

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics

Elective - ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

- 1. One of the roots of the reciprocal equation 6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0 is (a) 4 (b) -2 (c) -1 (d) 0
2. The equation whose roots are 10 times those of the equation x^3 + 3x - 5 = 0 is (a) x^3 + 3x - 50 = 0 (b) x^3 + 30x - 50 = 0 (c) x^3 + 30x - 500 = 0 (d) x^3 + 300x - 5000 = 0

- 3. If the equation 2x^3 - 3x^2 + 2x - 3 = 0 has one root i, then its real root is (a) 2/3 (b) -2/3 (c) 3/2 (d) 1
4. One root of x^4 - 3x + 1 = 0 lies between (a) 2 and 3 (b) 2 and 2.5 (c) 2.5 and 3 (d) 1 and 2
5. Horner's method can be used to get the (a) rational (b) integers (c) positive integers (d) irrational roots of the equation f(x) = 0.
6. The characteristic equation of the matrix A = (1 0 / 0 1) is (a) x^2 + 2x + 1 = 0 (b) x^2 - 2x + 1 = 0 (c) x^2 + 2x - 1 = 0 (d) x^2 - 2x - 1 = 0
7. The general solution of the differential equation y - P(x+1) = P is (a) y = P(x+2) (b) y = cx (c) y = cx + 2c (d) y = c(x+1)

- 8. The value of L(cosh at) is (a) s / (s^2 + a^2) (b) a / (s^2 + a^2) (c) s / (s^2 - a^2) (d) a / (s^2 - a^2)

- 9. The value of L^T(1/s^2) is (a) t (b) 2t^2 (c) t^2/2 (d) 2/t^2

- 10. The solution of 2p + 3q = 1 is (a) phi(2x - 3y, y + 3z) = 0 (b) phi(2x - 3y, y - 3z) = 0 (c) phi(3x - 2y, y - 3z) = 0 (d) phi(3x - 2y, 3y - z) = 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

- 11. (a) Solve the equation x^4 - 5x^3 + 4x^2 + 8x - 8 = 0 if one of whose roots is 1 - sqrt(5). Or (b) Form the equation with rational coefficients one of whose roots is sqrt(2) + sqrt(3).

- 12. (a) Diminish the roots of the equation x^3 + x^2 + x - 100 = 0 by 4. Or (b) Find by Newton's method the root of the equation x^3 - 3x + 1 = 0 which lies between 1 and 2.
13. (a) Find the inverse of the matrix A = (2 2 0 / 2 1 1 / -7 2 -3) using Cayley-Hamilton theorem.

Or

- (b) Calculate A^4 when A = (1 3 / 2 4).

- 14. (a) Solve : P^3 - 6P^2 + 11P - 6 = 0. Or (b) Solve : y = p sin p + cos p.

- 15. (a) Find L(e^-ax cos bx). Or (b) Find L^T(1 / ((s+3)^2 + 25)).

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation

$$8x^4 - 90x^3 + 315x^2 - 405x + 162 = 0 \quad \text{given that the roots are in geometric progression.}$$

Or

(b) Solve : $6x^6 - 5x^5 - 44x^4 + 44x^2 + 5x - 6 = 0$.

17. (a) Find the positive root of $x^3 - x - 3 = 0$ by Horner's method correct to two places of decimals.

Or

- (b) Find by Newton's method correct to 3 places of decimals the root between 0 and 1 of the equation $3x - \cos x - 1 = 0$.

18. (a) Find the eigen values and eigen vectors of

the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

Or

- (b) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

using Cayley-Hamilton theorem.

19. (a) Solve : $3x - y + \log p = 0$.

Or

- (b) Solve : $p \cot x + q \cot y = \cot z$.

20. (a) Find $L(t^2 + \cos 2t + \cos t + \sin^2 2t)$.

Or

- (b) Find $L^T \left[\log \left(\frac{s+a}{s+b} \right) \right]$.
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