

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2023.

First/Third Semester

Mathematics – Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 – 2020)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $3 + \sqrt{2}$ is a root of $x^3 - 11x^2 + 37x - 35 = 0$ then _____ is also a root of it.
- (a) $\sqrt{3} + \sqrt{2}$ (b) $\sqrt{3} - \sqrt{2}$
(c) $3 - \sqrt{2}$ (d) none

2. If α, β, γ are the roots of $3x^3 + 6x^2 - 9x + 2 = 0$ then $\alpha\beta\gamma =$ _____.

- (a) $\frac{2}{3}$ (b) $-\frac{2}{3}$
(c) 2 (d) -2

3. The equation obtained by multiplying the roots of the equation $x^3 + 3x^2 + x - 4 = 0$ by 10 is _____.

- (a) $x^3 - 30x^2 + 10x - 40 = 0$
(b) $10x^3 + 30x^2 + 100x - 4 = 0$
(c) $x^3 + 30x^2 + 100x - 400 = 0$
(d) $x^3 + 30x^2 + 100x - 4000 = 0$

4. A positive root of $x^3 - 3x + 1 = 0$ lies between _____.

- (a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 4 and 5

5. If $\text{rank}(A) = \text{rank}(A, B)$ = the number of unknowns, then the system of equations $AX = B$ has _____ solutions.

- (a) infinite (b) no
(c) unique (d) none

6. The sum of the eigen values of the matrix

$$\begin{pmatrix} 3 & 1 & 5 \\ 0 & 4 & 2 \\ 0 & 0 & -1 \end{pmatrix} \text{ is } \underline{\hspace{2cm}}.$$

- (a) 3 (b) 2
(c) 4 (d) 6

7. The general solution of the equation $y = px + p^2$ is

- (a) $y = cx + p^2$ (b) $y = cx + c^2$
(c) $y = cx - c^2$ (d) none

8. An equation solvable for x is of the form

- (a) $x = px + f(p)$ (b) $y = f(x, p)$
(c) $x = f(y, p)$ (d) none

9. $L[x] =$ _____.

- (a) $\frac{1}{s}$ (b) s
(c) s^2 (d) $\frac{1}{s^2}$

10. $L^{-1}\left[\frac{1}{s-a}\right] =$ _____.

- (a) e^{ax} (b) e^{-ax}
(c) x (d) x^2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the equation $x^3 - 11x^2 + 37x - 35 = 0$ if one of its roots is $3 + \sqrt{2}$.

Or

- (b) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ given that the roots are in A.P.

12. (a) Diminish the roots of the equation $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$ by 3.

Or

- (b) Show that the equation $x^3 + 3x - 1 = 0$ has only one real root and calculate it correct to two places of decimals.

13. (a) Test the consistency of the following system of linear equations. If consistent solve it.
 $x + y + z = 3; 2x + y - z = 2; 4x - y + 2z = 5$

Or

- (b) Find the eigen values of the matrix

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}.$$



14. (a) Solve $p^2 - 5p + 6 = 0$.

Or

(b) Solve $\sin px \cos y = \cos px \sin y + p$.

15. (a) Find $L[x^2 e^{-ax}]$.

Or

(b) Find $L^{-1}\left[\frac{s+1}{s^2+2s+2}\right]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in A.P. iff $2q^3 + 27p^2s = 9pqr$.

Or

(b) Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

17. (a) Find the root of the equation $x^4 - 3x + 1 = 0$ that lies between 1 and 2 correct to two decimal places using Newton's method.

Or

(b) Find the positive root of $x^3 + 2x^2 - 5x - 7 = 0$ correct to 2 decimal places by Horner's method.

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18. (a) Test the consistency of the equations $x - y + z = 2$; $3x - y + 2z = -6$; $3x + y + z = -18$ and solve if consistent.

Or

(b) Find the eigen values and the eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

19. (a) Solve : $x^2 - 2yp + x = 0$.

Or

(b) Solve : $y - 2px = x^2 p^4$.

20. (a) Find $L\left(\frac{1 - \cos x}{x}\right)$.

Or

(b) Find :

(i) $L^{-1}\left[\frac{s}{(s+2)^2}\right]$

(ii) $L^{-1}\left[\frac{1}{(s+3)^2 + 25}\right]$

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