AREAS RELATED TO CIRCLES

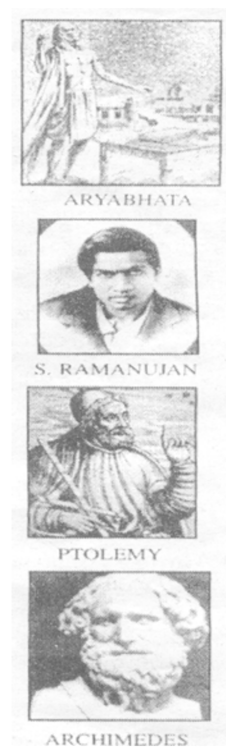
★ INTRODUCTION

In earlier classes, we have studied methods of finding perimeters and area of simple plane figures such as rectangles, squares, parallelograms, triangle and circles. In our daily life, we come across many objects which are related to circular shape in some form or the other. For example, cycle wheels, wheel arrow, drain cover, bangles, flower beds, circular paths etc. That is why the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall discuss problems on finding the area of some combinations of plane figures involving circles or parts of circles. Let us first recall the concepts related to their perimeter and area of a circle.

★ HISTORICAL FACTS

Mensuration is that branch of mathematics which studies the method of measurements. Measurement is a very important human activity. We measure the length of a cloth for stitching. The area of a wall for painting, the perimeter of a plot for fencing. We do many other measurements of similar nature in our daily life. All these measurements, we shall study in this capter called Mensuration.

π (pi) occupies the most significant place in measurement of surface area as well as volume of various solid and plane figures. The value of π is not exactly known. The story of the accuracy by which the value of π was extimated is an interesting one.



Mathematically $\pi = \dfrac{Cirumferecne\ of\ \ a\ circle}{Diameter\ of\ \ the\ circle}$

$$\pi = \frac{(10+4)\ x\ 8 + 62000}{20{,}000} = \frac{62832}{20{,}000} = 3.1416$$

According to **S. Ramanujan,** the value of π

$$\pi = \frac{355}{113}$$

According to **Archimedes** ; the value of π is given below :

$$\pi = \frac{Cirumference\ of\ \ a\ circle}{Diameter\ of\ \ the\ circle} = 3\frac{10}{71} < \pi < 3\frac{10}{70}$$

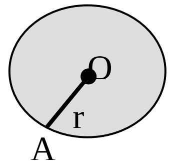
Ptolemy, $\pi = 3\dfrac{17}{120}$

The **Egyptians,** π = $\left(\dfrac{16}{9}\right)^2 = 3.160$

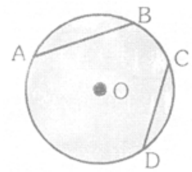
Note : π (pi) is an irrational number. It cannot be expressed as the ratio of whole numbers. However, the ratio 22 : 7 is ofter used as approximation for it.

★ **RECALL**

(A) **Circle :** Circle is the locus of a point which moves in such a manner that its distance from a fixed point O remains constant (the same). The fixed point is **called the centre** O and the constant distance OA is **called its radius.**

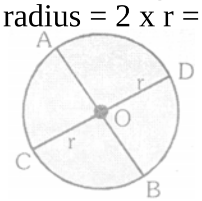


(B) **Chord :** A line segment joining two points on a circle is called a chord of the circle. If fig. AB and CD are two chords of the circle.



(C) **Diameter :** A chord passing through the centre of the circle is called the diameter. In fig, AOB and COD are diameter of the circle i.e., the diameter is the largest chord of the circle.

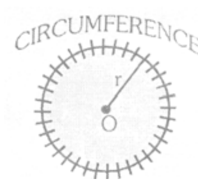
Length of diameter = Twice the radius = 2 x r = AOB = COD



(D) **Circumference :** The perimeter of the circle or the length of boundary of the circle is called its circumference i.e. the distance covered by traveling once around a circle is called the perimeter or circumference. The circumference of a circle is given by $2\pi r$. It is well-known fact that the ratio of the circumference of a circle to its diameter bears a constant ratio.

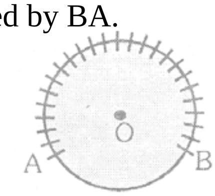
$$\pi = \frac{Circumference\ of\ \ a\,circle}{Diameter\ of\ \ the\ circle}$$

⇒ Circumference = π x diameter = π x 2r = 2πr where r is the radius of the circle.



(E) **Arc.** Any part of a circle is called an arc of the circle. Two points A and B on a circle divides it into two arcs. In general one arc is greater than other. The smaller arc is called minor arc and greater arc is called major arc.

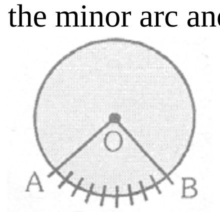
In the given fig, AB is an arc of a circle with centre O, denoted by $\widehat{AB}$. The remaining part of the circle shown by the dotted lines is represented by BA.



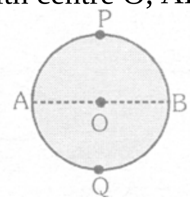
(F) **Central Angle :** Angle subtended by an arc at the centre of a circle is called its central angle. In fig. the centre of the circle is O.

Central angle made by $\widehat{AB}$ at the centre O = $\angle$AOB - θ

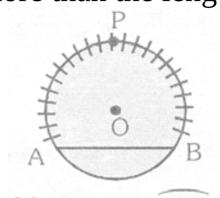
If $\theta^{o} < 180^{o}$ then the arc $\widehat{AB}$ is called the minor arc and the arc $\widehat{AB}$ is called major arc.



(G) **Semi-circle :** A diameter divides a circle into two congruent arcs. Each of these two arc is called a semicircle. In the given fig. of circle with centre O, $\widehat{APB}$ and $\widehat{BQA}$ are semicircles. Is the half of the circle.

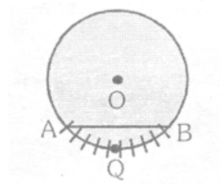


(H) **Major arc :** An arc whose length is more than the length of the semi-circle is called a major arc:



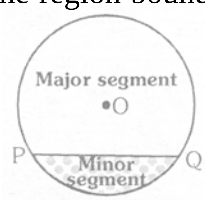
Major arc = $\widehat{APB}$

(I) **Minor arc :** An arc whose length is less than the length of semi-circle is called a minor arc.



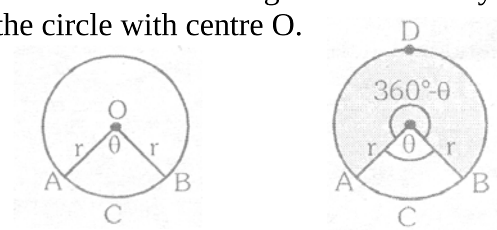
Minor arc = $\widehat{BQA}$

(J) **Segment :** A segment of a circle is the region bounded by an arc and its chord, including the arc and the chord.



The shaded segment containing the minor arc is called a minor segment, while the unshaded segment containing the major arc is called the major segment.

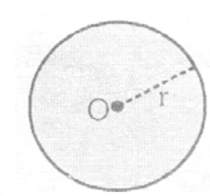
(K) **Sector of a circle :** A sector of a circle is a region enclosed by an arc and its two bounding radii. In the fig OACBO is a sector of the circle with centre O.



If arc AB is a minor arc then OACBO is a called the minor segment of the circle. The remaining part OADBO of the circle is called the major sector of the circle.

