PRASHANTH'S MATHEMATICS CLASSES

Comprehensive and Superlative coaching

for FIRST and SECOND PU

SPECIAL COACHING PROGRAMME FOR

CET/ JEE-M

II PU MATHEMATICS PRACTICE PROBLEMS

PART-A: (One Mark Questions)

1) A relation R on $A = \{1,2,3\}$ defined by $R = \{(1,1), (1,2), (3,3)\}$ is not symmetric why?
2) Given an example of a relation which is symmetric and transitive but not reflexive.
3) Define bijective function.
4) Given an example of a relation which is symmetric only.
5) State with reason whether the function $h: \{2,3,4,5\} \to \{7,9,11,13\}$ has inverse with $h = \{(2,7), (3,9), (4,11), (5,13)\}$.
6) Given an example of a relation which is reflexive and symmetric but not transitive.
7) Find the number of all one-one functions from set $A = \{1,2,3\}$ to itself.
8) Given an example of a relation defined on set $A = \{1,2,3\}$, which is symmetric but neither reflexive nor transitive.
9) Let * be a binary operation defined on the set of rational numbers Q defined by $a * b = ab + 1$, prove that * is commutative.
10) Let * be a binary operation on the set of natural numbers given by $a * b =$ L. C. M. of a and b, find $5 * 7$.
11) An operator * on Z^+(the set of all non-negative integers) is defined as $a * b = |a - b|$, $\forall a, b \in Z^+$. Is * a binary operation on Z^+?
12) Let * be a operation defined on the set of rational numbers by $a * b = \frac{ab}{4}$, find the identity element.
13) Let * be the binary operation on N given by $a * b =$ LCM of a and b, Find 20*60.
14) Let * be the binary operation on N given by $a * b =$ L. C. M of a and b, find $20 * 16$.
15) Let * be a binary operation defined on Q, by $a * b = \frac{2ab}{5}$. Find the identity element.
16) Operation * is defined by $a * b = a$. Is * a binary operation on Z^+?
17) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

18) Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.
19) Write the domain of $f(x) = \sin^{-1} x$.
20) Write the domain of $f(x) = \cos^{-1} x$.
21) Write the values of x for which $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ holds.
22) Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$.
23) Write the domain of $f(x) = \sec^{-1} x$.
24) Find the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.
25) Write the set of values of x for which $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ holds.
26) Write the principal value branch of $f(x) = \sin^{-1} x$.
27) Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.
28) Write the domain of $f(x) = \tan^{-1} x$.
29) Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.
30) Find the value of $\cot(\tan^{-1} x + \cot^{-1} x)$.
31) Write the range of $f(x) = \sin^{-1} x$ in $[0, 2\pi]$.
32) Find the value of $\cos^{-1}\left(\frac{13\pi}{6}\right)$.
33) Define a diagonal matrix
34) Define a Scalar Matrix.
35) Define a symmetric matrix.
36) Define skew symmetric matrix.
37) Define singular matrix.
38) Define a non singular matrix.
39) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find the value of $|2A|$.
40) Find $|3A|$, if $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$.
41) If $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$, find $|2A\,\text{adj}(A)A^T|$.
42) If $A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, find $|2\,\text{adj}A|$.
43) If A is a square matrix with $|A| = 6$, find the values of $|\text{adj}A.\,A'|$
44) If $A = \begin{pmatrix} 4 & 7 \\ 6 & 5 \end{pmatrix}$ find $|3A|$.
45) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, find $|2AA^T|$.
46) Find $|3A|$ if $A = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$.
47) If A is a square matrix with $|A| = 8$, then find the value of $|\text{adj}A.\,A'|$.
48) If A is a non singular matrix then find $|A^{-1}|$.
49) If A is a square matrix of order 2 and $A^{-1} = \frac{\text{adj}A}{10}$, then find $|3A|$.
50) Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $\frac{1}{2}|-3i + j|$.
51) Construct a 2×3 matrix whose elements are given by $a_{ij} = |i - j|$.
52) Construct a 2×3 matrix whose elements are given by $a_{ij} = |2i + j|$.
53) Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = 2i - 3j$.
54) Construct a 3×3 matrix $A = (a_{ij})$ whose elements are given by $a_{ij} = \frac{i}{j}$.
55) Construct a 2×2 matrix $A = [a_{ij}]$, where $a_{ij} = \left(\frac{i-j}{2}\right)$.
56) Construct a 2×3 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = -\frac{2i}{j}$.

57) Find the values of x for which $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$.

58) Find the values of x for which, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

59) Find the values of x for which, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 16 & 2 \\ 4 & 1 \end{vmatrix}$.

60) What is the number of the possible square matrices of order 3 with each entry 0 or 1?

61) If $\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = \begin{vmatrix} x & 2 \\ -3 & -x \end{vmatrix}$, find the value of x.

62) If $\begin{vmatrix} x & 8 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 2 & 8 \\ 8 & 2 \end{vmatrix}$, find the values of x.

63) If the matrix $\begin{bmatrix} 5-x & 2y-8 \\ 2 & 3 \end{bmatrix}$ is a symmetric matrix, find the values of x and y.

64) Find the values of k if the matrix $\begin{bmatrix} k & 4 \\ 3 & 2 \end{bmatrix}$ has no inverse.

65) Find $\frac{dy}{dx}$, if $y = \sin(x^2+5)$.

66) Find $\frac{dy}{dx}$, if $y = \cos(1-x)$.

67) Find $\frac{dy}{dx}$, if $y = \cos(\sqrt{x})$.

68) If $y = \tan(2x+3)$, find $\frac{dy}{dx}$.

69) If $y = \log(\sin x)$, find $\frac{dy}{dx}$.

70) If $y = \cos(\sqrt{x})$, find $\frac{dy}{dx}$.

71) Find the derivative of $\cos(x^2)$ with respect to x.

72) If $y = \tan(2-3x)$ find $\frac{dy}{dx}$.

73) The function $f(x) = \frac{1}{x-5}$ is not continuous at $x = 5$. Justify the statement.

74) Write the points of discontinuity for the function $f(x) = |x|, -3 \le x < 3$.

75) If $y = e^{3\log x}$, then show that $\frac{dy}{dx} = 3x^2$.

76) Differentiate $\log(\cos e^x)$ w r t to x.

77) The greatest integer function a not differentiable at integral points give reason.

78) Differentiate $\sin\sqrt{x}$ with respect to x.

79) Differentiate $e^{\log_e x}, x > 0$, with respect to x.

80) Find the derivative of $e^{\log(\cos^{-1} x)}$ with respect to x.

81) Evaluate: $\int (2x - 3\cos x + e^x)\, dx$.

82) Evaluate: $\int \sec x(\sec x + \tan x)\, dx$.

83) Find $\int (2x^2 + e^x)\, dx$.

84) Evaluate: $\int (\sin x + \cos x)\, dx$.

85) Evaluate: $\int e^x \left(\frac{x-1}{x^2}\right) dx$.

86) Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$.

87) Evaluate: $\int (1-x)\sqrt{x}\, dx$.

88) Write the integral of $\frac{1}{x\sqrt{x^2-1}}$, $x > 1$ with respect to x.

89) Write the anti-derivative of e^{2x} with respect to x.

90) Evaluate: $\int \operatorname{cosec} x(\operatorname{cosec} x - \cot x)\, dx$.

91) Find the anti-derivative of $x^2\left(1 - \frac{1}{x^2}\right)$ w.r.t x.

92) Evaluate: $\int \tan^2 2x\, dx$.

93) Evaluate: $\int \sin(2+5x)\, dx$.

94) Evaluate: $\int \frac{1-x}{\sqrt{x}}\, dx$.

95) Evaluate: $\int \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) dx$.

96) Evaluate: $\int e^x \sec x(1 + \tan x)\, dx$.

