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B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Sixth Semester

Mathematics — Core

DYNAMICS

(For those who joined in July 2017-2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Time of flight of the projectile is ————

(a) $\frac{2n\sin\alpha}{g}$ (b) $\frac{2n}{g}$

(c) $\frac{2n^2\sin\alpha}{g}$ (d) $\frac{u\sin\alpha}{g}$

2. Maximum horizontal range of a projectile is ————

(a) $\frac{u^2}{g}$ (b) $\frac{u^2}{g}$

(c) $\frac{u}{g}$ (d) $\frac{u^2\sin\alpha}{g}$

3. For inelastic body e = ————

(a) 2 (b) 1

(c) 0 (d) 4

4. An elastic ball of mass m falls from a height 'h' on a fixed horizontal plane and rebounds. Then the loss in K.E. is ————

(a) $mgh(1+e^2)$ (b) $mg(1-e^2)$

(c) $mg(1+e^2)$ (d) $mgh(1-e^2)$

5. The periodic time of a simple pendulum is ————

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{g}{l}}$

(c) $\sqrt{\frac{2\pi g}{l}}$ (d) $\sqrt{\frac{2\pi l}{g}}$

6. In a S.H.M. at time t, displacement x = ————

(a) $\cos\sqrt{\mu}t$ (b) $\sin\sqrt{\mu}t$

(c) $a\sin\sqrt{\mu}t$ (d) $a\cos\sqrt{\mu}t$

7. The magnitude of the radial component of velocity is ————

(a) $\dot{r}$ (b) $r\dot{\theta}$

(c) $\ddot{r}-r\dot{\theta}^2$ (d) $\dot{r}\theta$

8. The magnitude of the radial component of acceleration along the radius vector is ————

(a) $\dot{r}\theta$ (b) $\ddot{r}\dot{\theta}$

(c) $\dot{r}\theta^2$ (d) $\ddot{r}-r\dot{\theta}^2$

9. $p-r$ equation of the parabola-pole of focus is ————

(a) $p^2=r^2$ (b) $p=ar$

(c) $p^2=ar$ (d) $p^2=a^2r$

10. $p-r$ equation of the equiangular spiral is ————

(a) $p=kr$ (b) $p=kr^2$

(c) $p^2=r^2$ (d) $p^2=\sqrt{kr}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) A stone is thrown with a velocity of 39.2 m/sec at 30° to the horizontal. Find at what time will be at height of 14.7 m (g = 9.8 m/s²).

Or

(b) Prove that the latus rectum of the path of a projectile is $\frac{2}{g}\times$(horizontal velocity)².

12. (a) Explain the oblique impact of two smooth spheres.

Or

(b) A sphere of mass m moving on a horizontal plane with velocity impinges obliquely on a sphere of mass M at rest on the same plane if $m=eM$, prove that the direction of motion are at right angle after impact.

13. (a) Show that the displacement of a particle moving in a straight line is expressed by the equation $x=a\cos nt+b\sin nt$ then it describes a S.H.M. whose amplitude is $\sqrt{a^2+b^2}$ and period $\frac{2\pi}{n}$.

Or

(b) Find the composition of two S.H.M. of the same period in two perpendicular directions.

14. (a) The velocities of a particle along and perpendicular in the radius from a fixed origin are λr and $\mu\theta$ respectively. Find the path and the acceleration along and perpendicular to the radius vector.

Or

(b) A particle describes the equiangular spiral $r = ae^{\theta \cot a}$ whose pole is 0. It its radial velocity at distance from 0 is $\frac{K}{r}$. Prove that the acceleration of the particle is $K\sec^2 \alpha / r^3$ towards 0.

15. (a) Explain : Velocities in central orbit.

Or

(b) Find the law of force towards the pole under which the curve $\frac{a}{r} = e^{n\theta}$ can be described.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Obtain the range of a projectile on an inclined plane and find its maximum.

Or

(b) Show that for a given velocity of projection the maximum range down an inclined plane of inclination α bears to the maximum range up the inclined plane the ratio $\frac{1+\sin\alpha}{1-\sin\alpha}$.

17. (a) A particle is projected from a point on an inclined plane and at the r^{th} impact is strikes the plane perpendicularly and at the n^{th} impact is at the point of projection. Show that $e^n - 2e^r + 1 = 0$.

Or

(b) A particle of elasticity 'e' is dropped from a vertical height 'a' ∞ upon the highest point of a plane which is of length b and is inclined at an angle α to the horizon and descends to the bottom in three jumps. Show that

$$b = 4ae(1+e)(1+e^2)(1+e+e^2)\sin\alpha$$

18. (a) A particle describing SHM has distance x_1, x_2, x_3 in successive intervals of time from its centre of oscillation. Show that its period is $\frac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)}$.

Or

(b) Find the differential equation of SHM and solve it.

19. (a) Show that the path of a point P which posses two constant velocities u and v the first of which is in a fixed direction and the second of which its perpendicular to the radius vector OP drawn from a fixed point 0 is a conic whose focus is 0 and whose eccentricity is $\frac{u}{v}$.

Or

(b) A particle is describing the ellipse $\frac{l}{r} = 1 + e\cos\theta$ with uniform angular velocity u about the pole. Show that when the particle is at one end of the lautus rectum through the pole the component of the acceleration towards the pole is $(1-2e^2)w^2 l$.

20. (a) A particle describe $r^n = a^n \cos n\theta$ under a central force. Find the law of force.

Or

(b) A particle moves in an ellipse under a force which is always directed towards its focus. Find the law of force, the velocity at any point and its periodic time.
