Open Text-based Assessment 2016-17

Mathematics

Class IX

Central Board of Secondary Education

Shiksha Kendra, 2, Community Centre, Preet Vihar, Delhi-110092, India

OPEN TEXT BASED ASSESSMENT 2016-17

Mathematics (041) Class - IX

Theme 1: Solving Mystery of messed up fields

Learning objectives

The learner shall be able to

- Differentiate the quadrilaterals on the basis of the properties of sides and angles.
- Differentiate the quadrilaterals on the basis of the properties of their diagonals.
- Identify the types of quadrilateral on the basis of properties possessed by them.
- Apply the following properties of various types of quadrilaterals and triangles in real life situations:

1. The diagonal of a parallelogram divides it into two congruent triangles.
2. In a parallelogram opposite sides are equal and the converse of this statement is also true i.e. in a quadrilateral if opposite sides are equal, quadrilateral is parallelogram.
3. In a parallelogram opposite angles are equal and the converse of this statement is also true.
4. A quadrilateral is a parallelogram if one pair of opposite sides are parallel and equal.
5. In a parallelogram, the diagonals bisect each other and the converse of this statement is also true.
6. In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and is half of it and the converse of this statement is also true.

A note to Readers

The reader should design and discuss some more questions which are open ended

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Mathematics (041) Class - IX

Theme 1: Solving Mystery of messed up fields

Abstract

You have studied the properties of different types of quadrilaterals and learnt to identify the type of quadrilateral when some conditions on sides or angles or diagonals are given. Generally, such topics are theoretical and dry since you are not exposed to their practical applications. This text is created to show the practical applications of the concepts learnt in geometry. You will know how a girl was able to use her knowledge of geometry to sort out the land issues of farmers in a village with the help of clues given by them in their common language. Such exposures will motivate you to have more interest in the subject and to understand and interpret the concepts learnt in context.

Roshni visited her native village in Assam during summer vacations. She asked her grandfather to show the fields. The grandfather told her very sadly that there was no use of going to farms as all farms were submerged in water and hence, destroyed in flood.

He said, “Landowners gave their field to farmers on lease for farming. But after flood no one is able to identify the boundaries of their farms and hence they are very upset. Roshni asked her grandpa, “Why are they upset. They must know the dimensions of their fields. According to the dimensions they can draw new boundaries and start working". The grandfather said, “You may be right. But some poor illiterate farmers are not able to tell the dimensions of the field and hence they are cheated by the shrewd ones. Every day there is a fight amongst farmers and our peaceful village is now becoming the land of controversies”. Roshni noticed that her grandfather was very much worried about farmer’s welfare. She was very anxious to help the poor farmers. She asked her grandfather about the shape of field of any one land owner. Grandpa told her that his neighbour Ramkishan had the largest field which was four sided and all sides were equal.

She came to know that Ramkishan had given his field on lease to 12 farmers.

Then Roshni talked to some villagers and enquired further and found that all farms were four sided one is of three sided. She thought for a moment and there was a glitter in her eyes.

Roshni:“Grandpa, I have come to know that all farms were four sided except one and hence,there were 11 quadrilaterals and 1 triangle. I have learnt about the properties of quadrilaterals in my class. I think I can help all farmers to relocate the boundaries of their fields.”

Grandpa (surprisingly): “How?”

Roshni: “I need some information from the farmers. Think of me as a detective who has come to the village to solve the mystery. Please, arrange a meeting with them. I will interrogate them. Also, arrange for one measuring tape.”