97) Define coplanar vectors.
98) Define negative of a vector.
99) Define Collinear vectors.
100) Define a unit vector.
101) Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$.
102) Find unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$.
103) Find the direction cosines of the vectors $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.
104) Find the direction cosines of a line which makes equal angles with positive co-ordinate axes.
105) Find the DRs of the vector, joining the points $P(2,3,0)$ and $Q(-1,-2,-3)$, from P to Q.
106) If a line makes angle $90^0, 60^0$ and 30^0 with positive direction of x, y and z axis respectively, find its direction cosines.
107) Write the direction cosines of X-axis.
108) Write the direction cosines of Z-axis.
109) Show the vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = -4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear.
110) If the vectors $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $4\hat{\imath} - m\hat{\jmath} - 12\hat{k}$ are parallel find m.
111) Write the vector joining the points $A(2,3,0)$ and $B = (-1,-2,-4)$.
112) Find the angle between the two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 1, |\vec{b}| = 1$ and $\vec{a}.\vec{b} = 1$.
113) If $\vec{a}$ is a non zero vector of magnitude a and $\lambda\vec{a}$ is a unit vector, find the value of λ.
114) Find the vector components of the vector with initial point (2,1) and terminal point (−5,7).
115) For what value of λ, the vectors $\vec{a} = 2\hat{\imath} - 3\lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ are perpendicular to each other?
116) If the vectors $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $8\hat{\imath} - m\hat{\jmath} - 24\hat{k}$ are parallel find m.
117) Find the intercepts cut off by the plane $2x + y - z = 5$.
118) Find the equation of the plane having intercept 4 on the X-axis and parallel to ZOY plane.
119) Find the equation of the plane having intercept 3 on the y axis and parallel to ZOX plane.
120) Find the equation of the plane having intercept −5 on the z-axis and parallel to XOY plane.
121) Find the equation of the plane which makes intercept 1, −1, 2 on the x, y and z axes respectively.
122) Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.
123) Find the equation of the plane with the intercept 2, 3 and 4 on x, y and z axes respectively.
124) Find the direction ratio of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$.
125) Find the equation of the plane having intercept 3 on the x axis and parallel to ZOY plane.
126) Find the distance of the plane $2x + 3y + 6z - 14 = 0$ from the origin.
127) What is the equation of the plane that cuts the coordinate axes at $(a, 0,0), (0, b, 0)$ and $(0,0, c)$.
128) Find the direction ratios of the line $2x = \frac{1-y}{2} = \frac{z+4}{6}$.
129) Find the intercepts cut off by the plane $2x + 3y + 4z = 12$.
130) Find the intercepts cut off by the plane $2x + y + z - 6 = 0$.
131) In linear programming problem, define linear objective function.
132) Define feasible region.
133) Define optimal solution in linear programming problem.
134) Define Feasible region in linear programming problem.
135) Define linear objective function in linear programming problem.
136) Define the term constraints in the LPP.
137) Define the term corner point in the L. P. P.
138) If $P(A) = 0.6, P(B) = 0.3$ and $(A \cap B) = 0.2$, find $P(A/B)$.
139) If $P(A) = 0.8, P(B) = 0.5$ and $P(B/A) = 0.4$ then find $P(A \cap B)$.
140) If $P(A) = \frac{7}{13}, P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, find $P(A/B)$.
141) If $P(A) = \frac{4}{5}$ and $P(B/A) = \frac{2}{5}$, find $P(A \cap B)$.
142) If $P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$ find $P(A \cap B)$ if A and B are independent events.
143) An urn contains 5 red and 2 blackballs. Two balls are randomly selected. If X represents the number of black balls, what are the possible values of X?

144) If $P(B) = 0.5$ and $P(A \cap B) = 0.32$, find $P(A/B)$.
145) A fair die is rolled. Consider events $E = \{2,4,6\}$ and $F = \{1,2\}$. Find $P(E/F)$.
146) If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$ find $P(A \cap B)$.
147) If $P(E) = 0.6$ and $P(E \cap F) = 0.2$ then find $P(E/F)$.
148) A fair die is rolled. Consider the events $E = \{1,3,5\}$ and $F = \{2,3\}$, find $P(E/F)$.
149) If E is an event of a sample space S of an experiment then find $P(S/F)$.
150) If $P(A) = 0.3$, P(not B) = 0.4 and A and B are independent events, find P(A and not B).

PART-B: (Two Mark Questions)

1) Verify whether the operation * defined on Q by $a * b = \frac{ab}{2}$ is associative or not.
2) Verify whether the operation * defined on Q by $a * b = \frac{ab}{4}$ is associative or not.
3) A binary operation * on the set $\{1,2,3,4,5\}$ is defined by $a * b = \max\{a, b\}$. Write the operation table for the operation *.
4) Define binary operation on a set. Verify whether the operation * defined on Z, by $a * b = ab + 1$ is commutative or not.
5) A binary operation $\wedge$ on the set $\{1,2,3,4,5\}$ defined by $a \wedge b = \min\{a, b\}$, write the operation table for operation $\wedge$.
6) Define binary operation on a set. Verify whether the operation * defined on Z, by $a * b = ab + 1$ is binary or not.
7) Show that the relation R in the set of integers given by $R = \{(a, b): 5 \text{ divides } (a - b)\}$ is symmetric and transitive.
8) A relation R is defined on the set $A = \{1,2,3,4,5,6\}$ by $R = \{(x, y): y \text{ is divisible by } x\}$. Verify whether R is symmetric reflexive or not. Give reason.
9) If $f : R \to R$ is given by $f(x) = (3 - x^3)^{1/3}$ then find $(f \circ f)(x)$.
10) Find $f \circ g$, if $f: R \to R$ and $g: R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$.
11) Find the gof and fog if $f(x) = 8x^3$ and $g(x) = x^{1/3}$.
12) If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.
13) Prove that the greatest integer function, $f: R \to R$ defined by $f(x) = [x]$, where $[x]$ indicates the greatest integer not greater than x, is neither one-one nor onto.
14) If $f: R \to R$ defined by $f(x) = 1 + x^2$, then show that f is neither $1 - 1$ nor onto.
15) Show that if $f: A \to B$ and $g: B \to C$ are one-one, then $g \circ f: A \to C$ is also one-one.
16) Show that the function $f: N \to N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$, is onto but not one-one.
17) Prove that $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in R$.
18) Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1,1]$.
19) Prove that $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.
20) Prove that $\sin^{-1}(2x\sqrt{1 - x^2}) = 2 \sin^{-1} x$, for $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.
21) Prove that $2 \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.
22) Show that $\sin^{-1}(2x\sqrt{1 - x^2}) = 2 \cos^{-1} x,\ \frac{1}{\sqrt{2}} \le x \le 1$.
23) Prove that $2 \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.
24) Write the simplest form of $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), 0 < x < \pi$.
25) Write $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$ in the simplest form.
26) Write the simplest form of $\tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right), 0 < x < \pi$.
27) Find the simplest form of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.
28) Write the simplest form of $\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right), 0 < x < \frac{\pi}{2}$.
29) Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right), x > 1$ in the simplest form.
30) Write the function $\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$ $x \neq 0$ in the simplest form.

31) Find the simplest form of $\cot^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$ $0<x<\pi$.

32) Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

33) Evaluate $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$.

34) Evaluate $\sin\left\{\frac{\pi}{3}+\sin^{-1}(-1)\right\}$.

35) Find $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

36) If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$, then find the value of x.

37) Find the value of $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$.

38) Simplify: $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right]$, if $\frac{a}{b}\tan x>-1$.

39) Evaluate $\sin\left\{\frac{\pi}{6}+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right\}$.

40) Solve the equation, $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\ (x>0)$.

41) If $\sin\left\{\sin^{-1}\frac{1}{5}+\cos^{-1}x\right\}=1$, find x.

42) Solve $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$.

43) Solve the equation: $\sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x$.

44) Find the equation of line joining (3,1) and (9,3) using determinants.

45) Find the equation of line joining (1,2)and (3,6) using determinants.

