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B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023

Second/Fourth Semester

Mathematics – Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2017-2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answers.

1. The value of $\vec{i} \cdot \vec{i} =$ ————.

   (a) $\vec{0}$ (b) 0

   (c) 1 (d) $\vec{j}$

2. A vector $\vec{f}$ is harmonic vector if ————.

   (a) $\nabla \vec{f} = 0$ (b) $\nabla^2 \vec{f} = 0$

   (c) $\nabla \times \vec{f} = 0$ (d) $\nabla^2 \times \vec{f} = 0$

3. The value of $\int_0^2 \int_0^5 xy\,dx\,dy$

   (a) $\frac{21}{2}$ (b) $\frac{63}{4}$

   (c) $\frac{64}{3}$ (d) $\frac{22}{2}$

4. The value of $\int_0^2 \int_1^3 \int_1^2 xy^2 z\,dz\,dy\,dx$

   (a) 26 (b) 24

   (c) 42 (d) 62

5. If $\vec{f} = x^2 i - xy\vec{j}$ and C is the straight line joining the points (0, 0) and (1, 1) then $\int_C \vec{f} \cdot d\vec{r}$ is ————.

   (a) 0 (b) 1

   (c) x^2 (d) xy

6. If $\vec{A} = curl\,\vec{F}$, then $\iint_S \vec{A} \cdot \vec{n}\,dS =$ ———— for any closed S.

   (a) 0 (b) S

   (c) V (d) $3V$

7. The value of $\iiint_V \nabla \varphi\,dV$ is

   (a) $\int_S \vec{f} \cdot dS$ (b) $\iint_S \varphi \vec{n}\,dS$

   (c) $\iiint_V \varphi\,dV$ (d) 0

8. If $curl\,\vec{F} = 0$, then $\int_C \vec{F}\,d\vec{r} =$

   (a) $\vec{n}$ (b) 0

   (c) 1 (d) $\vec{r}$

9. Which one of the following is an even function?

   (a) x (b) $\sin x$

   (c) e^x (d) x^2

10. If $f(x)$ is odd, then $\int_{-a}^{a} f(x)\,dx$

    (a) $2\int_a^a f(x)\,dx$ (b) $\int_{-a}^{a} f(x)\,dx$

    (c) 0 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\nabla f(r) = \left(\frac{f'(r)}{r}\right)\vec{r}$.

    Or

    (b) Show that $div\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.

12. (a) Evaluate $\int_0^{+1} \int_0^{1-\cos\theta} r\,dr\,d\theta$.

    Or

    (b) Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z\,dz\,dy\,dx$.

13. (a) Evaluate $\int_C \vec{f} \cdot d\vec{r}$ where $\vec{f} = (x^2 + y^2)\vec{i} + (x^2 + y^2)\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1, 1).

    Or

    (b) Find the common area between $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem.

14. (a) Show that $\iint_S \bar{f}\cdot\bar{n}d\bar{S} = \iiint_V a^2 dV$ where $\bar{r} = \varphi a$ and $a = \nabla\varphi$ and $\nabla^2\varphi = 0$.

Or

(b) Evaluate using Stoke's theorem $\int_C (yzdx + zxdy + xydz)$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

15. (a) In $-\pi < x < \pi$, express $\sinh ax$ as in Fourier series of period 2π.

Or

(b) Explain the Half range sine series.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $\nabla\varphi = 2xyz^3\bar{i} + x^2z^3\bar{i} + 3x^2yz^2\bar{k}$, find $\varphi(x, y, z)$ if $\varphi(1, -2, 2) = 4$.

Or

(b) If $\bar{f}$ is solenoidal prove that $curl\,curl\,curl\ curl\ \bar{f} = \nabla^2\bar{f}$.

17. (a) Find the value of $\iint xy\ dx\,dy$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Or

(b) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\ dx\ dy$.

18. (a) Evaluate $\int_C \bar{f}\cdot d\bar{r}$ where $\bar{f} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ and the curve C is the rectangle in the $x - y$ plane bounded by $y = 0$, $y = b$, $x = 0$, $x = a$.

Or

(b) Evaluate $\iint_S \bar{f}\cdot\bar{n}\,dS$ where $\bar{f} = (x + y)\bar{i} - 2x\bar{j} + 2yz\bar{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

19. (a) Verify using Gauss divergence theorem for $\bar{f} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ over the rectangular parallopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

Or

(b) Evaluate $\int_C (e^x\ dx + 2y\ dy - dz)$ by using stokes theorem where C is the curve $x^2 + y^2 = 4$, $z = 2$.

20. (a) Expand in Fourier series of $f(x) = x\sin x$ for $0 < x < 2\pi$.

Or

(b) Find the half range Fourier cosine series for $f(x) = x$ in $0 < x < \pi$.

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