★ **FORMULA**

1. **For a circle of radius = r units, we have**



(a) Circumference of the circle = $(2\pi r)$ units = (πd) units, Where d is the diameter.

(b) Area of the circle = (πr^2) sq. units.

II. **For a semi-circle of radius = r units, we have**

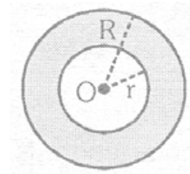
(a) Area of the semi-circle = $(\frac{1}{2}\pi r^2)$ sq. units

(b) Perimeter of the semi-circle = $(\pi r + 2r)$ units.

III. Area of a Circular Ring :

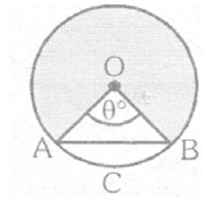
If R and r be the outer and inner radii of a ring, then

Area of the ring = $\pi(R^2 - r^2)$ sq. units



IV Results on Sectors and Segments :

Suppose an arc ACB makes an angle θ at the centre O of a circle of radius = r units. Then :



(a) Length of arc ACB = $\left(\dfrac{2\pi r\theta}{360}\right)$ units

(b) Area of sector OACBO = $\left(\dfrac{\pi r^2\theta}{360}\right)$ sq. units

$= \dfrac{1}{2} x\, r\, x \left(\dfrac{2\pi r\theta}{360}\right)$ sq. units = $\left(\dfrac{1}{2} x \text{ radius } x \text{ arc length}\right)$ sq. units

(c) Perimeter of sector OACBO = length of arc ACB + OA + OB = $\left(\dfrac{2\pi r\theta}{360} + 2r\right)$ units.

(d) Area of segment ACBA = (Area of sector OACBO) – (Area of ΔOAB) = $\left(\dfrac{\pi r^2\theta}{360} - \dfrac{1}{2} r^2 \sin\theta\right)$ sq. units.

(e) Perimeter of segment ACBA = arc ACB + chord AB) units.

(f) Area of Major segment BDAB = (Area of circle) – (Area of segment ACBA).

V. Rotations Made By a Wheel:

(a) Distance moved by a wheel in 1 revolution = Circumference of the wheel

(b) Number of rotations made by a wheel in unit time = $\dfrac{\text{Distance moved by it in unit time}}{\text{Circuference of the wheel}}$

VI. Facts About Clocks:

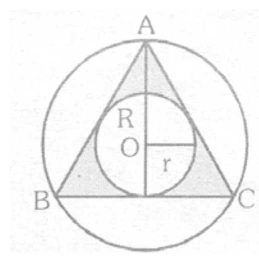
(a) Angle described by minute hand in 60 minutes = 360^o

(b) Angle described by minute hand in 5 minutes = $\left(\dfrac{360}{60} x\ 5\right)^o = 30^o$

(c) Angle described by hour hand in 12 hours = 360^o.

Angle described by hour hand in 1 hour = 30^o.

VII. In an equilateral triangle of side a units, we have:



(a) Height of the triangle, h = $\dfrac{\sqrt{3}}{2}$ a units.

(b) Area of the triangle = $\left(\dfrac{\sqrt{3}}{4} a^2\right)$ sq. units.

(c) Radius of incircle, r = $\dfrac{1}{3}$h = $\left(\dfrac{1}{3}.\dfrac{\sqrt{3}}{2} a\right) = \left(\dfrac{a}{2\sqrt{3}}\right)$ units.

(d) Radius of circumcircle, R = $\dfrac{2}{3} h = \left(\dfrac{2}{3}.\dfrac{\sqrt{3}}{2} a\right) = \left(\dfrac{a}{\sqrt{3}}\right)$ units.

Thus, r = $\dfrac{a}{2\sqrt{3}}$ and R = $\dfrac{a}{\sqrt{3}}$

Ex.1 Calculate the circumference and area of a circle of radius 5.6 cm.

Sol. We have :

$$Circum\ ference\ of\ the\ circle = 2\pi r = \left(2\ x \frac{22}{7} x 5.6\right)\ cm = 35.2\ cm.$$

$$\text{Area of the circle} = \pi r^2 = \left(\frac{22}{7} x 5.6 x 5.6\right) cm^2 = 98.56\ cm^2.$$

Ex.2 The circumference of a circle is 123.2 cm. Calculate :
(i) the radius of the circle in cm,
(ii) the area of the circle, correct to nearest cm^2.

Sol. (i) Let the radius of the circle be r cm.
Then, its circumference = $(2\pi r)$ cm.

$\therefore$ $2\pi r = 123.2 \Rightarrow 2\ x\ \frac{22}{7}\ x\ r = 123.2 \Rightarrow r = (123.2\ x\ \frac{7}{44}) = 19.6$ cm.

$\therefore$ Radius of the circle = 19.6 cm.

(ii) Area of the circle = $\pi r^2 = (\frac{22}{7}\ x\ 19.6\ x\ 19.6)\ cm^2 = 1207.36\ cm^2$.

$\therefore$ Area of the circle, correct to nearest cm^2 =m 1207 cm^2.

Ex.3 The area of a circle is 301.84 cm^2. Calculate:
(i) the radius of the circle in cm.
(ii) the circumference of the circle, correct to nearest cm.m

Sol. (i) Let the radius of the circle be r cm.
Then, its area = $\pi r^2\ cm^2 = 301.84$

$\Rightarrow$ $\frac{22}{7}\ x\ r^2 = 301.84$

$\Rightarrow$ $r^2 = \left(301.84\ x\ \frac{7}{22}\right) = 96.04 \Rightarrow r = \sqrt{96.04} = 9.8$ cm.

$\therefore$ Radius of the circle = 9.8 cm.

(ii) Circumference of the circle = $2\pi r = \left(2 x \frac{22}{7} x\ 9.8\right)$ cm = 61.6 cm.

$\therefore$ Circumference of the circle, correct to nearest cm = 62 cm.

Ex.4 The perimeter of a semi-circular protractor is 32.4 cm. Calculate :
(i) the radius of the protractor in cm,
(ii) the area of the protractor in cm^2.

Sol. (i) Let the radius of the protractor be r cm.
Then, its perimeter = $(\pi r + 2r)$ cm.

$\therefore$ $\pi r + 2r = 32.4 \Rightarrow (\pi + 2)r = 32.4$

$\Rightarrow \left(\frac{22}{7} + 2\right) r = 32.4 \Rightarrow \frac{36}{7} r = 32.4 \Rightarrow r = \left(32.4 x \frac{7}{36}\right)$ cm = 6.3 cm.

Radius of the protractor = 6.3 cm.

(ii) Area of the protractor = $\frac{1}{2}\pi r^2 = \left(\frac{1}{2} x \frac{22}{7} x\ 6.3\ x\ 6.3\right)\ cm^2 = 62.37\ cm^2$.

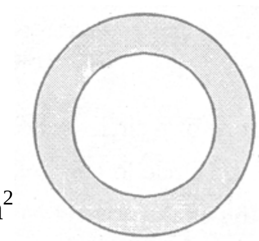
$\therefore$ Area of the protractor = 62.37 cm^2.

Ex.5 The area enclosed by the circumferences of two concentric circles is 346.5 cm^2. If the circumference of the inner circle is 88 cm, calculate the radius of the outer circle.

Sol. Let the radius of inner circle be r cm.

The, its circumference = $(2\pi r)$ cm.

$\therefore 2\pi r = 88 \Rightarrow 2 \ x \ \frac{22}{7} x r = 88 \Rightarrow r = \left(88 x \frac{7}{44}\right)$ = 14 cm.