46) Find the equation of the line passing through (−3,2) and (3,1) using the determinants.

47) Find the equation of line passing through the points (3,2) and (−1,−3) by using determinants.

48) Find the area of the triangle whose vertices are (3,8), (−4,2) and (5,1) using determinants.

49) Find the area of the triangle whose vertices are (1,8), (−4,3) and (3,1) using determinants.

50) Find the area of the triangle whose vertices are (1,3), (2,5) and (7,5) using determinant.

51) Find the area of the triangle whose vertices are (2,0), (−1,0) and (0,3) by using determinants.

52) If each element of a row is expressed as sum of two elements then verify for a third order determinant that the determinant can be expressed as sum of two determinants.

53) If area of the triangle with vertices (−2,0), (0,4) and (0, k) is 4 sq units, find the values of k using determinants

54) If area of the triangle with vertices (−1,0), (2,4) and (0, k) is 4 sq units, find the values of k using determinants

55) Find values of k, if area of triangle is 4 sq. units and vertices are (k, 0), (4,0), (0,2) using determinants.

56) If the area of the triangle with vertices (2, −6) , (5,4) and (k, 4) is 35sq. units. Find the value of k using determinant .

57) Let A(1,3), B(0,0) and C(k, 0) be the vertices of triangle ABC of area 3 sq. units. Find k using determinant method.

58) If the matrix $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ and $A^2=kA$, then write the value of k.

59) If $\sqrt{x}+\sqrt{y}=\sqrt{10}$, show that $\frac{dy}{dx}+\sqrt{\frac{y}{x}}=0$.

60) If $y+\sin y=\cos x$, find $\frac{dy}{dx}$.

61) Find $\frac{dy}{dx}$, if $x^2+xy+y^2=100$.

62) Find $\frac{dy}{dx}$, if $ax+by^2=\cos y$.

63) If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, prove that $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$.

64) If $y=\sin(\log_e x)$, prove that $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{x}$.

65) Find $\frac{dy}{dx}$, if $2x+3y=\sin y$.

66) Find $\frac{dy}{dx}$, if $\sin^2 x+\cos^2 y=k$, where k is constant.

67) Find the derivative of $\sqrt{x}+\sqrt{y}=9$ at (4,9).

68) If $\sqrt{x}+\sqrt{y}=\sqrt{5}$, prove that $\frac{dy}{dx}=-\frac{3}{2}$ when $x=4$ and $y=9$.

69) Find $\frac{dy}{dx}$, if $y=(\log x)^{\cos x}$.

70) Find $\frac{dy}{dx}$, if $y=x^{\sin x}, x>0$.

71) If $y=x^x$, find $\frac{dy}{dx}$.

72) Differentiate $\left(x+\frac{1}{x}\right)^x$ with respect to x.

73) Find the derivative of $x^x-2^{\sin x}$ with respect to x.

74) If $y=(\sin^{-1}x)^x$, find $\frac{dy}{dx}$.

75) Find $\frac{dy}{dx}$, if $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$.

76) If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0<x<1$ find $\frac{dy}{dx}$.

77) Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to x.

78) If $y=\tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right], \frac{-1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$ then find $\frac{dy}{dx}$.

79) If $y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then prove that $\frac{dy}{dx}=\frac{1}{2}$.

80) If $x=at^2, y=2at$ show that $\frac{dy}{dx}=\frac{1}{t}$.

81) Find $\frac{dy}{dx}$, if $x=4t$ and $y=\frac{4}{t}$.

82) Find $\frac{dy}{dx}$, if $y=\log_7(\log x)$.

83) Find the derivative of $(3x^2-7x+3)^{5/2}$ with respect to x.

84) Prove that greatest integer function defined by $f(x)=[x], 0<x<3$ is not differentiable at $x=1$.

85) Differentiate $(x^2-5x+8)(x^2+7x+9)$ with respect to x, by logarithmic differentiation.

86) Show that the function defined by $f(x)=|\cos x|$ is continuous function.

87) Approximate $\sqrt{36.6}$ by using differential.

88) Using differentials, find the approximate value of $\sqrt{49.5}$.

89) Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

90) Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 3%.

91) If the radius of a sphere is measured as 9 cm with an error, 0.03 cm, then find the approximate error in calculating its volume.

92) If the radius of a sphere is measured as 9 cm with an error, 0.03 cm, then find the approximate error in calculating its Surface area.

93) Find the approximate change in the volume V of a cube of a side x meters caused by increasing by 4%.

94) If the radius of a sphere is measured as 7 cm with an error, 0.02 m, then find the approximate error in calculating its volume.

95) Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly increasing.

96) Find the interval in which the function f given by $f(x)=2x^2-3x$ is strictly increasing.

97) Find the slope of the tangent to the curve $y=\frac{x-1}{x-2}, x\neq 2$ at $x=10$.

98) Find the equation of the tangent to the parabola $y^2=4ax$ at $(at^2, 2at)$.

99) Find a point on the curve $y=x^3-11x+5$ at which the tangent is $y=x-11$..

100) Find the slope of the tangent to the curve $y=x^3-3x+2$ at the point whose x co ordinate is 3.

101) Find the local maximum value of the function $g(x)=x^3-3x$.

102) Verify Rolle's Theorem for the function $y=x^2+2x-8,\ x\in[-4,2]$.

103) Integrate $\sin x.\sin(\cos x)$ with respect to x.

104) Evaluate: $\int\frac{x^2}{1-x^6}dx$.

105) Evaluate: $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$.

106) Find $\int x^2\log x\,dx$.

107) Evaluate: $\int\frac{\sin^2 x}{1+\cos x}dx$.

108) Evaluate: $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$.

109) Evaluate: $\int \frac{dx}{x-\sqrt{x}}$

110) Integrate $\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$ with respect to x.

111) Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}$.

112) Evaluate: $\int \log x dx$.

113) Evaluate: $\int \log(\sin x).(\cot x)dx$.

114) Find $\int \frac{dx}{(x+1)(x+2)}$.

115) Evaluate: $\int \frac{x^2}{x^6+1} dx$.

116) Evaluate: $\int x^n \log x dx$.

117) Find $\int \frac{2x dx}{(x+1)(x+2)}$.

118) Evaluate: $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$.

119) Evaluate: $\int \frac{dx}{x^2-6x+13}$

120) Evaluate: $\int \sin 3x \cos 4x \, dx$.

121) Find $\int e^x (\sec x)(1+\tan x)dx$.

122) Evaluate: $\int \sin^3 x \, dx$.

123) Evaluate: $\int \frac{\cos 2x}{(\sin x+\cos x)^2} dx$.

124) Evaluate: $\int \tan^{-1} x \, dx$.

125) Evaluate: $\int \frac{3x^2}{1+x^6} dx$.

126) Evaluate: $\int e^x \left(\frac{x-1}{x^2}\right) dx$

127) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$.

128) Evaluate: $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

129) Evaluate: $\int_1^e \frac{1}{x} dx$

130) Evaluate: $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$

131) Find $\int_0^{\pi/2} \cos 2x \, dx$.

132) Find $\int_2^3 \frac{x}{x^2+1} dx$.

133) Find the order and degree of the differential equation, $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + y = 0$.

134) Find the order and degree(if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$.

135) Find the order and degree of the differential equation, $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

136) Find the order and degree of the differential equation, $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$.

137) Find the order and degree, if the differential equation. $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$.

138) Find the order and degree of the differential equation, $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 y = 0$.

139) Prove that the differential equation $x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$ is a homogeneous differential equation of degree 0.

140) Find the order and degree of the differential equation, $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$.

141) Find the order and the degree of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

142) Form the differential equation of the family of curves $\frac{x}{a} + \frac{y}{b} = 1$, by eliminating the constants "a" and "b".

143) Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

144) Form the differential equation of the family of parabolas having vertex at origin and axis along positive X-axis.

