(8 pages)

Reg. No. : ....

Code No. : 20652 E **Sub. Code : EMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics — Core

ALGEBRA AND TRIGONOMETRY

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. One root of $x^4 - 3x + 1 = 0$ lies between ————

(a) 2 and 3

(b) 2 and 2.5

(c) 2.5 and 3

(d) 1 and 2

2. If $f(x) = 0$ is a reciprocal equation of first type and odd degree, then ———— is a factor of $f(x)$.

(a) $x+1$ (b) $(x-1)$

(c) x^2-1 (d) x^2+1

3. The value of e is ————

(a) 2.718 (b) 2.738

(c) 2.371 (d) 2.387

4. The coefficient of x^n in the expansion of $(2-4x)(1-2x)^{-2}$ is ———— if $|x| < \frac{1}{2}$

(a) 2^n (b) 2^{n+1}

(c) 2^{2n} (d) 2^{2n+1}

5. $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ and $|A| \neq 0$ then $A^{-1} =$ ————

(a) $\frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (b) $\frac{1}{|A|}\begin{pmatrix} -d & b \\ c & -a \end{pmatrix}$

(c) $\frac{1}{|A|}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$ (d) $\frac{1}{|A|}\begin{pmatrix} -d & b \\ c & -a \end{pmatrix}$

6. The characteristic polynamial of I_2 is ————

(a) x^2+2x+1

(b) x^2-2x+1

(c) x^2-x-1

(d) x^2+x+1

7. If $x = \cos\theta + i\sin\theta$, then $x^n - \frac{1}{x^n} =$ ————

(a) $2\cos\theta$ (b) $2i\sin\theta$

(c) $2i\sin n\theta$ (d) $2\cos n\theta$

8. 1 degree = ———— minutes.

(a) 60 (b) 30

(c) 45 (d) 90

9. The value of $\cosh\left(\frac{i\pi}{2}\right)$ is ————

(a) 1 (b) 0

(c) -1 (d) $-i$

10. The value of $\log i$ is ————

(a) $i\frac{\pi}{2}$ (b) $i\left(\frac{\pi}{2}+2n\pi\right)$

(c) $i(4n-1)\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

Or

(b) Increase the roots of the equation $4x^5 - 2x^3 + 7x - 3 = 0$ by 2.

12. (a) Find the coefficient of x^n in the expansion of $(1+x)e^{(1+x)}$ in ascending powers of x.

Or

(b) Prove that

$$\log 2 - \frac{(\log 2)^2}{2!} + \frac{(\log 2)^3}{3!} - \ldots\ldots = \frac{1}{2}$$

[P.T.O.]

13. (a) Show that the non-singular matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$. Hence evaluate A^{-1}.

Or

(b) Find the characteristic roots of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

14. (a) Prove that

$$2^5 \cos^6 \theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

Or

(b) Expand $\sin 7\theta$ in powers of $\cos\theta$ and $\sin\theta$.

15. (a) Prove that $\dfrac{1+\tanh x}{1-\tanh x} = \cosh 2x + \sinh 2x$.

Or

(b) If $i^{a+ib} = a+ib$, then prove that $a^2 + b^2 = e^{-b(4n+1)\pi}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

Or

(b) Find the positive root of $x^3 - 3x + 1 = 0$ correct to three places of decimals.

17. (a) Show that the coefficient of x^n in the expansion of e^{e^x} is $\dfrac{1}{n!}\left(\dfrac{1^n}{1!} + \dfrac{2^n}{2!} + \dfrac{3^n}{3!} + \dots\right)$ also show that

(i) $\dfrac{1^3}{1!} + \dfrac{2^3}{2!} + \dots = 5e$

(ii) $\dfrac{1^4}{1!} + \dfrac{2^4}{2!} + \dfrac{3^4}{3!} + \dots = 15e$

Or

(b) Prove that $s = \displaystyle\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

18. (a) Using Cayley-Hamilton theorem, find the inverse of a matrix, $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$.

Or

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

19. (a) Prove that

$$\cos 8\theta = 128\cos^8\theta - 256\cos^6\theta + 160\cos^4\theta - 32\cos^2\theta + 1$$

Or

(b) Prove that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

$$\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$$

20. (a) Prove that $u = \log_e \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$ iff $\cosh u = \sec\theta$.

Or

(b) If $\cos(x+iy) = r(\cos\alpha + i\sin\alpha)$, prove that $y = \dfrac{1}{2}\log\left[\dfrac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$.
