DETERMINANTS

GRADE : XII

1. If A = $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ then find the value of k given that $|3A|$ = k $|A|$

2. Use the product PQ = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations x - y +2z = 1, 2y - 3z = 1, 3x -2y +4z = 2

3. Prove that using properties $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} = 0$

4. If A is a non singular matrix of order 3 such that $|Adj\ \mathrm{A}|$ = 64. Find $|A|$

5. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$ = (a+b+c)($a^2 + b^2 + c^2$)

6. Find A^{-1} where A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$. Hence solve the equations x+2y+z = 4, -x+y+z = 0, x-3y+z = 2.

7. A square matrix A has order 3 has $|A|$ = 5 Find $|A\ \mathrm{AdjA}|$

8. What is the value of $|3I_3|$

9. For what value of K, then the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible.

10. A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$.*Find* AB. Use this to solve the following equations x - y + z = 4, x-2y-2z = 9, 2x + y+3z = 1

11. solve by matrix method $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,\ \ \frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 10,\ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

12. For the matrix A=$\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ Find the numbers k and m such that $A^2 - kA - mI = 0$

13. Prove that $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ = $9b^2(a+b)$ using properties

14. If a,b and c are all positive distinct show that $\triangle = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value or $-(a^3+b^3+c^3 - 3abc) = (3abc-a^3-b^3-c^3)$.

15. Using properties prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

16. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3 - 3abc$

17.Using properies $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

18. A square matrix B, order 3, has $|B| = 6$. Find $|B \ AdjB|$

19. What is the value of a 3x3 skew symmetric matrix.

20. If A is an invertibe matrix of order 2, and detA=5 Find det.A^{-1}

21. (a) $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix} = -(a^3+b^3)^2$

(b). $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$ (c) $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

(d) $\begin{vmatrix} (b+c)^2 & a^2 & c+b \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

(e). $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ show that $\dfrac{p}{p-a} \ \dfrac{q}{q-b} + \dfrac{r}{r-c} = 2, \ (p \neq a, q \neq b, r \neq c)$

22. Find the quadratic function f(x) = ax^2+bx+c such f(0) =1 ,f(2) = 7 and f(1) =2 using matrices.

23. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method find the number of students in each group . Apart from the values, hard work honest and respect for law, vigilance and obedience, suggest one more value which in your opinion, the school should consider for awards.

24. If A is a square matrix and $|A| = 2$ then write the value of $|AA^T|$

25. Use elementary column operation $C_2 \to C_2 - 2C_1$ in the matrix equation $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

26 . Two schools P and Q want to award their selected students on the values of tolerance , kindness and leadership . The school P wants to award Rs. X each , Rs y each and Rs. Z each for the three respective values to 3, 2 and 1 students respectively with a total award money of 2200. School Q wants to spend 3100 to award its 4 , 1 and 3 students on the respective values (by giving the same award money to the three values as school P) If the total amount of award for one prize on each value is 1200. Using matrices find the award money for each value. Apart from these three values. Suggest one more value which should be considered for award.

27. Using preperties prove that $\begin{vmatrix} x+a & 2x & 2x \\ 2x & x+a & 2x \\ 2x & 2x & x+a \end{vmatrix} = (5x+a)(a-x)^2$

28.If f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$ using properties of determinants find the value of f(2x) – f(x)

29.Using properties of determinants $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

30. Using properties of determinants $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} =$ $6x^2(1-x^2)$

31. If x , y and z are in G.P. then using properties of determinants show that

$\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix} = 0$ where $x \neq y \neq z$.

32. $\begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find adj A and verify that A adjA = adjA A = $|A|\, I$