Grandpa called all 12 farmers whose fields were affected. Roshni asked the names of all the farmers and started taking the information from the farmers. She translated the local language of the farmers and prepared the list of mathematical properties possessed by their respective farms in the table given below:

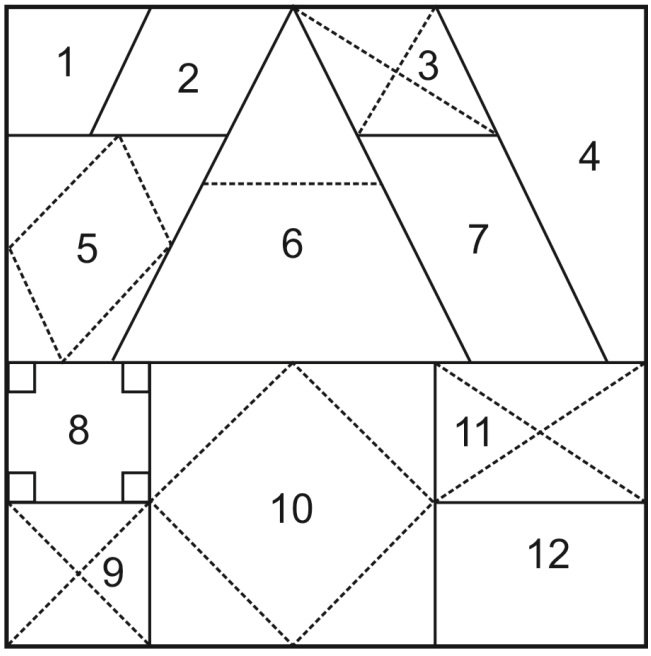
TABLE 1

S.No.	Name of the farmer	Interpretations of statements shared by farmers
1	Krishna	My farm was the corner one. One side was parallel to the boundary.
2	Dorjee	My farm was adjacent to Krishna's farm and one side was along the boundary. In my farm all sides were equal.
3	Dhondoop	One side of my farm was along the boundary but the lengths of the ropes joining opposite corners were not equal. But, the ropes at the point of intersection made exact "L" shape.
4	Rehman	Two sides were along the boundary. The third side was common with those of Dhoondoop and Uttapa's field. The fourth side was the smallest side.
5	Jeevan	I used to divide the field in five parts by joining the mid points of the adjacent sides of the field to grow five different varieties of crops.
6	Oonkar	My farm was a big three sided field. One side was common with Dorjee's and Jeevan's field and the other was common with Dhoondoop and Uttapa's field. I used to divide the field along the rope joining the mid points of these sides to obtain two parts whose areas were in the ratio 1:3
7	Uttapa	In my field opposite sides were equal. One was common with Dhoondoop's field.
8	Yousuf	In my field all sides were equal and eachangles measured 90^0.
9	Nekchand	In my field when I used to join the opposite corners with ropes, the lengths of the ropes required were equal and the two ropes bisected each other.
10	Laxminarayan	We are five brothers working on the same field and have a big field with equal sides. I used to divide the field in five parts by joining the mid points of the adjacent sides of the field. The lengths of ropes required to join the midpoints of the adjacent sides were also equal.
11.	Ram	In my field I used to join the opposite corners with ropes of equal length and the areas of the opposite triangles formed were equal.
12.	David	In my field opposite sides were equal. It used to look like Ram's field.

After collecting all information she made a table of possible shapes and a rough sketch of the land used by farmers and showed them the shape of their fields.

S.No.	Name of the farmer	Mathematical Interpretations of statements sharedby farmers
1	Krishna	Trapezium
2	Dorjee	Square or Rhombus
3	Dhondoop	Rhombus or Kite
4	Rehman	Any Quadrilateral
5	Jeevan	Any Quadrilateral
6	Oonkar	Triangle
7	Uttapa	Parallelogram
8	Yousuf	Square
9	Nekchand	Square or rectangle
10	Laxminarayan	Square
11.	Ram	Rectangle
12.	David	Rectangle

Roshni found out from the farmers that the rope-segments used to either join two points, corner points, the mid points of the sides of the field or to divide the field in different parts were all straight when their lengths were determined. When they confirmed about the shapes, she asked for few more clues and finalized the following division shapes of the field of the farmers.



Sample questions

1. Read the statement of Oonkar and write the relation between the lines joining the mid-point of two sides with the third side of the triangle. State the theorem to justify your answer. Why are the areas of the two fields is in the ratio of 1:3 exactly. [3marks]

Oonkar	My farm was a big three sided field. One side was common with Dorjee's and Jeevan's field and the other was common with Dhoondoop and Uttapa's field. I used to divide the field along the rope joining the mid points of these sides to obtain two parts whose areas were in the ratio 1:3

2. Listening to Dhoondoop's Statement, Roshni concluded that his farm might be a Rhombus or a kite in shape. Do you agree with her opinion? Justify.