145) Find the angle θ between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

146) If the position vectors of the points A and B respectively are $\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{j}-\hat{k}$ find the direction cosines of $\overrightarrow{AB}$.

147) Find a vector of magnitude 3units in the direction of the vector, $\vec{a}=5\hat{i}-3\hat{j}+2\hat{k}$.

148) Find a vector in the direction of the vector $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$ that has magnitude 7 units.

149) If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

150) If either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ then $\vec{a}.\vec{b}=0$, but the converse need not be true. Justify your answer with an example.

151) Obtain the projection of the vectors $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

152) Find the projection of the vectors $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$..

153) Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio 2:1 (i)internally (ii)externally.

154) If the position vectors of the points A and B respectively are $\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{j}-\hat{k}$ find the direction cosines of $\overrightarrow{AB}$.

155) Find a vector of magnitude 8 units in the direction of the vector, $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$.

156) Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the positive direction of the axes.

157) For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}.\vec{b}|\le|\vec{a}||\vec{b}|$.

158) If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+a)=4$ find $|\vec{x}|$.

159) Find $|\vec{a}|$ and $|\vec{b}|$ if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$ and $|\vec{a}|=8|\vec{b}|$.

160) If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$, then find $|\vec{x}|$

161) If two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}|=2, |\vec{b}|=3$ and $\vec{a}.\vec{b}=4$, find $|\vec{a}-\vec{b}|$.

162) If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$, find $|\vec{x}|$.

163) Find $|\vec{b}|$, if $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$.

164) Let $|\vec{a}|=3, |\vec{b}|=\frac{\sqrt{2}}{3}$ and $|\vec{a}\times\vec{b}|=1$, Find the angle between $\vec{a}$ and $\vec{b}$.

165) Find the area of the parallelogram whose adjacent sides determine by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

166) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

167) Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

168) Find the area of parallelogram whose adjacent sides are given by the vectors $\vec{a}=4\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=-\hat{i}-\hat{j}+\hat{k}$.

169) Find k if the vectors $\hat{i}+3\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}-\hat{k}$ and $k\hat{i}+7\hat{j}+3\hat{k}$ are coplanar.

170) Show that the points $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$ are the vertices of a right angled triangle.

171) Show that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2(\vec{a}\times\vec{b})$.

172) Find the vector equation of the line, passing through the points $(1,0,-2)$ and $(2,1,6)$

173) Find the vector equation of the line passing through the points $(-1,0,2)$ and $(3,4,6)$.

174) Find the vector and Cartesian equation of the line that passing through the points $(3,-2,-5)$ and $(3,-2,6)$.

175) Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.

176) Find the angle between the pair of lines $\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$, $\vec{r}=\hat{i}-2\hat{j}+\mu(3\hat{i}-2\hat{j}-6\hat{k})$.

177) Find the angle between the pair of lines $\vec{r}=3\hat{i}+\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}-2\hat{k})$, $\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}-2\hat{j}+6\hat{k})$.

178) Find the angle between the pair of lines $\vec{r}=3\hat{i}+5\hat{j}-\hat{k}+\lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}=7\hat{i}+4\hat{k}+\mu(2\hat{i}+2\hat{j}+2\hat{k})$.

179) Find the distance of the point $(2,3,-5)$ from the plane $\vec{r}.(\hat{i}+2\hat{j}-2\hat{k})=9$.

180) Find the distance of a point $(2,5,-7)$ from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$.

181) Find the distance between the parallel lines $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$, $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$.

182) Find the coordinates of the point where the line through the points A(3,4,1) and B(5,1,6) crosses XY- plane.

183) Find the distance of the point $(1,3,-6)$ from the plane $\vec{r}.(\hat{3i}+2\hat{j}+2\hat{k})=9$.

184) Find equation of the plane passing thru the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z-5=0$ and the point $(1,1,1)$.

185) Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$ and $x+y-z=0$ and the point $(2,2,1)$.

186) A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F is the event 'the number appearing is

even', then prove that E and F are independent events.

187) Find the probability distribution of the number of tails in simultaneous tosses of three coins.

188) Find the probability distribution of number of heads in two tosses of a coin.

189) Let X denotes the number of hours you study during a randomly selected school day. The probability that X can take the values of x, has the following form, where k is some constant .

190) Find k given Probability distribution of x is

x	0	1	2	3	4
$P(x_1)$	0.1	k	2k	2k.	k

191) Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

192) Two coins are tossed once, find (E/F) where E: no tail appears, F: no head appears.

193) Given that the event A and B are such that $P(A) = \frac{1}{2}, P(A \cap B) = \frac{3}{5}$ and $P(B) = k$, find k if A and B are independent.

194) Assume that each born child is equally likely to be a boy or girl. If a family has two children. What is the conditional probability of both are girls given that at least one is girl?

195) A fair die is rolled. Consider the events $E = \{1,3,4\}, F = \{2,3\}$ and $G = \{2,3,4,5\}$. Find (i)P(E/F) (ii)P(E/G).

196) Find the probability distribution of number of heads in two tosses of a coin.

PART-C: (Three Mark Questions)

1) Show that the relation R in the set Z of integers given by $R = \{(a,b): 2 \text{ divides } a - b\}$ is an equivalence relation.
2) Show that the relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b): |a - b| \text{is even}\}$ is an equivalence relation.
3) Prove that the relation R in the set of integers Z defined by $R = \{(x,y): x - y \text{ is an integer}\}$ is an equivalence relation.
4) Determine whether is the relation R in the set $A = \{1,2,3,4,5,..,13,14\}$ defined as $R = \{(x,y): 3x - y = 0\}$ is reflexive, symmetric and transitive.
5) Show that the relation R in $\mathcal{R}$ (set of real numbers) defined as $R = \{(a,b): a \le b\}$, is reflexive and transitive but not symmetric.
6) Show that the relation R in the set of real numbers R defined as $\{(a,b): a \le b^2\}$ is neither reflexive nor symmetric nor transitive.
7) Show that the relation R in the set $A = \{x: x \in z \text{ and } 0 \le x \le 12\}$ given by $R = \{(a,b)/|a - b| \text{ is multiple of } 4\}$ is an equivalence relation?
8) Determine whether the relation R in the set $A = \{1,2,3,4,5,6\}$ as $R = \{(x,y): y \text{ divisible by } x\}$ is reflexive, symmetric and transitive.
9) Let Z be the set of all integers and R is the relation on Z defined as $R = \{(a,b): a,b \in Z \text{ and } a - b \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.
10) Find gof and fog if $f: R \to R$ $g: R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that gof $\neq$ fog.
11) Verify whether the function, $f: A \to B$ where $A = R - \{3\}$ and $B = R - \{1\}$, defined by $f(x) = \left(\frac{x-2}{x-3}\right)$ is one – one and onto or not. Give reason.
12) If $*$ is a binary operation defined on $A = N \times N$, by $(a,b) * (c,d) = (a + c, b + d)$, prove that $*$ is both commutative and associative. Find the identity if it exists.
13) Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, when $xy < 1$.
14) Prove that $\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), |x| < \frac{1}{\sqrt{3}}$.
15) Solve for x, $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}, x > 0$.
16) Prove that $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x),\ x \in \left[\frac{1}{2}, 1\right]$.
17) Find the value of x, if $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

18) Write $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ in the simplest form.

19) Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

20) Simplify: $\tan^{-1}\left[\frac{2\cos x - 3\sin x}{3\cos x + 2\sin x}\right]$, $\frac{2}{3}\tan x > -1$.

21) Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right], |x| < 1, y > 0$ and $xy < 1$.

22) Write $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

23) Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

24) Solve: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.

25) For any square matrix A with real number, prove that $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix.

26) Express $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as sum of a symmetric and skew symmetric matrix.

27) Express $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

28) Express matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

29) Express $\begin{bmatrix} 6 & 5 \\ 1 & 8 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

30) By using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$.

31) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find A^{-1} by elementary operations.

32) Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.

33) By using elementary transformation, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

34) By using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$.

