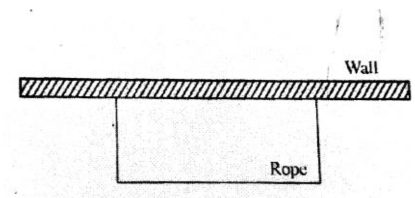
CHAPTER 6

APPLICATIONS OF DERIVATIVES

SAY 2018

1. A rectangular plot s to be fenced using a rope of length 20 metres with one of its sides is a wall as shown in the figure. Find the maximum area of such a rectangle. (3)



2. Consider the curve $y = x^3 + 8x + 3$.
   a) Find the point on the curve at which the slope of tangent is 20. (3)
   b) Does there exist a tangent to the above curve with negative slope? Justify your answer. (1)

MARCH 2018

3. a) $f(x)$ is strictly increasing if $f'(x)$ is ……..
   a) positive b) negative
   c) 0 d) None of these (1)
   b) Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$ is strictly incrasing. (2)
4. a) Find the slope of the tangent to the curve $y = (x-2)^2$ at $x = 1$. (1)
   b) Find a point at which the tangent to the curve $y = (x-2)^2$ is parallel to the chord joining the points A(2,0) and B(4,4). (2)
   c) Find the equation of the tangent to the above curve and parallel to the line AB. (1)

SAY 2017

5. a) Slope of the tangent to the curve $y = 5 - 10x^2$ at the point (-1,-5) is
   a) 10 b) -10
   c) 20 d) -20 (1)
   b) Show that of all rectangles inscribed in a fixed circle, the square has the maximum area (4)

   OR

   a) Maximum value of $f(x) = \log x$ in $[1, e]$ is
   a)1 b) e
   c) 1/ e d) 0 (1)
   b) Using differentials, find the approximate value of $(255)^{1/4}$ (4)

MARCH 2017

6. a) Slope of the normal to the curve at (1,2) is
   a) 1 b) $\frac{1}{2}$
   c) 2 d) -1 (1)
   b) Find the interval in which $2x^3 + 2x^2 + 12x - 1$ is strictly increasing. (4)

   OR

   a) The rate of change of volume of a sphere with respect to its radius when radius is 1 unit
   a) 4π b) 2π
   c) π d) $\frac{\pi}{2}$ (1)

b) Find positive number whose sum is 16 and the sum of whose cubes is minimum. (4)

SAY 2016

7. a) The slope of the normal to the curve $y = x^3 - x^2$ at $(1, -1)$ is

i) 0 ii) -1

iii) 1 iv) Not defined (1)

b) Find the intervals in which the function $f(x) = 2x^3 - 24x + 25$ is increasing or decreasing. (4)

OR

a) The rate of change of area of a circle with respect to radius r, when r = 5 cm

i) $25\pi cm^2 / cm$ ii) $25 cm^2 / cm$

iii) $10\pi cm^2 / cm$ iv) $10 cm^2 / cm$ (1)

b) Show that of all rectangles with a given area, the square has the last perimeter. (4)

MARCH 2016

8. a) Slope of the tangent to the curve given by

$x = 1 - \cos\theta,\ y = \theta - \sin\theta$ at $\theta = \frac{\pi}{2}$ is

i) 1 ii) -1

iii) 2 iv) 0 (1)

b) Find the intervals in which the function $f(x) = 2x^3 - 24x + 25$ is increasing or decreasing. (4)

OR

a) The rate of change of the area of a circle with respect to the radius 'r' when r=5 cm

i) $25\pi\, cm^2 / cm$ ii) $25 cm^2 / cm$

iii) $10\pi\, cm^2 / cm$ iv) $10 cm^2 / cm$ (1)

b) Show that of all rectangles with a given area, the square has the least perimeter. (4)

SAY 2015

9. a) Find the equation of the tangent to the curve

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1,1). (2)

b) Find two positive numbers whose sum is 15 and sum of whose squares is minimum.