   Give other properties of a Rhombus. What is the shape of quadrilateral formed by joining the mid- points of the adjacent sides of a Rhombus. [4marks]

Dhondoop	One side of my farm was along the boundary but the lengths of the ropes joining opposite corners were not equal. But, the ropes at the point of intersection made exact "L" shape.

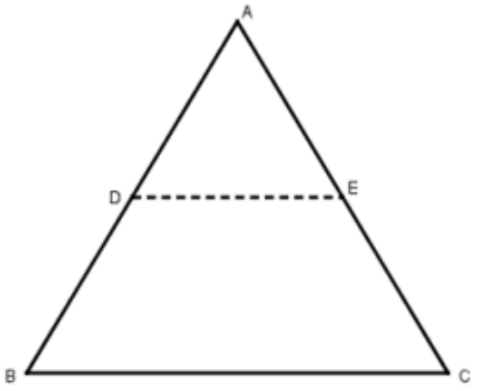
3. Listening to Laxminarayan's Statement, Roshni concluded that his farm might be a square in shape. Do you agree with her opinion? Justify.

   Give other properties of square. State any three properties of a square. [3 marks]

Laxminarayan	We are five brothers working on the same field and have a big field with equal sides. I used to divide the field in five parts by joining the mid points of the adjacent sides of the field.The lengths of ropes required to join the midpoints of the adjacent sides were also equal.

Marking scheme

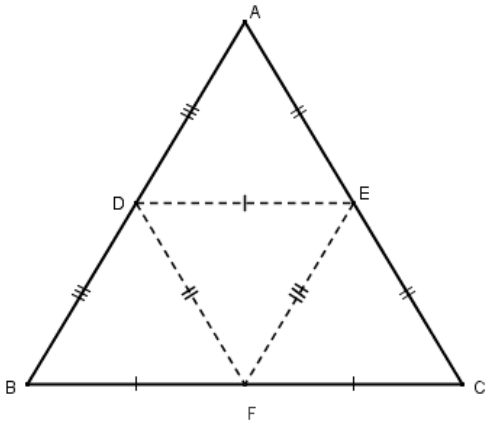
1.



Mid-point Theorem

(i) The line joining the mid pointsof two sides of a triangle is parallel to the third side and is half of it in length.

(ii) The four triangles formed by joining the mid points of the sides of a triangle are congruent to each other. Hence the area will be divided in the ratio 1:3.



2. (i) Yes, Dhoondoop had a four sided land, which was in the shape of a quadrilateral. Mathematically we can write the statement as follows:

If the diagonals of a quadrilateral are at right angles with one of them bisecting the other, but not equal in length then it is a Rhombus, or a Kite.

(ii) All sides are equal and diagonals bisect each other at right angles.

(iii) Rectangle

3. (i) The field is a rhombus as all its sides are equal. The quadrilateral formed by joining the mid points of the adjacent sides of the field is also a rhombus as this also has all four sides equal. This will further prove that the field is in the form of a square.

(ii) All sides are equal. All angles are equal. The diagonals bisect each other at right angles.

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Mathematics (041) Class - IX

Theme 2: 'Quadrilaterals in Architecture, WAH TAJ!

Learning objectives

The learner shall be able to

- Recognize different types of quadrilaterals used in architecture designs on the basis of their shapes, sides and angles.
- Understand the significance of diagonals of quadrilaterals in order to provide strength to buildings and symmetry in designs.
- Learn the new terms 'Golden ratio' and 'bilateral symmetry' and their applications with reference to quadrilaterals.
- To apply the knowledge of properties of different types of quadrilaterals and the different theorems in real situations:

    1. The diagonal divides a parallelogram into two congruent triangles.
    2. In a parallelogram opposite sides are equal and the converse of this statement is also true.
    3. In a parallelogram opposite angles are equal and the converse of this statement is also true.
    4. A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
    5. In a parallelogram, the diagonals bisect each other and the converse of this statement is also true.
    6. In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and is half of it and the converse of this statement is also true.

