(7 pages) Reg. No. : .....

Code No. : 20077 E **Sub. Code : SSMA 4 A/ ASMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Fourth Semester

Mathematics

Skill Based Subject — TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2017–2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. $\cos^2\theta =$ _____

(a) $\cos 2\theta - 1$ (b) $\cos 2\theta + 1$

(c) $\frac{1}{2}(\cos 2\theta + 1)$ (d) $\frac{1}{2}(\cos 2\theta - 1)$

2. $\frac{2\sin 2\theta - \sin 4\theta}{\sin\theta} =$ _____

(a) $8\sin^2\theta\cos\theta$ (b) $8\sin\theta\cos\theta$

(c) $8\cos^2\theta\sin\theta$ (d) $8\sin^3\theta\cos\theta$

3. $\cos ix =$ _____

(a) $\cos x$ (b) $\cosh x$

(c) $i\cosh x$ (d) $i\cos x$

4. $Log(-1) =$ _____

(a) $i\pi$ (b) $i(2n\pi)$

(c) $i(2n+1)\pi$ (d) $i(2n+1)\frac{\pi}{2}$

5. $L(1) =$ _____

(a) 1 (b) s

(c) $\frac{1}{s}$ (d) 0

6. $L^{-1}\left(\frac{a}{s^2+a^2}\right) =$ _____

(a) $\cos at$ (b) $\sin at$

(c) $\frac{1}{a}\sin at$ (d) $a\sin at$

7. $L(t^2) =$ _____

(a) $\frac{1}{s^3}$ (b) $\frac{1}{s}$

(c) $\frac{3!}{s^3}$ (d) $\frac{2}{s^3}$

8. $L^{-1}\left(\frac{1}{(s+2)^2}\right) =$ _____

(a) $e^{2t}\cdot t$ (b) $t\cdot e^{-2t}$

(c) e^{-2t} (d) e^{2t}

9. $f(x) =$ _____ is an even function.

(a) $\sin x\cos x$ (b) $\sin x$

(c) $\sin^2 x\cos x$ (d) $\sin 2x$

10. The Fourier coefficient a_0 for $f(x) = x$ in $(-l, l)$ is _____

(a) 0 (b) 1

(c) x^2 (d) $\frac{x^2}{2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Prove $\sin^5\theta = \frac{1}{2^4}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$.

Or

(b) Compute $\cos 5\theta$ in terms of powers of $\cos\theta$.

12. (a) Show that $\cos h^{-1}x = \log_e\left(x + \sqrt{x^2-1}\right)$.

Or

(b) Estimate :

(i) $Log(i)$

(ii) $Log(-e)$

13. (a) Evaluate : $L\left(te^{-t}\sin t\right)$.

Or

(b) Find $L^{-1}\left(\frac{s}{s^2+2s+5}\right)$.

14. (a) Show that $L^{-1}\left(\frac{F(s)}{s}\right)=\int_0^t L^{-1}\left(F(s)\right)dt$.

Or

(b) Obtain the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$.

15. (a) Find the Fourier series for $f(x)=x^2$ in $(-1,1)$.

Or

(b) Expand $f(x)=\pi-x$ as a Fourier cosine series in $(0,\pi)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Express $\sin 7\theta$ interms of powers of $\sin\theta$.

Or

(b) Derive a formula for the expansion of $\sin^n\theta$.

17. (a) If $\sin(\theta+i\phi)=\tan\alpha+i\sec\alpha$, prove that $\cos 2\theta\cosh 2\phi=3$.

Or

(b) If $\cos\alpha+i\sin\alpha=\cos(\theta+i\phi)$, show that $\sin^2\theta=\pm\sin\alpha$.

18. (a) Estimate $L\left(f(t)\right)$ if $f(t)=\begin{cases} t, & 0<t<b \\ 2b-t, & b<t<2b \end{cases}$, $f(t)=f(t+2b)$.

Or

(b) Compute :

(i) $L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$

(ii) $L^{-1}\left(\frac{1+2s}{(s+2)^2(s-1)^2}\right)$.

19. (a) Solve : $\frac{d^2y}{dt^2}+4\frac{dy}{dt}-5y=5$ given that $y=0, \frac{dy}{dt}=2$ when $t=0$.

Or

(b) Solve : $3\frac{dx}{dt}+\frac{dy}{dt}+2x=1$; $\frac{dx}{dt}+4\frac{dy}{dt}+3y=0$, given $x=y=0$ at $t=0$.

20. (a) Obtain the Fourier series expansion of $f(x)=e^x$ in $(0,2\pi)$.

Or

(b) Write $f(x)=x$ as a Fourier sine series in $(0,\pi)$ and hence compute the sum $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$

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