35) If A and B are invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$.

36) If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, that is $AB = BA$.

37) Find the values of x, y and z in the following matrices $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$.

38) Find the value of x and y in $\begin{pmatrix} x+2y & 2 \\ 4 & x+y \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} = 0$ where O is null matrix.

39) Find the value of x and y $\begin{vmatrix} x+y & 3 \\ x-y & -6 \end{vmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix}$.

40) If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, prove that $\frac{dy}{dx} = \frac{1}{2(1+x^2)}$.

41) If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ find $\frac{dy}{dx}$.

42) If $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ find $\frac{dy}{dx}$.

43) If $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then prove that $\frac{dy}{dx} = \frac{1}{2}$.

44) If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$, then Find $\frac{dy}{dx}$.

45) If $x = at^2$ and $y = 2at$, find $\frac{dy}{dx}$.

46) If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, Prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$.

47) Find $\frac{dy}{dx}$, if $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$.

48) If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$ then prove that $\frac{dy}{dx} = -\cot\left(\frac{\theta}{2}\right)$.

49) Find $\frac{dy}{dx}$ if $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

50) If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

51) Differentiate $(\log x)^{\cos x}$ with respect to x.

52) Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.

53) Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$ with respect to x.

54) Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t. x.

55) If $y^x + x^y = a^b$, find $\frac{dy}{dx}$.

56) Differentiate $\sin^2 x$ with respect to $e^{\tan x}$.

57) Verify Mean value theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$.

58) Verify Mean value Theorem for the function $f(x) = x^2 + 4x - 3$, in the interval $[2,6]$.

59) Verify mean value theorem for the function $f(x) = x^2$ in the interval $[2,4]$

60) Verify Rolle's theorem for the function $y = x^2 + 2, [-2,2]$.

61) Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4,2]$.

62) Verify Mean value theorem for the function $f(x) = x^3 - 5x^2 - 3x$, in the interval $[a, b]$ where $a = 1$ and $b = 3$. Find all $c \in (1,3)$ for which $f'(c) = 0$.

63) Find two numbers, whose sum is 24 and whose product is as large as possible.

64) Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

65) Find two positive numbers whose sum 15 is and the sum of whose squares is minimum.

66) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

67) Find the absolute maximum value and the absolute minimum value of the function $f(x) = \sin x + \cos x, x \in [0, \pi]$.

68) Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$.

69) Using differentials, find the approximate value of $(25)^{1/3}$.

70) Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is (i) strictly increasing; (ii)strictly decreasing.

71) Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly increasing and strictly decreasing.

72) If $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a)strictly increasing (b) strictly decreasing.

73) If a function $f(x)$ is differentiable at $x = c$. prove that it is continuous at $x = c$.

74) Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

75) Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$.

76) Find the equation of tangent and normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0,5)$.

77) Evaluate: $\int_0^2 e^x \, dx$ as the limit of sum.

78) Evaluate: $\int_0^5 (x + 1)dx$ as a limit of sum.

79) Find $\int e^x \left(\frac{1-\sin x}{1+\cos x}\right) dx$.

80) Find $\int \frac{(x^2+1)e^x}{(x+1)^2} dx$

81) Evaluate: $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

82) Evaluate: $\int \frac{x}{(x+1)(x+2)} dx$.

83) Find $\int \frac{x^2}{(x+1)(x+2)} dx$.

84) Evaluate: $\int \frac{2x}{x^2+3x+2} dx$.

85) Evaluate: $\int \frac{dx}{x(x^n+1)}$.

86) Evaluate: $\int \frac{dx}{x(x^n-1)}$.

87) $\int \frac{(x-1)e^x}{(x)^2} dx$.

88) Find $\int \frac{x}{(x-1)(x+3)} dx$.

89) Evaluate: $\int \frac{\cos x \, dx}{(1-\sin x)(2-\sin x)}$.

90) Integrate $x^2 e^x$ with respect to x.

91) Evaluate: $\int \tan^{-1} x \, dx$.

92) Evaluate: $\int x \tan^{-1} x \, dx$.

93) Evaluate: $\int e^x \sin x dx$.

94) Evaluate: $\int \cot^{-1} x\, dx$.
95) Evaluate: $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$.
96) Evaluate: $\int e^{-x} \sin 2x\, dx$.
97) Find $\int e^x \cos x\, dx$..
98) Evaluate: $\int \sin(ax+b).\cos(ax+b)dx$.
99) Evaluate: $\int \sin 3x \cos 4x dx$.
100) Find the anti- derivative of $f(x)$ given by $f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$.
101) Evaluate: $\int \frac{dx}{x + x\log x}$.
102) Find $\int \frac{1}{1+\cot x} dx$.
103) Evaluate: $\int \frac{(1+\log x)^2}{x} dx$.
104) Evaluate: $\int \frac{x+2}{2x^2+6x+5} dx$.
105) Integrate $\frac{\sin x}{\sin(a+x)}$ with respect to x.
106) Evaluate: $\int \frac{1}{1+\tan x} dx$.
107) Evaluate: $\int \frac{1}{1-\tan x} dx$.
108) Determine the area of the region bounded by the $y^2 = x$ and the lines $x = 1, x = 4$ and the x-axis in the first quadrant.
109) Find the area lying between the curve $y^2 = 4x$ and the line $y = 2x$.
110) Find the area of the region bounded by $y^2 = 9x$, $x = 2, x = 4$ and the x-axis in the first quadrant.
111) Find the area of the region bounded by the curve $y = x^2$ and the lines $y = 4$.
112) Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.
113) Find the area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 2$.
114) Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 4, x = 9$ and the x-axis in the first quadrant.
115) Find the area of the region bounded by the curve $y^2 = x$ and the line $y = 2$.
116) Find the area bounded by the parabola $y^2 = 5x$ and the line $y = x$.
117) Find the area between the curves $y = x^2$ and $y = x$.
118) Find the area of the circle $x^2 + y^2 = 4$ bounded by the lines $x = 0$ and $x = 2$ which is lying in the first quadrant.
119) Find the area of the region bounded by the curve $y = x^2 + 2$, $y = x, x = 0$ and $x = 3$.
120) Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.
121) Form the differential equation of the family of circles touching the X-axis at origin.
122) Form the differential equation of the family of circles touching the y-axis at origin.
123) Form the differential equation representing family of curves $y = a\sin(x+b)$ where a and b are arbitrary constants.
124) Form the differential equation representing the family of curves $y = a\cos(x+b)$ where a and b arbitrary constants.
125) Find the differential equation representing family of curves $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are arbitrary constants.
126) Form the differential equation representing the family of curves $y = mx$, where m is arbitrary constant.
127) Form the differential equation of the family of circles having centre on y-axis and radius 3 units.
128) Find the equation of the curve passing through the point $(-2,3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.
129) In a bank, principal p increases continuously at the rate of 5% / year. Find the principal in terms of time t.
130) Prove that the equation $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is a homogeneous differential equation.
131) Find the equation of the curve passing thru the point (1,1) whose differential equation is $xdy = (2x^2+1)dx, (x \neq 0)$.
132) Find a unit vector perpendicular to the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$.
133) Find a vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$.
134) Find the unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = \hat{i}+3\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}+4\hat{j}-3\hat{k}$.
135) Find a vector perpendicular to each of the vectors $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}, \vec{b} = 3\hat{i}+5\hat{j}-2\hat{k}$ which has magnitude 10 units.

136) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu = (\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$ if $|\vec{a}| = 1, |\vec{b}| = 4$ and $|\vec{c}| = 2$.

137) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

138) If two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 2, |\vec{b}| = 3$, and $\vec{a}.\vec{b} = 4$, find $|\vec{a}-\vec{b}|$.

139) Show that the position vector of the point P, which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio m:n is $\left(\frac{m\vec{a}+n\vec{b}}{m+n}\right)$.

140) If $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}, \vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find λ.