$\therefore$ Radius of the inner circle is, r = 14 cm.

Let the radius of the outer circle be R cm.

Than, area of the ring = $(\pi R^2 - \pi r^2)$ cm^2

$= \pi (R^2 - r^2)\ cm^2 = \frac{22}{7}$ x $[R^2 - (14)^2]\ cm^2 = \left(\frac{22}{7} R^2 - 616\right) cm^2$

$\therefore \quad \frac{22}{7} R^2 - 616 = 346.5 \Rightarrow \frac{22}{7} R^2 = 962.5$

$\Rightarrow \quad R^2 = \left(962.5 x \frac{7}{22}\right) = 306.25 \Rightarrow R = \sqrt{306.25} = 17.5$ cm.

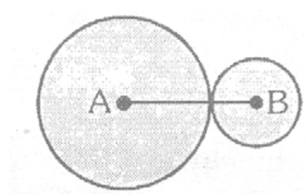
Hence, the radius of the outer circle is 17.5 cm.

Ex.6 Two circles touch externally. The sum of their areas is 130π sq. cm and distance between their centres is 14 cm. Determine the radii of the circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch externally, distance between their centres = (R + r) cm.

$\therefore \quad R + r = 14 \quad \ldots(i)$

Sum of their areas = $(\pi R^2 + \pi r^2)\ cm^2 = \pi (R^2 + r^2)\ cm^2$.



$\therefore \quad \pi (R^2 + r^2) = 130\pi$

$\Rightarrow \quad R^2 + r^2 = 130 \quad \ldots.(ii)$

We have the identity, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$(14)^2 + (R - r)^2 = 2 \times 130$ [From (i) and (ii)]

$(R - r)^2 = 64$

$R - r = 8 \quad \ldots.(iii)$

On solving (i) and (iii), we get R = 11 and r = 3.

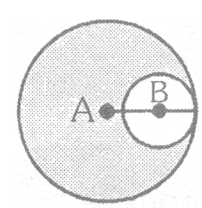
Hence, the radii of the given circles are 11 cm and 3 cm.

Ex.7 Two circles touch internally. The sum of their areas is 116π sq. cm and the distance between their centres is 6 cm. Find the radii of the given circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch internally, distance between their centres = (R – r) cm.

$\therefore \quad R - r = 6 \quad \ldots.(i)$

Sum of their areas = $(\pi R^2 + \pi r^2)\ cm^2 = \pi (R^2 + r^2)\ cm^2$



$\therefore \ \pi (R^2 + r^2) = 116\ \pi \Rightarrow R^2 + r^2 = 116 \quad \ldots(iii)$

We have the identity, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$\Rightarrow \quad (R + r)^2 + 6^2 = 2 \times 116$ [Using (i) and (ii)]

$\Rightarrow \quad (R + r)^2 = 196$

$\Rightarrow \quad R + r = \sqrt{196} = 14 \quad \ldots.(iii)$

On solving (i) and (iii), we get R = 10 and r = 4.

Hence, the radii of the given circles are 10 cm and 4 cm.

Ex.8 The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Give your answer, correct to nearest km.

Sol. Radius of the wheel = 42 cm.

Circumference of the wheel = $2\pi r = \left(2 x \frac{22}{7} x\ 42\right)$ cm = 264 cm.

Distance moved by the wheel in 1 revolution = 264 cm.
Distance moved by the wheel in 5 revolutions = (264 x 5) cm = 1320 cm.

$\therefore$ Distance moved by the wheel in 1 second = 1320 cm.
Distance moved by the wheel in 1 hour = (1320 x 60 x 60) cm.

$= \left(\frac{1320\ x\ 60\ x\ 60}{100\ x\ 1000}\right)$ km

$\therefore$ Speed of the cart = $\left(\frac{1320\ x\ 60\ x\ 60}{100\ x\ 1000}\right)$ km/hr = 47.52 km/hr.

Hence, the speed of the cart, correct to nearest km/hr is 48 km/hr.

Ex.9 The diameter of the driving wheel of a bus is 140 cm. How many revolutions must the wheel make in order to keep a speed of 66 km/hr?

Sol. Distance to be covered in 1 min. = $\left(\frac{66 x 1000}{60}\right)$ m = 1100 m.

Radius of the wheel = $\left(\frac{140}{2}\right)$ cm = 70 cm = 0.70 m.

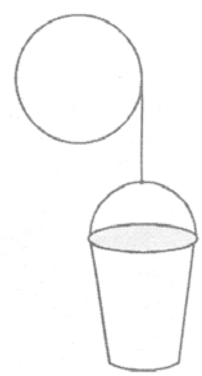
Circumference of the wheel = $2\pi r = \left(2 x \frac{22}{7} x 0.70\right)$ m = 4.4 m.

$\therefore$ Number of revolutions per minute = $\left(\frac{1100}{4.4}\right)$ = 250.

Hence, the wheel must make 250 revolutions per minute.

Ex.10 A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 min. 28 seconds with a uniform speed of 1.1 m / sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

Sol. Time taken by bucket to ascend = 1 min. 28 sec. = 88 sec. Speed = 1.1 m/sec.
Length of the rope = Distance covered by bucket to ascend
= (1.1m x 88) m = (1.1 x 88 x 100) cm = 9680 cm.



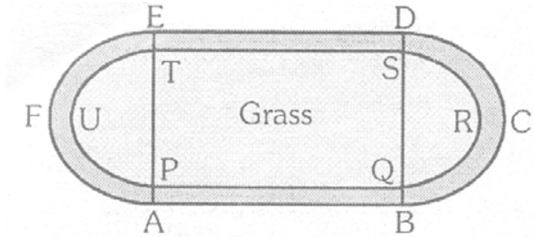
Radius of the wheel = $\frac{77}{2}$ cm.

Circumference of the wheel = $2\pi r = 2 x \left(\frac{22}{7} x \frac{77}{2}\right)$ cm = 242 cm.

` $\therefore$ Number of revolutions = $\frac{\textit{Length of the rope}}{\textit{Circumference of the wheel}} = \left(\frac{9680}{242}\right)$ = 40.

Hence, the wheel makes 40 revolutions to raise the bucket.

Ex.11 The figure shows a running track surrounding a grass enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semi-circular region at each end. Given, PQ = 200 m and PT = 70 m.



(i) Calculate the area of the grassed enclosure in m^2.

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF to the track.

Sol. (i) Diameter of each semi-circular region of grassed enclosure = PT = 70 m,

∴ Radius of each one of them = 35 m.

Area of grassed enclosure

$= \text{(Area of rect. PQST)} + 2 \text{ x } \frac{1}{2}\pi r^2 = \left[(200 x 70) + \frac{22}{7} x 35 x 35\right] \text{ m}^2 = 17850 \text{ m}^2.$

(ii) Diameter of each outer semi-circle of the track = AE = (PT + 7 + 7) m = 84 m.

∴ Radius of each one of them = 42 m.

Outer perimeter ABCDEF = (AB + DE + semi-circle BCD + semi-circle EFA)

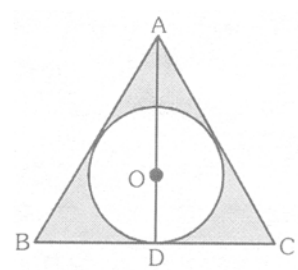
= (2PQ + 2 x circumference of semi-circle with radius 42 m)

$= (2 \text{ x } 200 + 2 \text{ x } \pi \text{ x } 42) \text{ m} = \left[2 \ x \ 200 + 2 \ x \frac{22}{7} x \ 42\right] \text{ m} = 664 \text{ m}.$

Ex.12 In an equilateral triangle of side 24 cm, a circle is inscribed, touching its sides. Find the area of the remaining portion of the triangle. Take $\sqrt{3}$ = 1.73 and π = 3.14.