A note to Readers!

The case study shall be used as an opportunity to create interest of students in Geometry and to show them practical aspect of problems. On careful examination of Geometrical drawing of Taj Mahal every type of quadrilateral can be traced out and numerous problems of Geometry can be framed. All these problems will lead to better understanding of symmetry of Taj Mahal.

The questions on quadrilaterals will catch the attention of students as they will now appear to be of practical importance.

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Theme 2: 'Quadrilaterals in Architecture, WAH TAJ!

Abstract

Knowledge of Euclidean Geometry and specially the properties of various types of quadrilaterals plays very special role in work of architects as it helps them to design any building by making effective utilization of space. This case study will help you to appreciate and understand the complex structure of famous monuments like TajMahal in simplified way with the knowledge gained by you in class IX about quadrilaterals.

This text is created to show the practical applications of the concepts learnt in geometry about quadrilaterals.

Note: Taj is specially chosen for this case study as it is embodiment of perfect symmetry. The simplest way to appreciate this symmetrical construction is to divide its geometrical or structural drawing into rectangular grids.

Quadrilaterals are second most popular shape used in architectural designs. With four corners and four edges it is possible to get many types of quadrilaterals such as trapezium, parallelogram, rectangle, square,rhombus and kite etc. by bringing variations in their angles and sides. This dynamic character of four sided polygon gives lots of possibilities and freedom to architects to create beautiful buildings.Quadrilaterals are preferred in constructing buildings over other polygons as they can make maximum utilization of space. Observe the following images of some buildings:





You can observe the prominent use of parallelograms, rectangles, squares, rhombus and trapezium etc. in designs of buildings. Architectural style of most of buildings can be understood using quadrilaterals. Right from foundation of the building to the vertical pillar and horizontal cross beams to the top roof one or the other type of quadrilateral is used. Very often it can be observed at construction sites that rectangular frame with a diagonal is used. It is done to provide strength to building as the diagonals provide rigidity in rectangular frame.

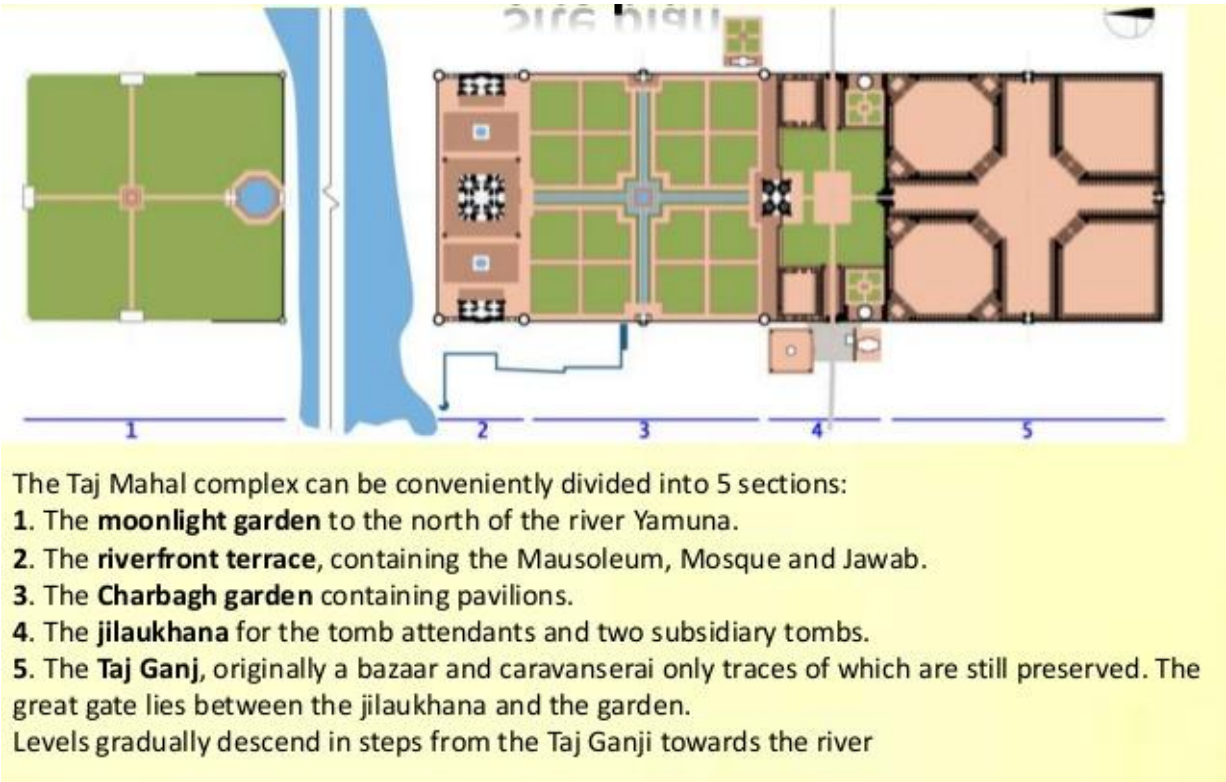
Before starting construction lay out plan of any land or construction site is worked out. Generally the layout of land is planned using quadrilaterals. For example when we talk about the TajMahal, first picture comes to our mind is of main Tomb with onion dome. But Taj is considered seventh wonder of the world due to perfect symmetry, harmony and order in its entire complex Pictures (i), (ii), (iii) of Taj complex demonstrates very clearly that the total land area of 42 acres on which Taj complex is built is divided into 5 main parts of rectangular shape.

