11323

B.Sc. III Semester Degree Examination, November/December 2015
MATHEMATICS
Paper – 3.1 : Vector Algebra and Solid Geometry

Time : 3 Hours Max. Marks : 60

***Instruction** : Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Show that
   $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$.

2. Find $\vec{a}\times(\vec{b}\times\vec{c})$ where $\vec{a}$ = i – 2j + k, $\vec{b}$ = 3i + j – k, $\vec{c}$ = 4i – 2j – k.

3. Find a set of vectors reciprocal to the set of vectors.

   $\vec{a}$ = 2i + 3j – k, $\vec{b}$ = 3i – j + k, $\vec{c}$ = 2i + j – 3k.

4. Find the distance between the points A (–2, 3, 5), B = (1, 2, 3).

5. Show that the three points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

6. Find the co-ordinates of the point that divides the line joining the points (2, –3, 1) and (3, 4, –5) in the ratio 1 : 3 internally.

7. If the direction ratios of a line are 6, 2, 3 find the direction cosines of the lines.

8. Show that the lines are at right angles whose direction ratios are (2, 3, 4) and (1, –2, 1).

9. Find the area of the triangle whose vertices are (2, 5, –4), (–1, 4, –3) and (4, 7, –6).

10. Find the equation of the plane passing through the points (1, 1, 0), (1, 2, 1), (–2, 2, –1).

11. Find the distance between the parallel planes
    2x – 2y + z + 6 = 0
    4x – 4y + 2z + 7 = 0

P.T.O.

12. Show that the lines

$$\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z-3}{1} \text{ and } \frac{x-2}{2}=\frac{y-4}{1}=\frac{z-6}{3} \text{ are coplanar.}$$

SECTION – B

Answer **any two** of the following : **(2×5=10)**

1. Define reciprocal system of vectors and prove that

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right].\left[\vec{a}'\ \vec{b}'\ \vec{c}'\right]=1$$

2. Find $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ where

$\vec{a} = i + 2j$

$\vec{b} = j + 2k$

$\vec{c} = i + 2k$

3. Show that

$$i\times(\vec{a}\times i)+j\times(\vec{a}\times j)+k\times(\vec{a}\times k)=2\vec{a}$$

4. Show that

$$\left(\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\right)=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$$

SECTION – C

Answer **any three** of the following : **(3×5=15)**

1. Find the angle between the diagonals of a cube.

2. Find the direction cosines of the two lines which are connected by the relations $l - 5m + 3n = 0$ and $7l^2 + 5m^2 - 3n^2 = 0$.

3. Find the value of a and b such that (a, 1, 1), (1, b, –1), (1, 3, –3) are collinear.

4. Find the angle between the two lines whose direction cosines satisfy the equation.

   $l + m + n = 0$ and $2l + 2m - nm = 0$.

5. Find the equation of the planes which passes through the points (0, 4, –3) and (6, –4, 3) and makes intercepts on the axes whose sum is zero.

SECTION – D

Answer **any three** of the following : **(3×5=15)**

1. Find the symmetrical form of the line of intersection of the planes.

   $2x + 3y + 5z - 1 = 0,\ 3x + y - z + 2 = 0.$

2. Derive the condition for a line to lie on a plane both in vector and Cartesian form.

3. Find the equation of plane passing through the points (2, 0, 2), (6, 1, 1) and (4, 2, 3) meat the coordinate axes at A, B and C. Find the coordinates of the centroid of the triangle ABC.

4. Find the length of the perpendicular from the point A (–3, 0, 1) to the plane $4x - 3y + 2z = 19$. Also find the coordinates of the foot of the perpendicular.

5. Find the shortest distance between the lines

   $\frac{x-3}{1} = \frac{y-4}{-2} = \frac{z+2}{-1}$ and $3x - y - 10 = 0 = 2x - z - 4$.