(3)

MARCH 2015

10. a) Which of the following functions is always increasing?

i) $x + \sin 2x$ ii) $x - \sin 2x$

iii) $2x + \sin 3x$ iv) $2x - \sin x$ (1)

(b) The radius of a cylinder increases at a rate of 1cm/s and its height decreases at a rate of 1cm/s. Find the rate of change of its volume when the radius is 5cm and the height is 15cm.

(2)

If the volume should not change even when the radius and height are changed. What is the relation between the radius and height?

(1)

(c) Write the equation of tangent at (1,1) on the curve $2x^2 + 3y^2 = 5$. (2)

SAY 2014

11. a) Find the slope of the tangent to the parabola

$y^2 = 4ax\ at\ (at^2, 2at)$ (1)

b) Find the intervals in which the function x^2-2x+5 is strictly increasing. (2)

c) A spherical bubble is decreasing in volume at the rate of 2 cm^3/sec. Find the rate of which the surface area is diminishing when the radius is 3cm. (2)

MARCH 2014

12. a) Which of the following function is increasing for all values of x in its domain?

A) Sin x B) log x

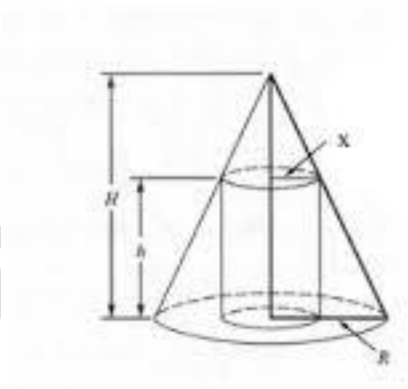
C) x^2 D) $|x|$ (1)

b) Find a point on the curve $y(x-2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4). (2)

c) Find the maximum profit that a company can make, if the profit function is given by $p(x)=41-24x-6x^2$. (2)

SAY 2013

13. A right circular cylinder is inscribed in a given cone of radius R cm and height H cm as shown in figure.



i. Find the curved surface area S of the circular cylinder as a function of x. (2)

ii. Find a relation connecting x and R when S is a maximum. (3)

MARCH 2013

14. a) Find the slope of the normal to the curve $y=\sin\theta$ at $\theta=\frac{\pi}{4}$ (1)

b) Show that the function $x^3-6x^2+15x+4$ is strictly increasing in R. (2)

c) Show that all rectangles with a given perimeter, the square has the maximum area. (2)

SAY 2012

15. a) Show that the function $f(x)=x^3-3x^2+6x-5$ is strictly increasing on R. (2)

b) Find the intervals in which the function $f(x)=\sin x+\cos x;\ 0\le x\le 2\pi$ is strictly increasing or strictly decreasing. (3)

MARCH 2012

16. a) The slope of the tangent to the curve $y=x^3-1$ at $x=2$ is .... (1)

b) Use differential to approximate $\sqrt{36.6}$ (2)

c) Find two numbers whose sum is 24 and whose product is as large as possible. (2)

SAY 2011

17. a) Find the approximate value of $(82)^{\frac{1}{4}}$ up to 3 places of decimals using differentiation. (3)

a) Find two positive numbers such that their sum is 8 and the sum of their squares is minimum. (3)

MARCH 2011

18. (a) The radius of a circle is increasing at the rate of 2 cm/s. Find the rate at which area of the circle is increasing when radius is 6 cm. (1)

(b) Prove that the function $f(x) = \log \sin x$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$. (2)

(c) Find the maximum value of the function $f(x) = x^3 - 6x^2 + 9x + 15$. (2)

SAY 2010

19. Consider the parametric forms: $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$ of a curve.

a) Find $\frac{dy}{dx}$ (2)

b) Find the equation of the tangent at $t = 2$. (3)

c) Find the equation of the normal at $t = 2$. (1)

MARCH 2010

20. a) A particle is moving along the curve $y = \frac{2}{3}x^3 + 1$. Find the point on the curve at which the y-coordinate is changing twice as fast as the x-coordinate. (3)

b) Consider the function $f(x) = \frac{\log x}{x}, x > 0$

i) Find $f'(x)$ *and* $f''(x)$ (1)

ii) For what value of x, the function f(x) has a maximum? (2)

SAY 2009

21. a) An open box is made by removing squares of equal size from the corners of a tin sheet of size 16cm x 10 cm and folding up the sides. Let V be the volume of the box so obtained.

i) With the help of figure, obtain the relation $V = x(16-2x)(10-2x)$. (1)

ii) What is the value of x for which V is maximum? (3)

(b) What is the slope of the tangent and normal at (1,1) on the curve $y = x^3$ (1)

OR

(a) A water tank is in the shape of a right circular cone with its axis vertical and vertex down. Its height and diameter are same. Water is poured into it at a constant rate of 2 m^3/minute.

