(8 pages) **Reg. No. :** ....

Code No. : 20066 E **Sub. Code : SMMA 54/ AMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Fifth Semester

Mathematics — Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017–2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $F(e^{iax}f(x)) =$ ————

(a) $F(s+a)$ (b) $F(s-a)$

(c) $\frac{1}{s-a}F(s+a)$ (d) $\frac{1}{s+a}F(s-a)$

2. $F\left(\frac{d^n f(x)}{dx^n}\right) =$ ————

(a) $(-is)^n F(s)$ (b) $(is)^n F(s)$

(c) $\left(\frac{1}{s}\right)^n . F(s)$ (d) $\left(\frac{s}{i}\right)^n F(s)$

3. $F_C(xf(x)) =$ ————

(a) $\frac{-d}{ds}F_s(f(x))$ (b) $\frac{-d}{ds}F_c(f(x))$

(c) $\frac{d}{ds}(F_s(f(x))$ (d) $\frac{d}{ds}(F_c(f(x))$

4. $F_s(f(ax)) =$ ————

(a) $\frac{1}{a}.F_s(s/a)$ (b) $\frac{1}{a}.F_s(a/s)$

(c) $a.F_s(s/a)$ (d) $a.F_s(a/s)$

5. The Fourier cosine transform of $f(x)=1$ in $(0, \pi)$ is ————

(a) 1 (b) –1

(c) 0 (d) none

6. The inverse finite Fourier cosine transform of $F_c(f(x))$ is $f(x) =$ ————

(a) $\frac{2}{l}\sum_{n=1}^{\infty}F_c(f(x))\cos\frac{n\pi x}{l}$

(b) $\frac{1}{l}F_c(0)+\frac{2}{l}\sum_{n=1}^{\infty}F_c(f(x))\cos\frac{n\pi x}{l}$

(c) $\frac{2}{l}\sum_{n=1}^{\infty}F_c(f(x))\sin\frac{n\pi x}{l}$

(d) none

7. $z \cdot f(n+1) =$.

(a) $z.F(z)-z.f'(0)$ (b) $z.F(z)-z.f(0)$

(c) $z.f(z)-z.f'(0)$ (d) $z.f(z)+z.f(0)$

8. $Z(a^{k-1}) =$

(a) $\frac{1}{a-z}$ (b) $\frac{1}{z-a}$

(c) $\frac{1}{a+z}$ (d) $\frac{a}{a+z}$

9. $Z^{-1}\left(\frac{Z^2}{(z-a)^2}\right) =$

(a) $(n+1)a^{n-1}$ (b) $(n+1)a^n$

(c) $n.a^{n+1}$ (d) na^{n-1}

10. $Z^{-1}\left(\frac{1}{z-2}\right) =$ ———— $(n \geq 1)$.

(a) 2^{n-1} (b) 2^n

(c) $\frac{1}{2^n}$ (d) $\frac{1}{2^{n-1}}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the Fourier transform of $f(x)=\begin{cases} x^2, & |x|<a \\ 0, & |x|>a \end{cases}$.

Or

(b) Show that $F(f(x-a)) = e^{ias} f(s)$.

12. (a) Find the Fourier cosine transform of $\frac{1}{a^2+x^2}$.

Or

(b) Show that if $F_s(f(x)) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$, $F_s(x.f(x)) = \frac{-d}{ds}F_c((f(x))$.

13. (a) Find the Fourier sine transform of $f(x) = x^3$ in $(0, l)$.

Or

(b) Find $f(x)$ if its finite sine transform is egien by $\frac{2\pi(-1)^{P-1}}{P^3}$ where P is a positive integer and $0 < x < \pi$.

14. (a) Find $Z(n^2)$.

Or

(b) Find $Z\left(\frac{1}{(n+1)(n+2)}\right)$.

15. (a) Using convolution theorem, find $Z^{-1}\left(\frac{z^2}{(z+2)^2}\right)$.

Or

(b) Find $Z^{-1}\left(\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the Fourier transform of $\frac{\sin ax}{x}$ and hence prove that $\int_{-\infty}^{\infty}\frac{\sin^2 ax}{x^2}dx = a\pi$.

Or

(b) Show that the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & if \quad |x| < a \\ 0 & if \quad |x| > a > 0 \end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - a\cos as}{s^3}\right)$ hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt = \pi/4$.

17. (a) Show that

(i) $F_c(x.f(x)) = \frac{d}{ds}(F_s(f(x)))$

(ii) $F_s(x.f(x)) = \frac{-d}{ds}(F_c(F_c f(x)))$

Or

(b) Find $f(x)$, its sine transform is $\frac{e^{-as}}{s}$. Hence deduce that the inverse sine transform of $\frac{1}{s}$.

18. (a) Find the Fourier sine and cosine transform of $f(x) = \begin{cases} 1 & if \quad 0 < x < \pi/2 \\ -1 & if \quad \pi/2 < x < \pi \end{cases}$.

Or

(b) Find the finite sine and cosine transform of $f(x) = \cos kx \ (0 < x < \pi)$.

19. (a) (i) Prove that $Z(f(n) * g(n)) = F(z).G(z)$ where $Z(f(n)) = F(z)$ and $Z(g(n)) = G(z)$

(ii) If $f(n) = u(n)$ and $g(n) = \delta(n) + \left(\frac{1}{2}\right)^n U(n)$, then find $Z(f(n) * g(n))$

Or

(b) State and prove initial value theorem and find the initial value of the function $F(z) = \frac{1+z^{-1}}{1-0.25z^{-2}}$.

20. (a) Find

(i) $Z^{-1}\left(\frac{1}{1-1.5z^{-1}+0.5z^{-2}}\right)$ and

(ii) $Z^{-1}\left(\frac{z^2}{z^2-z+0.5}\right)$ using the method of partial fractions.

Or

(b) Find $Z^{-1}\left(\frac{z^2+2z}{z^2+2z+4}\right)$ by the method of partial fraction.

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