(i)

See the aerial view and Layout of land use for Taj complex

(ii)

Site Plan



The Taj Mahal complex can be conveniently divided into 5 sections:
1. The **moonlight garden** to the north of the river Yamuna.
2. The **riverfront terrace**, containing the Mausoleum, Mosque and Jawab.
3. The **Charbagh garden** containing pavilions.
4. The **jilaukhana** for the tomb attendants and two subsidiary tombs.
5. The **Taj Ganj**, originally a bazaar and caravanserai only traces of which are still preserved. The great gate lies between the jilaukhana and the garden.
Levels gradually descend in steps from the Taj Ganji towards the river

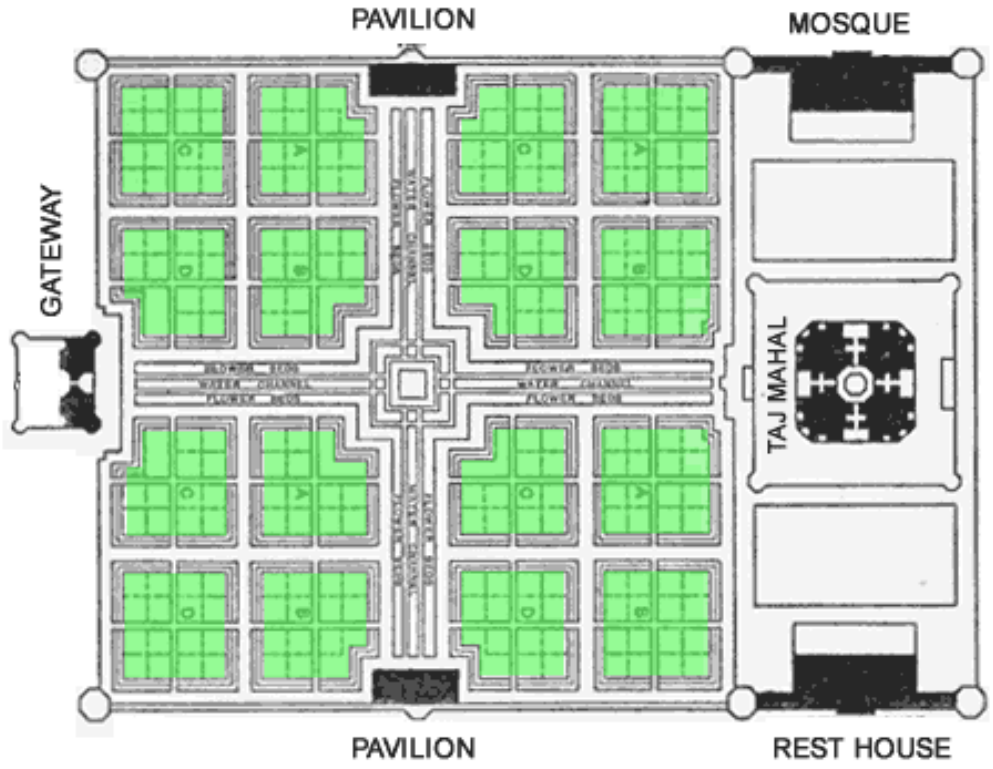
(iii)