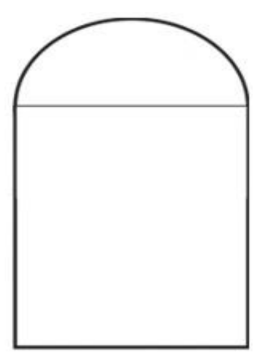
i) With the help of figure obtain the relation $V = \frac{1}{12}\pi h^3$ (1)

ii) Find the rate at which water level is increasing when depth of water in the tank is 6m. (3)

(b) Find the interval in which the function $x^3 - 6x^2 + 9x + 15$ is increasing. (1)

MARCH 2009

22. A window is in the form of a rectangle surmounted by a semicircle as shown in the figure. The perimeter of the window is 5 metres.



a) If r is the radius of the semicircle and x is the length of the larger side of the rectangle, find a relation between r and x. (1)

b) Find the area of the window in terms of r. (2)

c) Find the dimensions of the window so that the greatest possible light may be admitted. (3)

OR

a) Find the slope of the curve $x^2+3y=3$ at the point (1,2). (2)

b) Find the equation of the tangent to the curve $x^2+3y=3$ which is parallel to the line $y-4x+5=0$. Find also the equation of the normal to the curve at the point of contact. (4)

MARCH 2008

23. The total profit y (in rupees) of a drug company from the manufacture and sale of x bottles of drug is given by $y=-\frac{x^2}{300}+2x-50$.

a) How many bottles of drug must the company sell to obtain the maximum profit? (5)

b) What is the maximum profit? (1)

MARCH 2007

24. Consider the function $f(x)=x(x-2),\ x\in[1,3]$

a) Verify the Mean Value Theorem for the function in [1,3] (2)

b) Find the minimum value of the function by using differentiation. (2)

c) Find the equation of the tangent to the above function at (1,3) (2)

MARCH 2006

25. a) Verify truth of the mean value theorem for the function given by $f(x)=x^2-1, x\in[2,3]$ (2)

b) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$ (2)

c) Find the equation of the tangent to the curve $\sqrt{x}+\sqrt{y}=a$ at a point $\left(\frac{a^2}{4},\frac{a^2}{4}\right)$ (2)

MARCH 2005

26. i) Rolle's theorem is not applicable for $f(x)=|x|$ in $[-1,1]$ because:

a) $f'(-1)$ does not exist

b) $f(-1)\neq f(1)$

c) $f(x)$ is not continuous at x=0

d) $f'(0)$ does not exist. (1)

ii) Using differentials find the approximate value of $\sqrt{25.2}$ (2)

iii) A spherical bubble is decreasing in volume at the of 2 c.c/s. Find the rate at which the surface area is diminishing when the radius is 3 cm. (2)

iv) An open tank with square base and vertical sides is to be constructed so as to hold a given quantity of water. Show that the cost of material is least when the depth of the tank is half the width. (5)

JUNE 2004

27. i) The value of 'a' for which the function $f(x)=a\sin x+\frac{1}{3}\sin 3x$ has an extreme at

$x = \frac{\pi}{3}$ is

a) 1 b) –1 c) 2 d) –2 (1)

28. Verify Rolle's theorem for $f(x) = x^2 - 6x + 8 \ \ in \ [2,4]$ (2)

29. Prove that the function $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on R. (2)

30. The slope of the tangent of the curve $y = e^{-x^2}$ $at \ x = 1$ is

a) 0 b) $-\frac{2}{e}$

c) –2e d) $\frac{2}{e}$ (1)