Sol. Let ΔABC be the given equilateral triangle in which a circle is inscribed.

Side of the triangle, a = 24 cm.



Height of the triangle, $h = \left(\frac{\sqrt{3}}{2} x a\right) cm = \left(\frac{\sqrt{3}}{2} x \ 24\right) cm = 12\sqrt{3}\,cm.$

Radius of the incircle, $r = \frac{1}{3}h = \left(\frac{1}{3} x 12\sqrt{3}\right) cm = 4\sqrt{3}\,cm.$

∴ Required Area = Area of the shaded region

= (Area of ΔABC) – (Area of incircle)

$= \left(\frac{\sqrt{3}}{4} x \ 24 \ x \ 24 - \pi \ x \ 4\sqrt{3} \ x \ 4\sqrt{3}\right) \text{cm}^2$

$= (144\sqrt{3} - 3.14 \text{ x } 48) \text{ cm}^2 = (144 \text{ x } 1.73 - 3.14 \text{ x } 48) \text{ cm}^2$

$= [48 \text{ x } (3 \text{ x } 1.73 - 3.14)] \text{ cm}^2 = (48 \text{ x } 2.05) \text{ cm}^2 = 98.4 \text{ cm}^2$

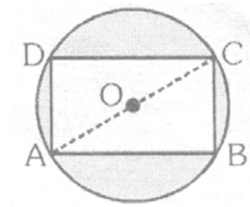
Ex.13 In the given figure, a circle circumscribes a rectangle with sides 12 cm and 9 cm. Calculate :

(i) the circumference of the circle to nearest cm,

(ii) the area of the shaded region, correct to 2 places of decimal, in cm^2.

Take $\pi = 3.14$.

Sol. Let ABCD be the rectangle with AB = 12 cm and BC = 9 cm.



$\therefore$ AC = $\sqrt{AB^2 + BC^2} = \sqrt{(12)^2 + 9^2} = \sqrt{225} = 15$ cm.

Let O be the mid-point of AC.

Then, O is the centre and OA, the radius of the circum-circle.

$\therefore$ Radius, OA = $\frac{1}{2}$AC = $\left(\frac{1}{2} x 15\right)$ cm = 7.5 cm.

$\therefore$ (i) Circumference of the circle = $2\pi r$ = (2 x 3.14 x 7.5) cm = 47.1 cm.

Hence, the circumference of the circle, correct to nearest cm is 47 cm.

(ii) Area of shaded region = (Area of the circle) – (Area of the rectangle)

$$= \left[\left(3.14 x \frac{15}{2} x \frac{15}{2}\right) - (12 x 9)\right] cm^2$$

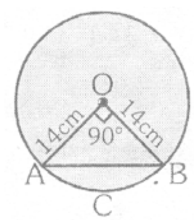
= (176.625 – 108) cm^2 = 68.625 cm^2 = 68.63 cm^2.

Ex.14 A chord of a circle of radius 14 cm makes a right angle at the centre. Calculate :

(i) the area of the minor segment of the circle, (ii) the area of the major segment of the circle.

Sol. Let AB be the chord of a circle with centre O and radius 14 cm such that $\angle$AOB = 90^o.

Thus, r = 14 cm and $\theta = 90^o$.



(i) Area of sector OACB = $\frac{\pi r^2 \theta}{360} = \left(\frac{22}{7} x 14 x \frac{90}{360}\right)$ cm^2 = 154 cm^2.

Area of ΔOAB = $\frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} x 14 x 14 x \sin 90^o\right)$ cm^2 = 98 cm^2.

$\therefore$ Area of minor segment ACBA = (Area of sector OACB) – (Area of ΔOAB) = (154 – 98) cm^2 = 56 cm^2.

(ii) Area of major segment BDAB

= (Area of the circle) – (Area of minor segment ACBA)

= $\left[\left(\frac{22}{7} x 14 x 14\right) - 56\right]$ cm^2 = (616 – 56) cm^2 = 560 cm^2.

Ex.15 The minute hand of a clock is 10.5 cm long. Find the area swept by it in 15 minutes.

Sol. Angle described by minute hand in 60 minutes = 360^o.

Angle described by minute hand in 15 minutes = $\left(\frac{360}{60} x 15\right)^o$ = 90^o.

Thus, required area is the area of a sector of a circle with central angle, $\theta = 90^o$. and radius, r = 10.5 cm.

Required area = $\left(\frac{\pi r^2}{360}\right) = \left(\frac{22}{7} x 10.5 x 10.5 x \frac{90}{360}\right)$ cm^2 = 86.63 cm^2.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. If the radii of two circles are 7 cm and 24 cm, then the radius of circle having area equal to the sum of the areas of the two circles, is

(A) 31 cm (B) 25 cm
(C) 17 cm (D) 28 cm

2. The cost of fencing a circular field at the rate of Rs. 24 per metre is RS. 5280. Then the cost of ploughing the field, at the rate of 50 paise/m^2, is

(A) Rs. 2875 (B) Rs. 3850
(C) Rs. 1925 (D) Rs. 1825

3. The inner circumference of a circular track is 220 m, and the track is 14 m wide. The cost of leveling the track, at 50 paise/m^2, is

(A) Rs. 1848 (B) Rs. 1663.2
(C) Rs. 1478.4 (D) None of these

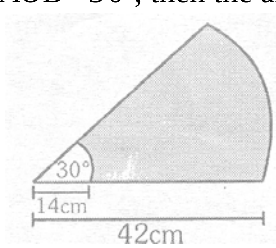
4. The area of a sector, of a circle with radius 7 cm and angle of the sector is 60^o. is

(A) $\frac{144}{3}$ cm^2 (B) $\frac{154}{21}$ cm^2
(C) $\frac{150}{7}$ cm^2 (D) $\frac{77}{3}$ cm^2

5. In a circle of radius radius 21 cm, an are subtends an angle of the centre. The area of the segment formed by the corresponding chord of the arc is

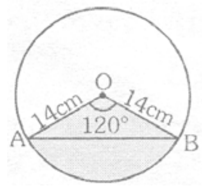
(A) 40.63 cm^2 (B) 421.73 cm^2
(C) 429.43 cm^2 (D) 40.27 cm^2

6. AB and CD are respectively arcs of two concentric circles of radii 42 cm and 14 cm and centre O as shown in the adjoining figure. If $\angle AOB = 30^o$, then the area of the shaded region is



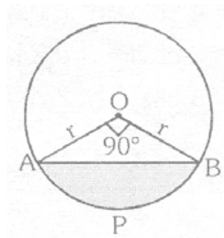
(A) $\frac{1232}{3} cm^2$ (B) $\frac{1220}{3} cm^2$
(C) 411cm^2 (D) None of these

7. In the given figure, the shaded area is



(A) 205.03 cm^2
(B) 205.04 cm^2
(C) 205.33 cm^2
(D) 205.35 cm^2

8. In the given figure, the area of the segment APB is



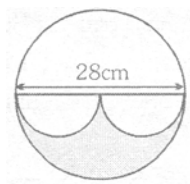
(A) $\frac{1}{4}\pi r^2$

(B) $\frac{1}{4}(\pi - 2)r^2$

(C) $\frac{1}{4}(\pi - 1)r^2$

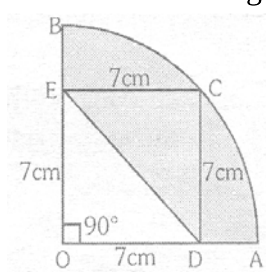
(D) None of these

9. In the given figure, the area of shaded region is



(A) 462 cm^2
(B) 308 cm^2
(C) 616 cm^2
(D) 154 cm^2

10. In the given figure, ODCE is a square then the area of shaded region is



(A) 52.5 cm^2 (B) 24.5 cm^2 (C) 49 cm^2 (D) None of these

OBJECTIVE					ANSWER KEY				EXERCISE	
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	A	D	D	A	C	B	D	A

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Remark : Take $\pi = \frac{22}{7}$, unless mentioned otherwise.