In this article we will be focusing more on part 2 comprising of Tomb, Mosque and Jawab and part 3 i.e. Charbagh gardens. Part 1 is the area lying behind the Tomb.

Part 2 of the site plan is divided into three parts- a square in the middle surrounded by two identical rectangles. The main tomb of Taj Mahal stands on this square platform .Dimension of

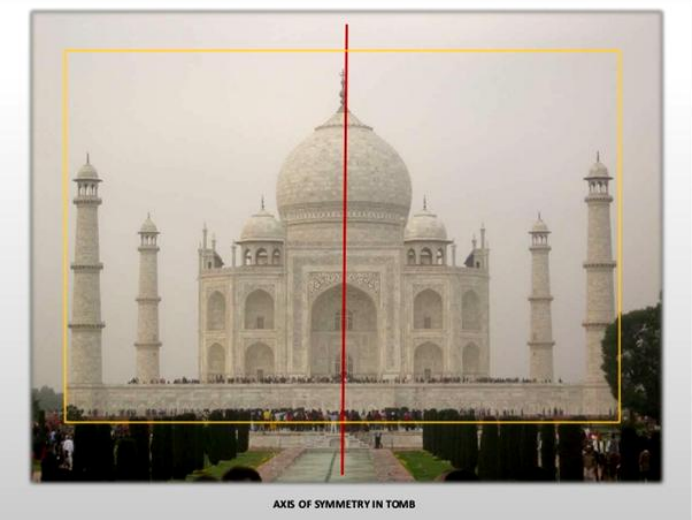
square platform is 186 feet x 186 feet and its height is 22 feet. This platform raised 50 meter above the river bank and was leveled with dirt in order to reduce seepage from the river. On four corners of the square platform four minarets each of height 137 feet are standing. The height of the tomb standing on square platform is equal to the height of spherical dome over it. It is 35 metre high. The dome is placed on a truncated drum to retain volume, and it locates the exact centre of the building. Because of its shape, the dome is often called an onion dome .The dome is topped by a gilded finial, whose tip rises to a height of 240 Feet above the ground.

Part 3 covers the total area of 580 meter by 300 meter, the garden alone covers 300 meter by 300 meter..The immaculate symmetry is maintained in the designing of this garden. The path grid consists of two sets of parallel paths that intersect at equal distances and create square rectangular fields of space. The four main sections are separated by two water channels which bisect at right angles. Each of these sections are further divided into quarters by sandstone paths.



(iv)

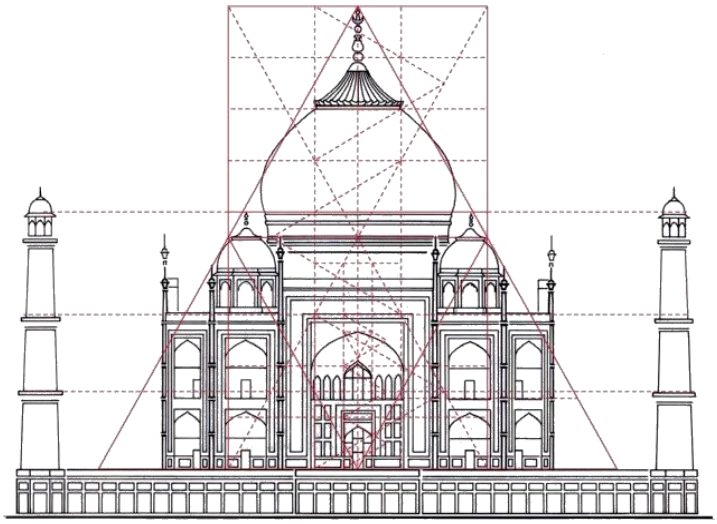
The ground plans of the main chamber at the center in octagonal. To maintain symmetry, the base of the four minarets uses an octagonal template also.



