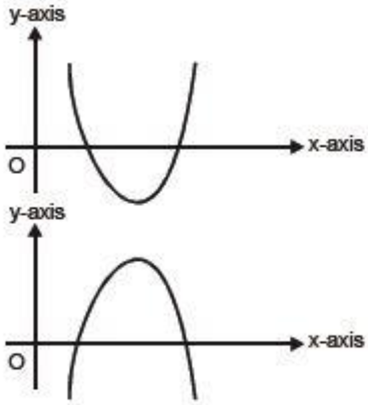
CLASS: 10^{th}	SUBJECT: MATHEMATICS
CHAPTER: POLYNOMIALS	DATE: 26.03.2020

Basic Concepts

- Zeroes of a polynomial. $k$is said to be zero of a polynomial $p(x)\ if\ p(k) = 0$
- Graph of polynomial.
  - (i) Graph of a linear polynomial $ax + b$ is a straight line.
  - (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like $\cup$, if a > 0.
  - (iii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$, if a < 0.
  - (iv) In general a polynomial $p(x)$ of degree n crosses the $x - axis$ at atmost n points.



- Relationship between the zeroes and the coefficients of a Polynomial.
  - (i) If $\alpha, \beta$are zeroes / roots of $p(x) = ax^2 + bx + c$, then

$$\text{Sum of roots } \alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} \Rightarrow \ \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

  - (ii) If $\alpha, \beta\ and\ \gamma$ are zeroes / roots of $p(x) = ax^3 + bx^2 + cx + d$

$$\text{Then, sum of roots} = \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$$

Sum of product of roots taken, two at a time

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\text{Product of roots} = \frac{-d}{a} \Rightarrow \alpha\beta\gamma = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

  - (iii) If α, β are roots of a quadratic polynomial $p(x)$, then $p(x) = x^2 - (\alpha + \beta)\, x + \alpha\beta$

    $\Rightarrow p(x) = x2 - (sum\ of\ roots)\, x + product\ of\ roots$
  - (iv) If α, β and γ are zeroes of a cubic polynomial p(x),

    Then, $p(x) = x^3 - (\alpha + \beta + \gamma)\, x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)\, x - (\alpha\beta\gamma)$

$p(x) = x^3 - (sum\ of\ zeroes)\, x^2 + (sum\ of\ product\ of\ zeroes\ /\ roots\ taken\ two\ at\ a\ time)x - (product\ of\ zeroes)$

EXAMPLES TO SOLVE NUMERICALS

1. Find the zeroes of the quadratic polynomial and verify the relationship between the zeroes and coefficient of polynomial $p(x) = x^2 + 7x + 12$.

Sol. p(x) = $x^2 + 7x + 12$

⇒p(x) = (x + 3)(x + 4)

∴p(x) = 0 if x + 3 = 0 or x + 4 = 0

⇒ x = – 3 or x = – 4

∴– 3 and – 4 are zeros of the $p(x)$.

Now,

Sum of the zeroes = –3 + (–4) = –7 $= \frac{-7}{1} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

Product of the zeroes = (– 3) × (– 4) = 12 = $\frac{12}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

2. Find the zeroes of $4x^2 - 7$ and verify the relationship between the zeroes and its coefficients.
Sol. Let p(x) = $4x^2 - 7$

Here coefficient of $x^2 = 4$,
Coefficient of x = 0 and constant term = –7.

Now p(x) = 4x2 – 7 = $(2x - \sqrt{7})(2x + \sqrt{7})$

$\therefore$ p(x) = 0, if $2x - \sqrt{7} = 0$ or $2x + \sqrt{7} = 0$

$\Rightarrow x = \frac{\sqrt{7}}{2}$ or $x = \frac{-\sqrt{7}}{2}$

$\therefore \frac{\sqrt{7}}{2}$ and $\frac{-\sqrt{7}}{2}$ are zeroes of p(x).