1. A sheet is 11 cm long and 2 cm wide. Circular pieces 0.5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared.

2. Find the circumference and area of a circle of radius 17.5 cm.

3. Find the circumference and area of a circle of diameter 91 cm.

4. Find the circumference and area of a circle of radius 15 cm. (Take $\pi = 3.14$)

5. The circumference of a circle is 123.2 cm. Taking $\pi = \frac{22}{7}$, calculate:

(i) the radius of the circle in cm;
(ii) the area of the circle in cm^2, correct to the nearest cm^2;
(iii) the effect on the area of the circle if the radius is doubled

6. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 m^2.

7. The area of a circle is 394.24 cm^2. Calculate : (i) the radius of the circle, (ii) the circumference of the circle.

8. Find the perimeter and area of a semi-circular of a plate of radius 25 cm (Take $\pi = 3.14$).

9. The perimeter of a semi-circular metallic plate is 86.4 cm. Calculate the radius and area of the plate.

10. The circumference of a circle exceeds its diameter by 180 cm. Calculate
(i) the radius (ii) the circumference and (iii) the area of the circle.

11. A copper wire when bent in the form of a square encloses an area of 272.25 cm^2. If the same wire is bent into the form of a circle, what will be the area enclosed by the wire?

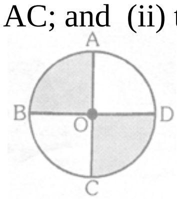
12. A copper wire when bent in the form of a equilateral triangle has an area of $121\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle, find the area enclosed by the wire.

13. The circumference of a circle field is 528 m.

14. The cost of levelling a circular field at Rs2 per sq. metre is Rs 33957. Calculate:
(i) the area of the field; (ii) the radius of then field; (iii) the circumference of the field;
(iv) the cost of fencing it at Rs 2.75 per metre.

15. The cost of fencing a circular field at Rs 9.50 per metre is Rs 2926. Find the cost of ploughing the field at Rs 1.50 per sq. metre.

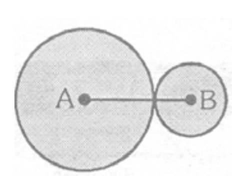
16. AC and BD are two perpendicular diameters of a circle ABCD. Given that the area of the shaded portion is 308 cm^2, calculate : (i) the length of AC; and (ii) the circumference of the circle.



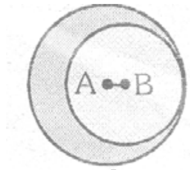
17. The sum of the radii of two circles is 140 cm and the difference of their circumference is 88 cm. Find the radii of the two circles.

18. The sum of the radii of two circles is 84 cm and the difference of their areas is 5544 cm^2. Calculate the radii of the two circles .

19. Two circles touch externally. The sum of their areas is 117π cm^2 and the distance between their centres is 15 cm. Find the radii of the two circles.

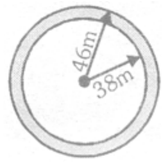


20. Two circles touch internally. The sum of their areas is and the distance between then centres is 4 cm. Find the radii of the circles.



21. Find the area of a ring whose outer and inner radii are 19 cm and 16 cm respectively.

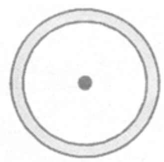
22. A path of width 8 m runs around a circular park whose radius is 38 m. Find the area of the path.



23. The areas of two concentric circles are 962.5 cm^2 and 1386 cm^2 respectively. Find the width of the ring.

24. The area enclosed between two concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, calculate the radius of the inner circle.

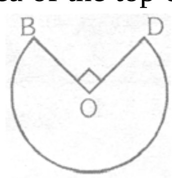
25. In the given figure, the area enclosed between two concentric circles is 808.5 cm^2. The circumference of the outer circle is 242 cm. Calculate : (i) the radius of the inner circle, (ii) the width of the ring.



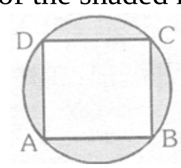
26. Find the area of a circle circumscribing an equilateral triangle of side 15 cm. [Take π = 3.14].

27. Find the area of a circle inscribed in an equilateral triangle of side 18 cm. [Take π = 3.14].

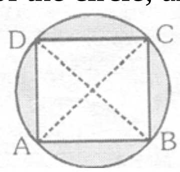
28. The shape of the top of a table in a restaurant is that of a segment of a circle with centre O and $\angle$BOD = 90^o. BO = OD = 60 cm. Find: (i) the area of the top of the table; (ii) the perimeter of the table. [Take π = 3.14].



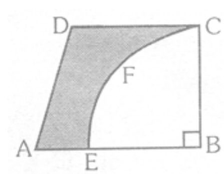
29. In the given figure, ABCD is a square of side 5 cm inscribed in a circle. Find:
(i) the radius of the circle, (ii) the area of the shaded region. [Take π = 3.14]



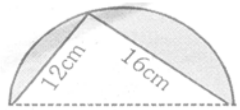
30. In the given figure, ABCD is a rectangle inscribed in a circle. If two adjacent sides of the rectangle be 8 cm and 6 cm, calculate : (i) the radius of the circle; and (ii) the area of the shaded region. [Take π = 3.14].



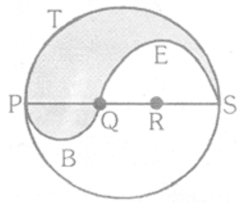
31. In the given figure, ABCD is a piece of cardboard in the shape of a trapezium in which AB || DC, $\angle$ABC = 90^o. From this piece, quarter circle BEFC is removed. Given DC = BC = 4.2 cm and AE = 2 cm. Calculate the area of the remaining piece of the cardboard.



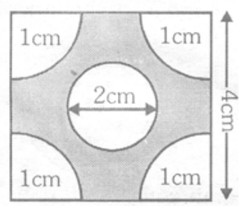
32. Find the perimeter and area of the shaded region in the given figure. (Take π = 3.142).



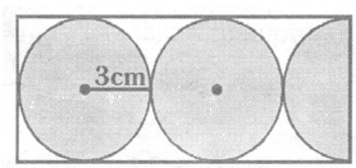
33. In the given figure, PQRS is a diameter of circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters. If PS = 12 cm, find the perimeter and the area of the shaded region. [Take π = 3.14].



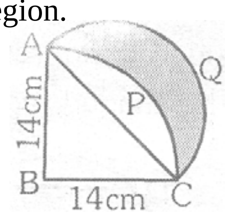
34. Find the perimeter and area of the shaded region shown in the figure. The four corners are circle quadrants and at the centre, there is a circle. [Take π = 3.14].