(v)

Good use of geometry is visible in design of Taj. It seems that even 400 years ago lots of geometrical planning to create bilateral symmetry was done by the architect of Taj Mahal. Bilateral symmetry refers to simultaneous pairing of same graphics and patterns.

A geometrical sketch of Taj is divided into rectangular grids. A careful and geometrical analysis of this sketch will help you to appreciate the symmetry in Taj.

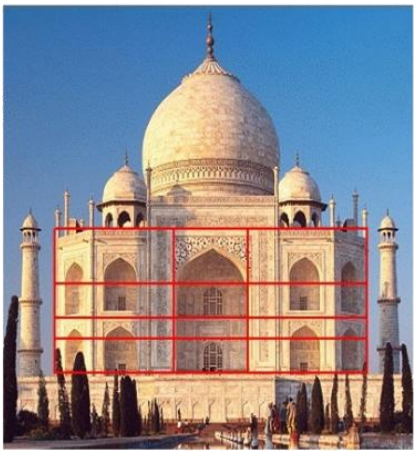


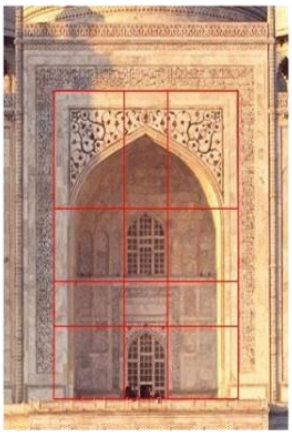
(vi)

The above picture clearly reflects that

- Tomb is comprised of rectangular building with dome over it. Height of the dome is equal to the height of building.
- Three chambers of side minarets are congruent trapezium.
- Simultaneous patterns on both sides of building of Tomb can be observed everywhere be it the wall or the windows or minarets.

- The concept of golden ratio was used in design of Taj Mahal.
- All rectangles used in the tomb building and its main arch are all Golden rectangle. That means Ratio of length to breadth in all rectangles is equal to 1.618. This ratio is known as Golden ratio and is represented by Φ





Using properties of quadrilaterals and mid-point theorem one can explore geometrical symmetry of Tajfrom different perspectives. You can try identifying all problems of quadrilaterals attempted in classroom hidden in picture (vi) and use them to appreciate the symmetry and beauty of Taj.

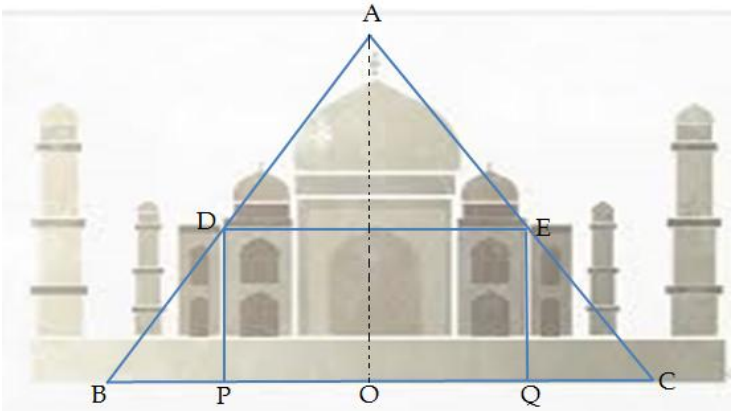
Sample Questions

Q.1 D and E are the mid-points of the sides AB and AC of a triangle ABC and O is any point on the side BC.O is joined to A If P and Q are the mid-points of side OB and OC respectively, then prove that