141) If $\vec{a} = 2\hat{i}-3\hat{j}+3\hat{k}, \vec{b} = -\hat{i}+3\hat{j}+\hat{k}$ and $\vec{c} = -\hat{i}+\hat{j}+5k$ such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find λ.

142) Find the area of the triangle ABC where position vectors of A,B and C are $\hat{i}-\hat{j}+2\hat{k}$, $2\hat{j}+\hat{k}$ and $\hat{j}+3\hat{k}$ respectively.

143) Find the area of a triangle having the points A(1,1,2), B(1,2,3) and C(2,3,1) as its vertices using vector method.

144) Find the area of triangle with vertices A(1,1,2), B(2,3,5), C(1,5,5).

145) The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

146) Find λ, if $\vec{a} = \hat{i}+3\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}-\hat{k}$ and $\vec{c} = \lambda\hat{i}+7\hat{j}+3\hat{k}$ are coplanar.

147) Show that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, if $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

148) Prove that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$.

149) Prove that $[\vec{a}, \vec{b}, \vec{c}+\vec{d}] = [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}]$.

150) Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k}$, $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

151) Find x, such that the four points A(3,2,1), B(4,x,5), C(4,2,−2) and D(6,5,−1) are coplanar.

152) For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ prove that $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}] = 0$.

153) If $\vec{a} = 2\hat{i}-3\hat{j}+4\hat{k}, \vec{b} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c} = 3\hat{i}+4\hat{j}-\hat{k}$ then find $\vec{a}\cdot(\vec{b}\times\vec{c})$ and $(\vec{a}\times\vec{b})\cdot\vec{c}$.

154) If $\vec{a} = -4\hat{i}-6\hat{j}-\lambda\hat{k}$, $\vec{b} = -\hat{i}+4\hat{j}+3\hat{k}$, $\vec{c} = -8\hat{i}-\hat{j}+3\hat{k}$ are coplanar, find λ.

155) Show that the points A(−1,4,−3), B(3,2,−5), C(−3,8,−5) and D(−3,2,1) are coplanar.

156) Find the equation of the line passing through the points (−1,0,2) and (3,4,6) in both vector and Cartesian forms.

157) Find the Cartesian and vector equation of the line that passes through the points (3,−2,−5) and (3,−2,6).

158) Find the equation of the line which passes through the point (1,−2,4) and is parallel to the vector $4\hat{i}-2\hat{j}+2\hat{k}$, both in vector form and Cartesian form.

159) Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$ both in vector form and Cartesian form.

160) Find the Cartesian and vector equation of the line that passes through the points (4,2,6) and (1,−2,7).

161) Find the Cartesian and vector equation of the line that passes through the points (1,−2,6) and (4,1,−3).

162) Find the distance of a point (1,4,−2) from the plane $\vec{r}.(\hat{i}+2\hat{j}+4\hat{k}) = 6$.

163) Find the distance of a point (2,5,−3) from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k}) = 4$.

164) Find the equation of the plane passing through the intersection of the planes $3x-y+2z=0, x+y+z+2=0$ and the point (2,2,1).

165) Find equation of the plane passing thru the line of intersection of $x+2y-z=2$ and $2x-3y+2z-7=0$ and perpendicular to $3x+y-2z-1=0$.

166) Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x+2y-11z=3$.

167) Find the equation of the plane that contains the point (1,−1,2) and is perpendicular to each of the planes $2x+3y-2z=5$ and $x+2y-3z=8$.

168) Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$.

169) Find the distance between the lines $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r} = (3\hat{i}+3\hat{j}-5\hat{k})+\mu(2\hat{i}+3\hat{j}+6\hat{k})$.

170) Find the shortest distance between the lines $\vec{r} = \hat{i}+2\hat{j}+\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = 2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$.

171) Find the distance between the lines $\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+m(\hat{i}+3\hat{j}+2\hat{k})$ and $\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+n(\hat{i}+3\hat{j}+2\hat{k})$.

172) Find the distance between lines $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + m(2\hat{i} - 3\hat{j} - 4\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + n(2\hat{i} - 3\hat{j} - 4\hat{k})$.

173) Find the shortest distance between $l_1: \vec{r} = \hat{i} + \hat{j} + k + \lambda(2\hat{i} + \hat{j} + \hat{k})$ and $l_2: \vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(3\hat{i} + 5\hat{j} - 2\hat{k})$.

174) Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.

175) A die is tossed thrice. Find the probability of getting an odd number at least once.

176) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident are 0.01,0.03 and 0.15 respectively. One of the insured people meets with an accident. What is the probability that he is a scooter driver?

177) Given that two number appearing on throwing two dice are different. Find the probability of the event the sum of numbers on the dice is 4.

178) Consider the experiment of tossing two fair coins simultaneously, find the probability that both are head given that at least one of them is a head.

179) Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. Find the probability that it is actually head.

180) A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

181) Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

182) Two groups are competing for the position on the board of directors of a corporation. The probability of I and II groups will win are 0.6 and 0.4 respectively. Further, if I group wins, the probability of introducing a new product is 0.7 and corresponding probability is 0.3 if the II group wins. Find the probability that new product introduced was by the II group.

183) A die is thrown. If E is the event "the number appearing is a multiple of 3" and F be the event. "The number appearing is even". Then find whether E and F are independent?

184) Find the mean of the number obtained on throw of an unbiased die.

PART-D: (FIVE Mark Questions)

1) Prove that the function, $f: N \to Y$ defined by $f(x) = x^2$, where $Y = \{y: y = x^2, x \in N\}$ is invertible. Also write the inverse of f(x).

2) Prove that the function, $f: R \to R$ defined by $f(x) = 5x + 7$ is invertible and find the inverse of f.

3) Prove that the function $f: N \to Y$ defined by $f(x) = 4x + 3$, where $Y = \{y : y = 4x + 3, x \in N\}$ is invertible. Also write the inverse of f(x).

4) Let R_+ be the set of all non-negative real numbers. Show that the function $f : R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$ is invertible. Also write the inverse of f.

5) Let $f : N \to R$ defined by $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$, where S is the range of function f is invertible. Find the inverse of f.

6) Consider $f : R^+ \to [5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, show that f is invertible with $f^{-1}(y) = \left\{\frac{\sqrt{y+6}-1}{3}\right\}$.

7) If $f : A \to A$ defined by $f(x) = \frac{4x+3}{6x-4}$, where $A = R - \left\{\frac{2}{3}\right\}$, show that f is invertible and $f^{-1} = f$.

8) Let $A : R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $(x) = \frac{x-2}{x-3}$. Show that f is invertible and write the inverse of f .

9) S.T. $f : [-1,1] \to R$ defined by $f(x) = \frac{x}{x+2}$ is invertible and write the inverse of f .

10) Let $f : W \to W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here W is the set of all whole numbers.

11) Let $f : R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function defined by $(x) = \frac{4x}{3x+4}$. Show that f is invertible and write the inverse of f .

12) Let $f : R - \left\{-\frac{5}{2}\right\} \to R - \left\{\frac{3}{2}\right\}$ be a function defined by $(x) = \frac{3x-5}{2x+5}$. Show that f is invertible and write the inverse of f .

13) If $A=\begin{bmatrix}1&2&3\\3&-2&1\\4&2&1\end{bmatrix}$, that show that $A^3-23A-40I=0$.

14) If $A=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}, B=\begin{bmatrix}3&-1&2\\4&2&5\\2&0&3\end{bmatrix}$ and $C=\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$ then compute $(A+B)$ and $(B-C)$. Also verify $A+(B-C)=(A+B)-C$.

15) If $A=\begin{bmatrix}0&6&7\\-6&0&8\\7&-8&0\end{bmatrix}, B=\begin{bmatrix}0&1&1\\1&0&2\\1&2&0\end{bmatrix}$ and $C=\begin{bmatrix}2\\-2\\3\end{bmatrix}$, calculate AC, BC and $(A+B)C$. Also verify $(A+B)C=AC+BC$.