Now,

Sum of zeroes $= \frac{\sqrt{7}}{2} + \left(\frac{-\sqrt{7}}{2}\right) = 0 = \frac{0}{4} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

Product of zeroes = $\frac{\sqrt{7}}{2} \times \frac{-\sqrt{7}}{2} = \frac{-7}{4} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

3. Find a quadratic polynomial, the sum of whose zeroes is 0 and one zero is 5.
Sol. Let zeroes are α and β.

⇒α + β = Sum of zeroes
⇒α + β = 0 ⇒ 5 + β = 0 ⇒β = –5
Now product of zeroes = αβ = 5 × (–5) = –25
Let polynomial p(x) = $ax^2 + bx + c$

$\therefore \alpha + \beta = 0 = \frac{-b}{a}; \alpha\beta = \frac{c}{a} = -25$

$\therefore$ a = 1, b = 0, c = –25
Hence, p(x) = $x^2 - 25$

4. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeroes are 5, –6 and –20 respectively.
Sol. Let p(x) = $ax^3 + bx^2 + cx + d$

and&alpha, &beta, γ are its zeroes.
∴ α + β + γ = Sum of zeroes = 5 = $\frac{-b}{a}$
αβ + αγ + βγ = Sum of the products of zeroes taken two at a time=$-6 = \frac{c}{a}$

$$\alpha\beta\gamma = -20 = \frac{-d}{a}$$

If a = 1, then b = –5, c = –6 and d = 20
∴ Polynomial, p(x) = $x^3 - 5x^2 - 6x + 20$.

Division Algorithm for polynomials.
If p(x) and g(x) are any two polynomials with g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that p(x) = $q(x) \times g(x) + r(x)$, where r(x) = 0 or degree of r(x) < degree of g(x).
Step–1. Divide the highest degree term of the dividend by the highest degree term of the divisor and obtain the remainder.
Step2. If the remainder is 0 or degree of remainder is less than divisor, then we cannot continue the division any further. If degree of remainder is equal to or more than divisor repeat step-1.

5. Divide $4x^3 + 2x^2 + 5x - 6$ by $2x^2 + 3x + 1$.

$$
\begin{array}{r|l}
 & 2x - 2 \\
2x^2 + 3x + 1 & 4x^3 + 2x^2 + 5x - 6 \\
 & 4x^3 + 6x^2 + 2x \\
 & - \quad - \quad - \\ \hline
 & -4x^2 + 3x - 6 \\
 & -4x^2 - 6x - 2 \\
 & + \quad + \quad + \\ \hline
 & 9x - 4
\end{array}
$$

∴ Quotient is $2x - 2$ and remainder is $9x - 4$.

6. Find all zeroes of polynomial

$4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeroes are 2 and 3.

Sol. Since two zeroes are 2 and 3.

∴ $(x - 2)(x - 3)$ is a factor of given polynomial.

$\Rightarrow x^2 - 5x + 6$ is a factor of given polynomial.

Now

$$
\begin{array}{r|l}
 & 4x^2 - 1 \\
x^2 - 5x + 6 & 4x^4 - 20x^3 + 23x^2 + 5x - 6 \\
 & 4x^4 - 20x^3 + 24x^2 \\
 & - \quad + \quad - \\ \hline
 & -x^2 + 5x - 6 \\
 & -x^2 + 5x - 6 \\
 & + \quad - \quad + \\ \hline
 & 0
\end{array}
$$

∴ $4x^4 - 20x^3 + 23x^2 + 5x - 6$

$= (x^2 - 5x + 6)(4x^2 - 1)$

$= (x - 2)(x - 3)(2x - 1)(2x + 1)$

∴ Zeroes of the given polynomial are 2, 3, $^1/_2$, $^{-1}/_2$.

ASSIGNMENT

1. For what value of $k, (-4)$ is a zero of the polynomial $x^2 - x - (2k + 2)$?

2. For what value of $p, (-4)$ is a zero of the polynomial $x^2 - 2x - (7p + 3)$?

3. If 1 is a zero of the polynomial $p(x) = x^2 - 3(a - 1)x - 1$, then find the value of a.

4. Write the zeroes of the polynomial $x^2 + 2x + 1$.

5. Write the zeroes of the polynomial $x^2 - x - 6$.

6. Write a quadratic polynomial, the sum and product of whose zeroes are 3 and –2 respectively.

7. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficient of the polynomial.

8. Find the zeroes of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeroes and the coefficient of the polynomial.

9. Find the quadratic polynomial, the sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

10. Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeroes are 2 and – 2.