(6 pages)

Reg. No. : _____

Code No. : 20083 E **Sub. Code : SEMA 5 B/ AEMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023

Fifth Semester

Mathematics – Major Elective

DISCRETE MATHEMATICS

(For those who joined in July 2017–2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer

1. Those statements which do not contain any connectives are called _____
   (a) Atomic (b) Primary
   (c) Simple statements (d) All of the above

2. If P and Q are two statements, then the statement $P \to Q$ is called _____
   (a) Conditional Statement
   (b) Biconditional Statement
   (c) Simple Statement
   (d) None

3. A product of the variables and their negations in a formula is called an _____ products
   (a) elementary (b) atomic
   (c) normal (d) none

4. Which of the following is an example of elementary product?
   (a) P (b) $P \wedge \neg P$
   (c) $Q \wedge \neg P$ (d) All of the above

5. Let S be a nonempty set and 0 be a binary operation on S. The algebraic system $< S, 0 >$ is called a _____ if the operation is associative
   (a) group (b) subgroup
   (c) semigroup (d) monoids

6. Hamming codes were constructed by introducing redundant digits called _____ digits
   (a) Parity (b) Checks
   (c) Error (d) None

7. A _____ is a partially ordered set $\langle L, \leq \rangle$ in which every pair of elements $a, b \in L$ has a greatest lower bound and a lease upper bound
   (a) lattice (b) duality
   (c) sublatices (d) direct product

8. Let $\langle B, *, \oplus, 1, 0, 1 \rangle$ and $\langle P, \cap, \cup, -, \alpha, \beta \rangle$ be two Boolean algebras. A mapping $f: B \to P$ is called a _____
   (a) Boolean homomorphism
   (b) Boolean algebra
   (c) Isomorphism
   (d) None

9. What are the numbers using for represent any decimal number?
   (a) 0–1 (b) 0–7
   (c) 0–9 (d) None

10. Add : 1011 and 1001
    (a) 10100 (b) 2012
    (c) 1100 (d) 11001

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Construct the truth table for $P \vee \neg Q$.

    Or

    (b) Write in symbolic form for the statement, "The Crop will be destroyed if there is a flood".

12. (a) Let $P(x) : x$ is a person. $F(x, y) : x$ is a father of y. $M(x, y) : x$ is the mother of y. Write the predicate "x is the father of the mother of y".

    Or

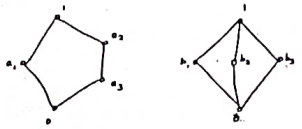
    (b) Show that
    $(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x))$
    $\Rightarrow (x)(P(x) \to R(x))$

13. (a) Let $\langle G, * \rangle$ be a finite cyclic group generated by an element $a \in G$. If G is of order n, that is, $|G| = n$, then prove that $a^n = e$, so that $G = \{a, a^2, a^3, \ldots\ldots a^n = e\}$. Furthermore, n is the least positive integer for which $a^n = e$

    Or

    (b) Prove that A code can detect all combinations of k or fewer errors $\Leftrightarrow$ the minimum distance between any two code words is at least $k+1$.

14. (a) Show that the lattices given by the diagram in following are not distributive.



    Or

    (b) Show that $(a + b)(b' + c)(a + c) = (a + b)(b' + c)$.

15. (a) Convert $(1111)_2$ to decimal.

Or

(b) Multiply : 1011.01×110.1

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Construct the truth table for $\neg (P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$

Or

(b) Does the formula $(P \rightarrow (\neg P)) \rightarrow \neg P$ is tautology or Contradiction?

17. (a) Show that the following are equivalent formulaes.

(i) $P \vee (P \wedge Q) \Leftrightarrow P$

(ii) $P \vee (\neg P \wedge Q) \Leftrightarrow P \vee Q$

Or

(b) Show that $P \vee Q$ follows from P.

18. (a) Show that if every element of a group is its own inverse then the group is abelian.

Or

(b) Define Hamming distance and minimum distance.

19. (a) Write the properties of Lattices

Or

(b) When $\langle B, *, \oplus \rangle$ becomes a distributive lattice?

20. (a) Convert the following to octal numbers

(i) 110101110_2 (ii) 111101.01101_2

Or

(b) Add :

(i) 100101+100101

(ii) 1011.01+1001.11

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