16) If $A=\begin{bmatrix}1&2\\2&1\end{bmatrix}, B=\begin{bmatrix}2&0\\1&3\end{bmatrix}$ and $C=\begin{bmatrix}1&1\\2&3\end{bmatrix}$, calculate AC, BC and (A+B)C. Also verify that $(A+B)C=AC+BC$.

17) If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=0$.

18) If $A=\begin{bmatrix}-2\\4\\5\end{bmatrix}, B=\begin{bmatrix}1&3&-6\end{bmatrix}$, verify that $(AB)'=B'A'$.

19) Verify $(B+C)A=BA+CA$ if $A=\begin{bmatrix}2&3\\4&-5\end{bmatrix}, B=\begin{bmatrix}3&8\\11&21\end{bmatrix}$ and $C=\begin{bmatrix}7&13\\5&19\end{bmatrix}$.

20) If $A=\begin{bmatrix}2&0&1\\0&-3&0\\0&0&4\end{bmatrix}$, verify $A^3-3A^2-10A+24I=0$, where O is zero matrix of order 3×3.

21) If $A=\begin{bmatrix}2&3&4\\0&-2&1\\3&-1&2\end{bmatrix}, B=\begin{bmatrix}2&0&-3\\4&0&-1\\3&4&5\end{bmatrix}$ and $C=\begin{bmatrix}5&6&7\\-1&2&3\\4&-5&4\end{bmatrix}$. Prove that $A(BC)=(AB)C$.

22) If $A=\begin{bmatrix}1&2&3\\-4&5&6\\7&8&0\end{bmatrix}, B=\begin{bmatrix}2&3&4\\5&-3&0\\4&5&-3\end{bmatrix}$ and $C=\begin{bmatrix}2&3&-1\\4&5&6\\-1&2&3\end{bmatrix}$. Prove that $A(B+C)=AB+AC$.

23) If $A=\begin{bmatrix}1&1&-1\\2&0&3\\3&-1&2\end{bmatrix}, B=\begin{bmatrix}1&3\\0&2\\-1&4\end{bmatrix}$ and $C=\begin{bmatrix}1&2&3&-4\\2&0&-2&1\end{bmatrix}$ then find A(BC) and (AB)C. Show that $A(BC)=(AB)C$.

24) If $A=\begin{bmatrix}1&2\\3&1\end{bmatrix}, B=\begin{bmatrix}2&1\\1&3\end{bmatrix}$ and $C=\begin{bmatrix}1&-1\\2&3\end{bmatrix}$, calculate AB, AC and $A(B+C)$, verify that $AB+AC=A(B+C)$.

25) If $A=\begin{bmatrix}0&6\\-6&0\end{bmatrix}, B=\begin{bmatrix}2&1\\1&2\end{bmatrix}$ and $C=\begin{bmatrix}3&4\\2&3\end{bmatrix}$, calculate AB, AC and $A(B+C)$, verify that $AB+AC=A(B+C)$.

26) If $A=\begin{bmatrix}1&2\\2&1\end{bmatrix}, B=\begin{bmatrix}2&0\\1&3\end{bmatrix}$ and $C=\begin{bmatrix}1&1\\2&3\end{bmatrix}$, calculate AB, AC and $A(B+C)$, verify that $AB+AC=A(B+C)$.

27) **Solve the following system of equations by matrix method:**

- $x-y+2z=7; 3x+4y-5z=-5$ and $2x-y+3z=12$.
- $2x+3y+3z=5; x-2y+z=4$ and $3x-y-2z=3$.
- $3x-2y+3z=8,\ 2x+y-z=1,\ 4x-3y+2z=4$.
- $x-y+z=4;\ 2x+y-3z=0$ and $x+y+z=2$.
- $x-y+2z=1, 3x-2y+4z=2, 2y-3z=1$.
- $x+y+z=6;\ y+3z=11$ and $x-2y+z=0$.
- $x+y+z=6, x-y-z=-4$ and $x+2y-2z=-1$.
- $x-y+3z=10, x-y-z=-2$ and $2x+3y+4z=4$.
- $x+y+z=6, x-2y+3z=6,\ x-y+z=2$.
- $x+2y+3z=2;\ 2x+3y+z=-1$ and $x-y+z=-2$.
- $2x+y+z=1;\ x-2y-z=\frac{3}{2}$ and $3y-5z=9$.
- $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$.

28) If $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x-3y+5z=11;\ 3x+2y-4z=-5$ and $x+y-2z=-3$.

29) The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

30) If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2y_2 + xy_1 + y = 0$.
31) If $y = 5\cos(\log x) + 7\sin(\log x)$, show that $x^2y_2 + xy_1 = 0$.
32) If $y = (\tan^{-1} x)^2$, show that $(1 + x^2)^2y_2 + 2x(1 + x^2)y_1 = 2$.
33) If $y = \sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.
34) If $y = e^{m\cos^{-1}x}; -1 \le x \le 1$, show that $(1 - x^2)y_2 - xy_1 - m^2y = 0$.
35) If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.
36) If $y = Ae^{mx} + Be^{nx}$, then prove that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + (mn)y = 0$.
37) If $y = 500e^{7x} + 600e^{-7x}$, then prove that $\frac{d^2y}{dx^2} = 49y$.
38) If $e^y(x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$ and hence prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.
39) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, show that $\frac{dy}{dx} = \tan\theta$ and $\frac{d^2y}{dx^2} = \frac{1}{a}\sec^4\theta\sin\theta$.
40) The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When $x = 10$cm and $y = 6$ cm, find the rates of change of (i) the perimeter and (ii) the area of the rectangle.
41) The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$cm and $y = 6$ cm, find the rates of change of (i) the perimeter and (ii) the area of the rectangle.
42) Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?
43) If ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
44) A ladder 25 ft long leans against a vertical wall. The lower end is moving away at the rate of 3 ft/sec, find the rate at which the top of the ladder is moving downwards, if the foot is 24 ft from the wall.
45) A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times a fast as the x- coordinate.
46) The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?
47) A man of height 2 meters walks at a uniform speed of 5 km/hour, away from a lamp post which is 6 meters high. Find the rate at which the length of the his shadow increases.
48) Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one- sixth of the radius of the base. How fast is the height of the sand increasing. When the height is 4cm?
49) A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.
50) Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect x and evaluate $\int\frac{dx}{\sqrt{7-6x-x^2}}$, $\int\frac{dx}{\sqrt{5-4x-x^2}}$.
51) Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and evaluate $\int\frac{1}{\sqrt{x^2+7}}dx$, $\int\frac{1}{\sqrt{x^2+2x+2}}dx$, $\int\frac{1}{\sqrt{x^2+2x+4}}dx$.
52) Find the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect x and evaluate $\int\frac{1}{\sqrt{x^2+6x-7}}dx$, $\int\frac{1}{\sqrt{x^2-2x}}dx$.
53) Find $\int\frac{dx}{x^2+a^2}$, and hence evaluate $\int\frac{dx}{x^2+7}$, $\int\frac{dx}{x^2+2x+4}$.
54) Find $\int\frac{dx}{x^2-a^2}$ and hence evaluate $\int\frac{dx}{3x^2+13x-10}$, $\int\frac{dx}{x^2-8x+5}$.
55) Find $\int\frac{dx}{a^2-x^2}$ and hence find $\int\frac{dx}{7-6x-x^2}$, $\int\frac{dx}{5-4x-x^2}$.
56) Find the integral of $\sqrt{a^2-x^2}$ w.r.t. x and hence find $\int\sqrt{5-x^2+2x}\,dx$, $\int\sqrt{1-4x-x^2}\,dx$.
57) Find the integral of $\int\sqrt{x^2+a^2}\,dx$ with respect x and evaluate $\int\sqrt{x^2+4x+6}\,dx$, $\int\sqrt{4x^2+9}\,dx$.
58) Find the integral of $\sqrt{x^2-a^2}$ with respect x and hence evaluate $\int\sqrt{x^2-8x+7}\,dx$.