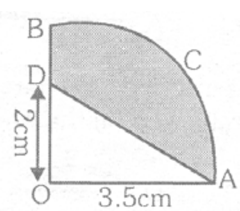
35. In the given figure, find the area of the unshaded portion within portion within the rectangle. [Take π = 3.14].



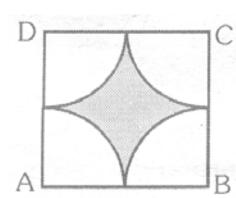
36. In the given figure, ABCP is a quadrant of circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded region.



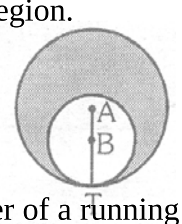
37. In the given, OACB is a quadrant of a circle. The radius OA = 3.5 cm, OD = 2 cm. Calculate the area of the shaded portion.



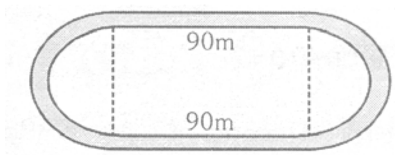
38. In the given figure, ABCD is a square of side 14 cm and A, B, C, D are centres of circular arcs, each of radius 7 cm. Find the area of the shaded region.



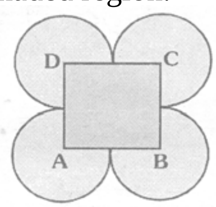
39. In the given figure, two circles with centres A and B touch each other at the point T. If AT = 14 cm and AB = 3.5 cm, find the area of the shaded region.



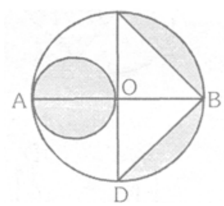
40. In the adjoining figure, the inside perimeter of a running track with semi-circular ends and straight parallel sides is 312 m. The lengths of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



41. In the given figure, ABCD is a square of side 7 cm and A, B, C, D are centres of equal circles which touch externally in pairs. Find the area of the shaded region.



42. In the given figure, AB is the diameter of a circle with centre O and OA = 7 cm. Find the area of the shaded region.



43. The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions?

44. The wheel of the engine of a train $4\frac{2}{7}$ m in circumference makes 7 revolutions in 3 seconds. Find the speed of the train in km per hour.

45. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 30 revolutions?

46. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

AREAS RELATED TO CIRCLES **ANSWER KEY** EXERCISE – 2 (X)-CBSE

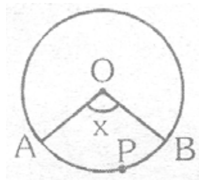
1. 88 **2.** 110 cm, 962.5 cm^2 **3.** 286 cm, 6506.5 cm^2 **4.** 94.2 cm, 706.5 cm^2
5. (i) 19.6 cm (ii)mm 1207 cm^2 (iii) area becomes four times. **6.** 56 m **7.** (i) 1.2 cm (ii) 70.4cm
8. 128.5 cm, 981.25 cm^2 **9.** 168. cm, 443.52 cm^2 **10.** (i) 42 cm (ii) 264 cm (iii) 5544 cm^2 **11.** 346.5 cm^2
12. 346.5 cm^2 **13.** (i) 84 m (ii) 22176 m^2 (iii) Rs 33264 **14.** (i) 16978.5 m^2 (ii) 73.5 m (iii) 462 m (iv) Rs 1270.50 **15.** Rs 11319 **16.** (i) 28 cm (ii) 88 cm **17.** 77 cm, 63cm **18.** 52.5 cm; 31.5 cm
19. 9 cm, 6 cm **20.** 11 cm, 7 cm **21.** 330 cm^2 **22.** 2112 m^2 **23.** 3.5 m^2 **24.** 14 cm
25. 35 cm, 3.5 cm **26.** 235.5 cm^2 **27.** 84.78 cm^2 **28.** (i) 8478 cm^2 (ii) 402.60 cm
29. (i) $\frac{5}{2}\sqrt{2}$ cm (ii) 14.25 cm^2 **30.** (i) 5 cm (ii) 30.5 cm^2 **31.** 7.28 cm^2 **32.** 59.4 cm, 61.1 cm^2
33. 37.68 cm, 37.68 cm^2 **34.** 20.56 cm, 9.72 cm^2 **35.** 19.35 cm^2 **36.** 98 cm^2 **37.** 6.125 cm^2
38. 42. cm^2 **39.** 269.5 cm^2 **40.** 636.57 m^2 **41.** 164.5 cm^2 **42.** 66.5 cm^2 **43.** 1980 m
44. 36 km/hr **45.** 50 **46.** 14 m

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

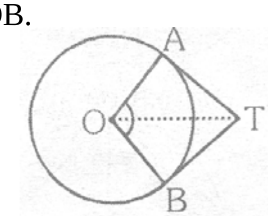
VERY SHORT ANSWER TYPE QUESTIONS

1. In the fig. O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.

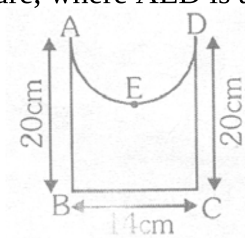


[Delhi-2008]

2. In fig., if $\angle ATO = 40^{o}$, find $\angle$ AOB. **[AI-2008]**



3. Find the perimeter of the given figure, where AED is a semi circle and ABCD is a rectangle. **[AI-2008]**

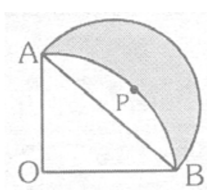


4. If the diameter of a semicircular protractor is 14 cm, then find it's perimeter. **[AI-2009]**

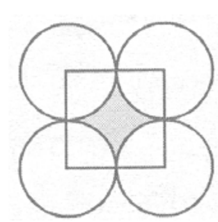
5. The length of the minute hand of a wall clock is 7 cm. How much area does it sweep in 20 minutes? **[Foreign-2009]**

SHORT ANSWER TYPE QUESTIONS

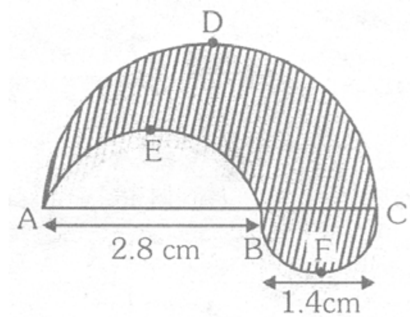
1. In fig. AOBPA is quadrant of a circle of radius 14 cm. A semicircle with AB as diameter is draw. Find the area of the shaded region.



