,b): 2\, divides\, a-b\}$ is an equivalence relation.

26. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute that is AB=BA.

27. Find $\frac{dy}{dx}$, if $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$.

28. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for $-1<x<1$ and $x\neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

29. Find the point at which the tangent to the curve $y=\sqrt{4x-3}-1$ has its slope $\frac{2}{3}$

30. Evaluate: $\int \tan^4 x\,dx$.

31. Evaluate: $\int \frac{x}{(x+1)(x+2)}dx$.

32. Evaluate: $\int_0^{\frac{\pi}{4}} \sin 2x\,dx$.

33. Find the area of the region bounded by the curve $y=x^2$ and the line $y=2$.

34. Solve: $y\log y dx - x dy = 0$.

35. Show that the position vectors of the point P which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ration m : n is $\frac{m\vec{b}+n\vec{a}}{m+n}$

36. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ Where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

37. Find the distance between the parallel lines $\vec{r}=\hat{i}+2j-4k+\lambda(2\hat{i}+3j+6k)$ and $\vec{r}=3\hat{i}+3j-5k+\mu(2\hat{i}+3j+6k)$.

38. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag-II

PART – D

Answer any six following questions **6 x 5 = 30**

39. Verify whether the function $f: N\to N$ defined by $f(x)=x^2$ is one-one, onto and bijective.

40. If A= $\begin{bmatrix}0\\1\\2\end{bmatrix}$, B= $\begin{bmatrix}1 & 5 & 7\end{bmatrix}$ verity that $(AB)'=B'A'$.

41. Solve $4x+3y+2z=60$, $2x+4y+6z=90$ and $6x+2y+3z=70$ by a matrix method..

42. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$,, then prove that $(1-x^2)y_2-xy_1-a^2y=0$.

43. A particle moves along the curve $6y = x^3 + 2$, find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

44. Find the integral $\frac{1}{x^2 - a^2}$ with respect to x and hence evaluate $\int \frac{1}{4x^2 - 9}\, dx$.

45. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$ by method of integration.

46. Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$.

47. Derive the equation of a plane in normal form both in vector and Cartesian form

48. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red
(ii) first ball is black and second is red.
(iii) one of them is black and other is red.

PART – E

Answer any ONE of the following question **10 x 1 = 10**

49. a) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ and hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$. 6

b) Find the values of *a* and *b* such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$ is continuous function. 4

50. a) Solve the following linear programming problem graphically:
Minimize and maximize $z = x + 2y$, subject to constraints
$x + 2y \ge 100,\ 2x - y \le 0,\ 2x + y \le 200$ *and* $x, y \ge 0$. 6

a) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfying the equation $A^2 - 4A + I = O$,
Where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Find A^{-1}. 4

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