31. Find the equation of the tangent to the curve $y = \sin^2 x$ at $x = \frac{\pi}{6}$ (2)

32. The radius of a circle is increasing at the rate of 3 cm./sec. What rate the area is increasing when the radius is 10 cm. (2)

33. Show that function $e^x \cos x$ satisfies the conditions of Rolle's theorem in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and find the point 'c' in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $f'(c) = 0$ (3)

34. Determine two positive integers whose sum is 15 and the product of the square of one with the other number is maximum. (5)

MARCH 2004

35. Verify Rolle's theorem for $f(x) = x^2 - 6x + 8 \ \ in \ [2,4]$ (2)

36. Show that the semi-vertical angles of the cone of maximum volume and of given slant height is $\tan^{-1}\left(\sqrt{2}\right)$ (3)

SAY 2003

37. The point on the curve $y = x^2$ at which the tangent makes an angle 45^0 with the X-axis is

a) $\left(\frac{1}{2}, \frac{1}{3}\right)$ b) $\left(\frac{1}{3}, \frac{1}{2}\right)$

c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ d) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (1)

38. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which y-coordinate changes as fast as the x-coordinate. (2)

39. Find the points on the curve $3x^2 - y^2 = 8$ at which the normals are parallel to the line $x + 3y = 4$ (2)

40. Find a point on the curve $y = (x-3)^2$ where the tangent is parallel to the chord joining (3,0) and (4,1). (3)

41. A wire of length 28m. is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum? (5)

MARCH 2003

42. Examine whether Mean Value Theorem is applicable for $f(x) = |x|$ in $[-1, 1]$. (2)

43. Show that of all rectangles of given perimeter, the square has the largest area. (3)

SAY 2002

44. The triangle to the curve $y = e^{2x}$ at the point $(0,1)$ is:

a) $2x + y + 1 = 0$ b) $2x + y - 1 = 0$

c) $2x - y + 1 = 0$ d) $x + 2y - 1 = 0$

(1)

45. Find the least value of "a" such that $f(x) = x^2 + ax + 1$ is increasing in $[1,2]$ (2)

46. State Lagrange's mean value theorem and verify the same for the function:

$f(x) = 2x^2 - 10x + 29$ in the interval $[2,7]$

(5)

47. The combined resistance R of two resistors R_1 and R_2 is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, R_1, R_2 > 0$. If $R_1 + R_2 = C$, where C is a constant, show that the maximum resistance R is obtained by setting $R_1 = R_2$. (5)

MARCH 2002

48. If the normal to the curve at a point is parallel to the x axis, then which of the following is true at that point?

a) $\frac{dy}{dx} = 0$ b) $\frac{dx}{dy} = 0$

c) $\frac{dy}{dx} = 1$ d) $\frac{dy}{dx} = -1$ (1)

49. Show that $y = x^3 + x^2 + x + 1$ does not attain the maximum or minimum value for any real number.

(2)

50. The radius of a circular plate is increasing at the rate of 0.2 cm/sec. At what rate the area is increasing when the radius of the plate is 2 cm?

(2)

51. Use differential to approximate $\sqrt{25.2}$ (3)

52. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent has the equation $y = x - 11$.

(5)

53. Show that of all rectangles with given perimeter, the square has the maximum area. (5)

MARCH 2001

54. The slope of the curve $x = 1 - \cos\theta$, $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$ is:

a) $\sqrt{2} + 1$ (b) $-\sqrt{2} + 1$

(c) $\sqrt{2} - 1$ (d) $-(\sqrt{2} + 1)$ (1)

55. If the radius of a circle is increased from 6 to 6.1 cm., find the approximate increase in its area.

(2)

56. Find the least value of 'a' such that the function $x^2 + ax + 1$ is increasing on (1,2). (3)

57. Show that of all rectangles with a given area, the square has the least perimeter. (5)

MARCH 2000

58. Find the equation of the tangent and normal to the curve $y = x^3 + 2x + 6$ at the point (2,18) (2)

59. Verify Lagrange's Mean Value theorem for the function $f(x) = x^2 - 1$ on [2,3] (3)

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