a) DEQP is a parallelogram

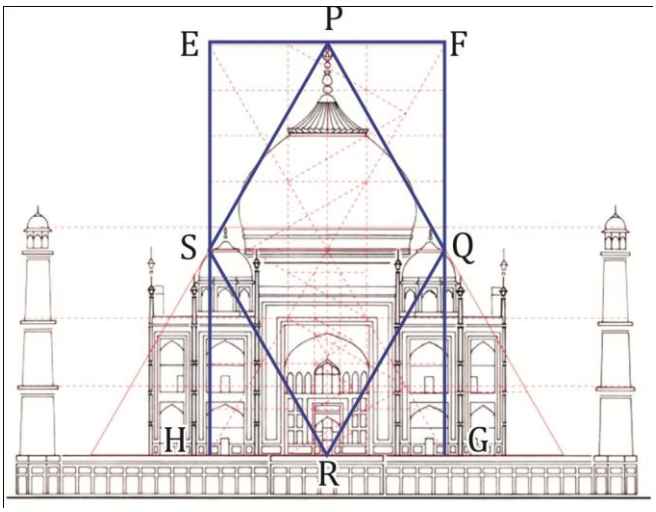
b) If AB = AC then DECB is an isosceles trapezium. [4 marks]

[Note: If trapezium has two non parallel sides of equal length then the trapezium is called isoceles trapezium]



Q.2 What properties of quadrilateral make them most popular choice for architects? [3 marks]

Q.3 Prove that quadrilateral formed by joining the mid points of the adjacent sides of a rectangle is a rhombus. In the figure below given that EFGH is a rectangle and P, Q, R, S are mid-points of sides EF, FG, GH & HE respectively. [3 marks]



Answers

1. (a) In ΔABC, D & E are respectively the mid points of AB & AC

$\Rightarrow$2DE =BC and DE is parallel to BC (Mid-Point Theorem).....(1)

In ΔAOC,

E & Q are respectively the mid points of AC & OC

$\Rightarrow$2EQ =AO and EQ is parallel to AO (Mid-Point Theorem) .. (2)

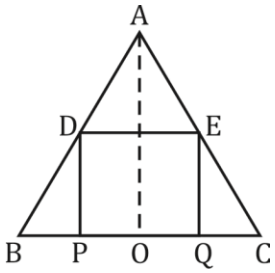
In ΔAOB,

D & P are respectively the mid points of AB & OB

$\Rightarrow$2DP = AO and DP is parallel to AO (Mid-Point Theorem)..(3)

From (2) & (3), EQ = DP AND EQ $\parallel$ DP.....(4)

From (1) & (4), DEQP is a. Parallelogram.



(b) If AB = AC

$\Rightarrow$ 2DB = 2EC ($\because$ D and E are mid points)

$\Rightarrow$ DB = EC

$\therefore$ DECB is an isosceles trapezium.

2. 1. A quadrilateral has four sides and four angles. By changing sides and angles various types of quadrilaterals can be formed such as trapezium, parallelogram, rectangle etc. Each of them has special properties.

2. A quadrilateral provides maximum utilization of space too.

3. Diagonals of a rectangle divide them into two congruent triangles, due to which rectangular frames have more strength.

3. EFGH is a rectangle

In $\Delta PQF \& \Delta RQG$

$\angle 1 = \angle 2$ (90°)

QF = QG ($\because$ Q is mid point of FG)

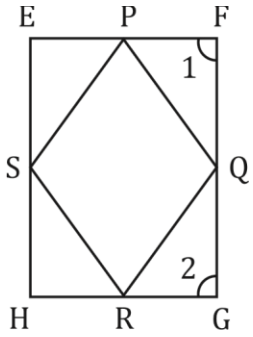
PF = RG (as EF = HG $\Rightarrow \frac{EF}{2} = \frac{HG}{2}$)

$\therefore \Delta PQF \cong \Delta RQG$ (SAS)

$\Rightarrow PQ = QR$

similarly PQ = QR = PS = SR

$\therefore$ PQRS is a rhombus.



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Shiksha Kendra, 2, Community Centre, Preet Vihar, Delhi-110092 India
Tel: 011-22509256-57 • Website: www.cbse.nic.in