2. Four circles are described about the four corners of a square of a square so that each touches two of the others as shown in fig. Find the area of the shaded region. Each side of the square is 14 cm. (Take π = 22/7) **[Delhi-2007]**

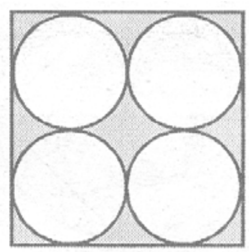


3. In the fig., find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively.

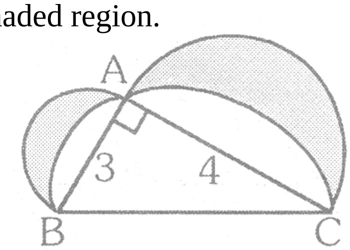


OR

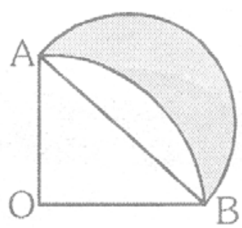
Find the area of the shaded region in the fig., where ABCD is a square of side 14 cm. **[Delhi-2008]**



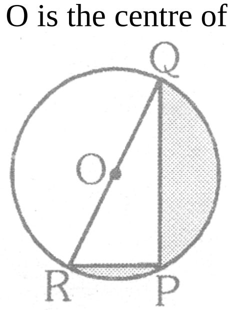
4. In fig., ABC is a right-angled triangle, right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. **[AI-2008]**



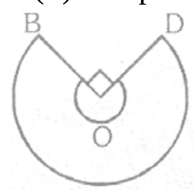
5. In the fig., ABC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region. **[Foreign-2008]**



6. In fig., PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Find the area of shaded region. (Take π = 3.14) **[Delhi-2009]**

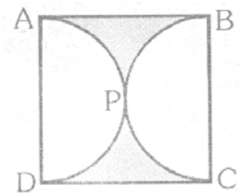


7. In figure, the shape of the top of a table in a restaurant is that of a sector of a circle with centre O and $\angle BOD = 90^o$. If BO = OD = 60 cm, find.
(i) the area of the top of the table (ii) The perimeter of the table top. (Take π = 3.14)



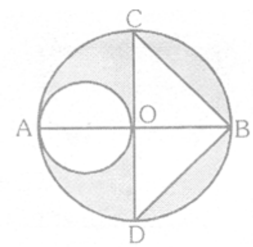
OR

In fig., ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area of shaded region.



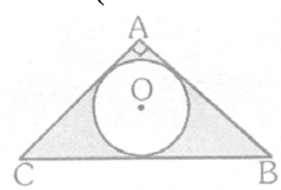
(Take π = 22/7) **[Foreign-2009]**

8. In fig., AB and CD are two perpendicular diameters of a circle with centre O. If OA = m7 cm. find the area of the shaded region.
(Take π = 22/7) **[AI-2010]**



LONG ANSWER TYPE QUESTIONS

1. In fig., ABC is a right triangle right angled at A. Find the area of shaded region if AB = 6 cm, BC = 10 cm and O is the centre of the incircle of ΔABC. (Take π = 3.14) **[Delhi-2009]**



2. The area of an equilateral triangle is $49\sqrt{3}\,cm^2$. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. (Take $\sqrt{3}$ = 1.73)

AREAS RELATED TO CIRCLES **ANSWER KEY** EXERCISE – 3 (X)-CBSE

- **VERY SHORT ANSWER TYPE QUESTIONS**

  1. 100^o **2.** 100^o **3.** $(7\pi + 54)$ cm **4.** 36 cm **5.** $\frac{154}{3}$ cm^2

- **SHORT ANSWER TYPE QUESTIONS**

  1. 98 cm^2 **2.** 42 cm^2 **3.** 13.2 cm or 42 cm^2 **4.** 6 sq. units **5.** 98 cm^2 **6.** 161.3 cm^2

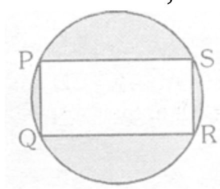
- **LONG ANSWER TYPE QUESTIONS**

  1. 11.44 cm^2 **2.** 7.77 cm^2

EXERCISE – 4 (FOR OLYMPIADS)

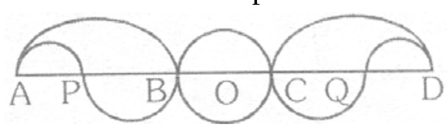
CHOOSE THE CORRECT ONE

1. In the adjoining figure PQRS is a rectangle 8 cm x 6 cm, inscribed in the circle. The area of the shaded portion will be :



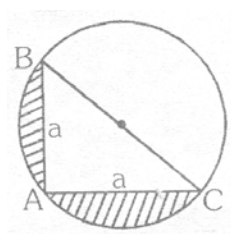
(A) 48 cm^2
(B) 42.50 cm^2
(C) 32.50 cm^2
(D) 30.5 cm^2

2. In the adjoining figure AB = CD = 2BC = 2BP = 2 CQ. In the middle, a circle with radius 1 cm is drawn. In the rest figure all are the semicircular arcs. What is the perimeter of the whole figure?



(A) 4π
(B) 8π
(C) 10π
(D) None of these

3. If BC passes through centre of the circle, then the area of the shaded region in the given figure is :



(A) $\frac{a^2}{2}(3-\pi)$
(B) $a^2\left(\frac{\pi}{2}-1\right)$
(C) $2a^2(\pi - 1)$
(D) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$

4. Two circles of unit radii, are so drawn that the centre of each lies on the circumference of the other. The area of the region common to both the circles, is :

(A) $\frac{(4\pi-3\sqrt{3})}{12}$ (B) $\frac{(4\pi-6\sqrt{3})}{12}$
(C) $\frac{(4\pi-3\sqrt{3})}{6}$ (D) $\frac{(4\pi-6\sqrt{3})}{6}$

5. The area of the largest possible square inscribed in a circle of unit radius (in square unit) is :

(A) 3 (B) 4 (C) $2\sqrt{3}\pi$ (D) 2

6. The area of the largest triangle that can be inscribed in a semicircle of radius r is:

(A) r^2 cm^2 (B) $\left(\frac{r}{3}\right)^2 cm^2$ (C) $r\sqrt{2}$ cm^2 (D) $3\sqrt{3r}$ cm^2

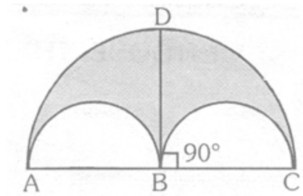
7. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is :

(A) $6\sqrt{3r}$ (B) 6r (C) 3r (D) 12r

8. If a regular circumscribes a circle of radius r, then its perimeter is :

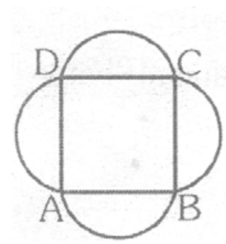
(A) $4\sqrt{3r}$ (B) $6\sqrt{3r}$ (C) 6r (D) $12\sqrt{3r}$

9. In the adjoining figure there are three semicircles in which BC = 6 cm and BD = $6\sqrt{3}$ cm. What is the area of the shaded region (in cm):



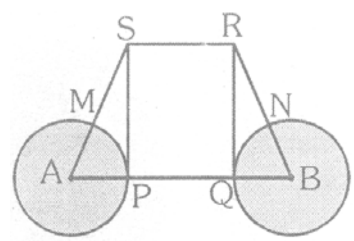
(A) 12π
(B) 9π
(C) 27π
(D) 28π

10. ABCD is a square of side a cm. AB, BC, CD and AD all are the chords of circles with equal radii each. It the chords subtends an angle of 120^o at their respective centres, find the total area of the given figure, where arcs are part of the circles:



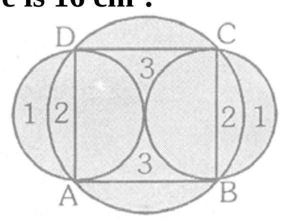
(A) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{3\sqrt{2}}\right)\right]$
(B) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}\right)\right]$
(C) $[9a^2 - 4\pi + 3\sqrt{3a^2}]$
(D) None of these

11. In the adjoining figure PQRS is a square and MS = RN and A, P, Q and B lie on the same line. Find the ratio of the area of two circles to the area of the square. Given that AP = Ms.