59) Find the area of the region enclosed between the two circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$.

60) Find the area bounded by the curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$ using integration method.

61) Find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the method of integration and hence, find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

62) Find the area of the region bounded by the two parabolas $y=x^2$ and $y^2=x$.

63) Using integration find the area of the triangular region whose sides have the equations
$y=2x+1,\ y=3x+1$ and $x=4$.

64) Using integration find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$.

65) Using integration find the area bounded by the triangle whose vertices are $A(2,0)$, $B(4,5)$ and $C(6,3)$.

66) Using integration find the area bounded by the circle $x^2+y^2=16$ and the parabola $y^2=6x$.

67) Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$.

68) Find the area of the region enclosed by the parabola $x^2=4y$ and the line $x=4y-2$ and the x axis.

69) Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ by the method of integration.

70) Find the area of the of the circle $x^2+y^2=a^2$ by the method of integration and hence find the area of $x^2+y^2=2$.

71) Solve the differential $\frac{dy}{dx}+y\sec x=\tan x, 0\le x<\frac{\pi}{2}$.

72) Find the particular solution of $\frac{dy}{dx}+y\cot x=(4x\,\mathrm{cosec}\,x)$, $x\neq 0$, given that $y=0$ when $x=\frac{\pi}{2}$.

73) Find the particular solution of the differential equation $\frac{dy}{dx}+\frac{2xy}{1+x^2}=1$ when $y=0$ and $x=2$.

74) Find the general solution of the differential equation $\frac{dy}{dx}+y.\cot x=2x+x^2.\cot x$.

75) Solve $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$.

76) Find the general solution of the differential equation, $(x+3y^2)\frac{dy}{dx}=y,\ (y>0)$.

77) Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2,\ (x\neq 0)$

78) Find the general solution of the differential equation $e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

79) Solve $\cos^2 x\frac{dy}{dx}+y=\tan x$, where $0<x<\frac{\pi}{2}$.

80) Solve the differential equation $ydx-(x+2y^2)dy=0$.

81) Find the equation of the curve passing through the point (0,2) given that the sum of the co-ordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

82) Derive the equation of a straight line in a space through a given point and parallel to a vector both in the vector and Cartesian form.

83) Derive the equation of a line in a space passing through two given points both in the vector and Cartesian form.

84) Derive the equation of a plane perpendicular to a given vector and passing through a given point in both the vector and Cartesian form.

85) Derive the equation of a plane in normal form (both in the vector and Cartesian form).

86) Derive the equation of a plane in the intercept form.

87) Derive the equation of a plane passing through 3 non-collinear points.

88) Derive the formula to find the shortest distance between the two skew lines $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ in the vector form.

89) If a fair coin is tossed 10 times. Find the probability of (i) exactly six heads (ii) at least six heads.

90) Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that (i) all the five cards are spades? (ii) only 3 cards are spades? (iii) none is a spade?

91) A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize exactly once?

92) A die is thrown 6 times. If "getting an odd number" is success, What is the probability of (i) 5 success? (ii) at most 5 success? (iii) at least 5 success?

93) The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs (i) none (ii) not more than one (iii)more than one will fuse after 150 days of use.

94) If a fair coin is tossed 8 times. Find the probability of (i) at least five heads and (ii) at most five heads.

95) There are 5% defective items in a large bulk of items. What is the probability that sample of 10 items will include not more than one defective item?

96) A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes..

97) A fair coin is tossed 8 times. Find the probability of at most 5 heads..

PART-E: (SIX Mark Questions)

1) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ and evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$, $\int_3^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}dx$

2) Prove that $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}$

Hence evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x\ dx$, $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x)\ dx$.

3) Prove that $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ and hence evaluate $\int_{-1}^{2} |x^3 - x|dx$, $\int_{-5}^{5} |x+2|dx$.

4) Prove that $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{when } f(2a-x) = f(x) \\ 0 & \text{when } f(2a-x) = -f(x) \end{cases}$ and hence evaluate $\int_0^{\pi} |\cos x| dx$.

5) Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. Hence evaluate

$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$. $\quad \int_0^{\pi/4} \log(1+\tan x)dx \quad \int_0^{\pi/2} (2\log\sin x - \log\sin 2x dx) \quad \int_{-\pi/2}^{\pi/2} \sin^7 x\,dx$.

$\int_{-\pi/2}^{\pi/2} \tan^9 x\,dx$; $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$

6) Minimize and Maximize $z = x + 2y$ subject to the constraints $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$, by the graphical method

7) Minimize and maximize $Z = 3x + 9y$. Subject to the constraints $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$ and $x \geq 0$, $y \geq 0$ by graphical method.

8) Minimize and maximize $Z = 600x + 400y$ subject to the constraints $x + 2y \leq 12$, $2x + y \leq 12$, $4x + 5y \leq 20$ and $x \geq 0$, $y \geq 0$ by graphical method.

9) Solve the following linear programming problem graphically: Maximize, $Z = 3x + 2y$ subjected to the constraints: $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0, y \geq 0$.

10) One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

11) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine. A and 3hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of RS.17.50 per package on nuts and RS. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit if he operates his machines for at most 12hours a day?

12) A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foodsF1 and F2 are available. Food F1 costs Rs 4 per unit food and F2 costs Rs 6 per unit. One unit of food F1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

13) A furniture dealer deals in only two items – tables and chairs. He has Rs 50,000 to invest and has storage space of at most 60 pieces. A table costs Rs 2500 and a chair Rs 500. He estimates that from the sale of one table, he can make a profit of Rs 250 and that from the sale of one chair a profit of Rs 75. How many tables and chairs he should buy from the available money so as to maximize his total profit assuming that he can sell all the items which he buys.

14) A co-operative society of famers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much should be allocated to each crop so as to maximize the total profit of the society?

15) A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food "I" contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food "II" contains 1 unit/kg of vitamin A and 2 uits/kg of vitamin C. It costs Rs. 50 per kg to purchase Food 'I' and Rs 70 per/kg to purchase Food

'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

(FOUR Mark Questions)

1) Determine the value of k, if $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

2) Find the values of a and b such that $f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$ is continuous function.

3) Find the value of K, if $f(x) = \begin{cases} Kx^2, & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$.

4) Determine the value of k, if $f(x) = \begin{cases} kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$ is continuous at $x = 5$.

5) Define a continuity of a function at a point. Find all the points of discontinuity of f defined by $f(x) = |x| - |x + 1|$.

6) Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

7) Find the points of discontinuity of the function $f(x) = x - [x]$, where $[x]$ indicates the greatest integer not greater than x. Also write the set of values of x, where the function is continuous.

8) Find all points of discontinuity of f(x), where f is defined by $f(x) = \begin{cases} x^3 - 3, & \text{if } x \geq 2 \\ x^2 + 1, & \text{if } x < 2 \end{cases}$

9) If the function $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, find the values of a and b.

10) Determine the value of k, if $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

11) Show that $\begin{vmatrix} x + y + 2z & x & y \\ z & y + z + 2x & y \\ z & x & z + x + 2y \end{vmatrix} = 2(x + y + z)^3$

12) Show that $\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$.

13) Prove that $\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3$.

14) Prove that $\begin{vmatrix} b + c & a & a \\ a & c + a & b \\ c & c & a + b \end{vmatrix} = 4abc$.

15) Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$.

16) Prove that $\begin{vmatrix} 1 + a & 1 & 1 \\ 1 & 1 + b & 1 \\ 1 & 1 & 1 + c \end{vmatrix} = ab + bc + ca + abc = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.

17) Show that $\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$.

18) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$.

19) Prove that $\begin{vmatrix} x + 4 & 2x & 2x \\ 2x & x + 4 & 2x \\ 2x & 2x & x + 4 \end{vmatrix} = (5x + 4)(4 - x)^2$.

20) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$ or $(x^3 - 1)^2$.