(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$
(C) $\frac{3\pi}{2}$ (D) $\frac{6}{\pi}$

Direction for questions number (12 to 14) : In the adjoining figure ABCD is a square. A circle ABCD is passing through all the four vertices of the square. There are two more circles on the sides AD and BC touching each other inside the square, AD and BC are the respective diameters of the two smaller circles. Area of the square is 16 cm^2.

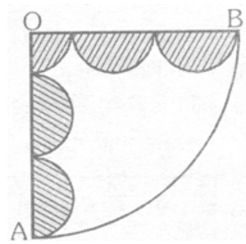


12. What is the area of region 1?
(A) 2.4 cm^2 (B) $\left(2 - \frac{\pi}{4}\right)cm^2$
(C) 8 cm^2 (D) $(4\pi - 2)$ cm^2

13. What is the area of region 2?
(A) $3(\pi - 2)$ cm^2 (B) $(\pi - 3)$ cm^2 (C) $(2\pi - 3)$ cm^2 (D) $4(\pi - 2)$ cm^2

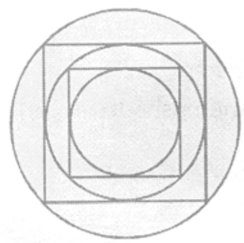
14. What is the area of region 3?
(A) $(4 - 4\pi)$ cm^2 (B) $4(4 - \pi)$ cm^2 (C) $(4\pi - 2)$ cm^2 (D) $(3\pi + 2)$ cm^2

15. A circular paper is folded along its diameter, then again it is folded to form a quadrant. Then it is cut as shown in the figure, after it the paper was reopened in the original circular shape. Find the ratio of the original paper to that of the remaining paper? (The shaded portion is cut off from the quadrant. The radius of quadrant OAB is 5 cm and radius of each semicircle is 1 cm) :



(A) 25 : 16 (B) 25 : 9 (C) 20 : 9 (D) None of these

Directions for questions number 16-18 : A square is inscribed in a circle then another circle is inscribed in the square. Another square is then inscribed in the circle. Finally a circle is inscribed in the innermost square. Thus there are 3 circles and 2 squares as shown in the fig. The radius of the outer-most circle is R.



16. What is the radius of the inner-most circle?

(A) $\frac{R}{2}$ (B) $\frac{R}{\sqrt{2}}$ (C) $\sqrt{2}R$ (D) None of these

17. What is the sum of areas of all the squares shown in the figure?

(A) $3R^2$ (B) $3\sqrt{2}R^2$ (C) $\frac{3}{\sqrt{2}}R^2$ (D) None of these

18. What is the ratio of sum of circumferences of all the circles to the sum of perimeters of all the squares?

(A) $(2+\sqrt{3})\pi R$ (B) $(3+\sqrt{2})\pi R$ (C) $3\sqrt{3}\pi R$ (D) None of these

Directions for questions number 19-21 : A regular hexagon is inscribed in a circle of radius R. Another circle is inscribed in the hexagon. Now another hexagon is inscribed in the second (smaller) circle.

19. What is the sum of perimeters of both the hexagons?

(A) $(2+\sqrt{3})R$ (B) $3(2+\sqrt{3})R$ (C) $3(3+\sqrt{2})R$ (D) None of these

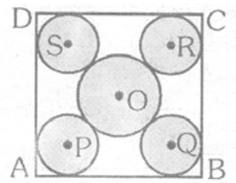
20. What is the ratio of area of inner circle to the outer circle?

(A) 3 : 4 (B) 9 : 16 (C) 3 : 8 (D) None of these

21. If there are some more circles and hexagons inscribed in the similar way as given above, then the ratio of each side of outermost hexagon (largest one) to that of the fourth (smaller one) hexagon is (fourth hexagon means the hexagon which is inside the third hexagon from the outerside.):

(A) $9 : 3\sqrt{2}$ (B) 16 : 9 (C) $8 : 3\sqrt{3}$ (D) None of these

22. In the adjoining diagram ABCD is a square with side 'a' cm. In the diagram the area of the larger circle with centre 'O' is equal to the sum of the areas of all the rest four circles with equal radii, whose centres are P, Q, R, and S. What is the ratio between the side of square and radius of a smaller circle?



(A) $(2\sqrt{2}+3)$ (B) $(2+3\sqrt{2})$ (C) $(4+3\sqrt{2})$ (D) Can't be determined.

23. There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?

(A) 32 cm^2 (B) 64π cm^2 (C) 36 cm^2 (D) 48 cm^2

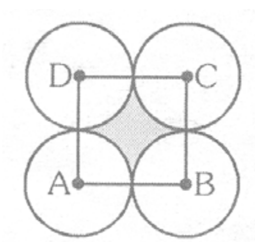
24. ABCD is a square, 4 equal circles are just touching each other whose centres are the vertices A, B, C, D of the square. What is the ratio of shaded to the unshaded area within square?

(A) $\frac{8}{11}$

(B) $\frac{3}{11}$

(C) $\frac{5}{11}$

(D) $\frac{6}{11}$



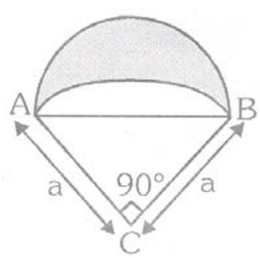
25. In the adjoining figure ACB is a quadrant with radius 'a'. A semicircle is drawn outside the quadrant taking AB as a diameter. Find the area of shaded region :

(A) $\frac{1}{4}(\pi - 2a^2)$

(B) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$

(C) $\frac{a^2}{2}$

(D) Can't be determined



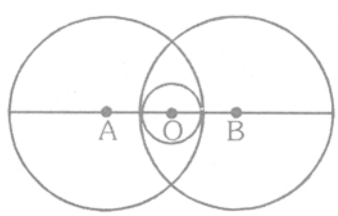
26. There are two circles intersecting each other. Another smaller circle with centre O, is lying between the common region of two larger circles. Centre of the circle (i.e., A, O and B) are lying on a straight line. AB = 16 cm and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?

(A) 4π cm^2

(B) 2π cm^2

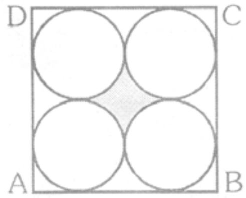
(C) $\frac{4}{\pi}$ cm^2

(D) $\frac{\pi}{4}$ cm^2



27. ABCD is a square, inside which 4 circles with radius 1 cm, each are touching each other. What is the area of the shaded region?

(A) $(2\pi - 3)\ cm^2$
(B) $(4 - \pi)\ cm^2$
(C) $(16 - 4\pi)\ cm^2$
(D) None of these



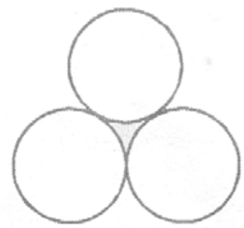
28. Three circles of equal radii touch each other as shown in figure. The radius of each circle is 1 cm. What is the area of shaded region?

(A) $\left(\frac{2\sqrt{3}-\pi}{2}\right) cm^2$

(B) $\left(\frac{3\sqrt{2}-\pi}{3}\right) cm^2$

(C) $\frac{2\sqrt{3}}{\pi}\ cm^2$

(D) None of these



OBJECTIVE						**ANSWER KEY**								EXERCISE – 4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	D	C	D	C	D	A	B	A	C	B	B	C	D	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
Ans.	A	A	D	B	A	C	B	B	B	C	A	B	A		