(7 pages) **Reg. No. :** ....

Code No. : 20431 E **Sub. Code : CSMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Fourth Semester

Mathematics

Skill Based Subject – TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2021 – 2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $1^\circ =$ ____________ radians.

(a) π (b) $\frac{\pi}{180}$

(c) $\frac{\pi}{90}$ (d) 2π

2. $-\frac{1}{32}(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10) =$ ____________.

(a) $\sin^6\theta$ (b) $\cos^6\theta$

(c) $\sin^3\theta\cos^3\theta$ (d) $\sin^2\theta\cos^4\theta$

3. When θ is expressed in radians, $\sin\theta =$ ____________.

(a) $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$

(b) $1 + \theta - \frac{\theta^2}{2} + \frac{\theta^3}{3} - \cdots$

(c) $1 + \theta + \frac{\theta^2}{2} + \frac{\theta^3}{3} + \cdots$

(d) $\theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \cdots$

4. If z, w are two complex numbers then $z^w =$ ____________.

(a) e^{wLogz} (b) $e^w \log z$

(c) $e^{z\log w}$ (d) $wLogz$

5. $L(e^{2t}) =$ ____________.

(a) $\frac{1}{s+2}$ (b) $\frac{1}{s-2}$

(c) $\frac{1}{s}$ (d) 1

6. $L^{-1}\left(\frac{1}{s(s+a)}\right) =$ ____________.

(a) $\frac{e^{-at}}{a}$ (b) $\frac{1-e^{-at}}{a}$

(c) $\frac{1+e^{at}}{a}$ (d) $\frac{e^{at}}{a}$

7. Value of $L(\sinh at) =$ ____________.

(a) $\frac{a}{s^2}$ (b) $\frac{a}{(s+a)^2}$

(c) $\frac{a}{s^2-a^2}$ (d) $\frac{a}{s^2+a^2}$

8. $L^{-1}\left(\frac{s}{a^2s^2+b^2}\right) =$ ____________.

(a) $\cos\frac{b}{a}t$ (b) $\frac{1}{a}\cos\frac{b}{a}t$

(c) $\frac{1}{a^2}\cos\frac{a}{b}t$ (d) $\frac{1}{a^2}\cos\frac{bt}{a}$

9. If $f(x)$ is an even function then $f(-x) =$ ____________.

(a) $f(x)$ (b) $-f(x)$

(c) $f(x^2)$ (d) $-f(x^2)$

10. $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots =$ ____________.

(a) $\frac{\pi}{8}$ (b) $\frac{\pi^2}{8}$

(c) $\frac{\pi}{12}$ (d) $\frac{\pi^2}{12}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove :

$$\frac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta.$$

Or

(b) Expand $\cos^5\theta\sin^3\theta$ in a series of sines of multiples of θ.

[P.T.O.]

12. (a) Prove that $\frac{1+\tanh x}{1-\tanh x} = \cosh 2x + \sinh 2x$.

Or

(b) If $i^{x+iy} = A+iB$, show that $A^2+B^2 = e^{(4n+1)\pi y}$.

13. (a) Find $L\left(\frac{1-\cos 2t}{t}\right)$.

Or

(b) Find $L^{-1}\left(\log\left(\frac{s+a}{s+b}\right)\right)$.

14. (a) Using Laplace transform, solve $y'+3y = e^{-2t}$, given $y(0)=4$.

Or

(b) Solve : $(D^2+5D+6)y = e^{-t}$, given $y(0)=0$ and $y'(0)=0$ by using Laplace transform.

15. (a) If $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x & 0 \le x \le \pi \end{cases}$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$.

Or

(b) Find the cosine series for the function $f(x) = \pi - x$ in the interval $(0, \pi)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{8}$.

Or

(b) Derive the expansion of $\cos^n\theta$ when n is a positive integer.

17. (a) If $A+iB = \tan^{-1}(x+iy)$ prove that $B = \frac{1}{4}\log\left(\frac{x^2+(1+y)^2}{x^2+(1-y)^2}\right)$.

Or

(b) Find the sum to infinity the series $1+\cos\theta\cos\theta+\cos^2\theta\cos 2\theta+\cos^3\theta\cos 3\theta+\cdots\infty$.

18. (a) (i) Derive an expression for $L(f''(t))$.

(ii) Find $L^{-1}\left(\frac{s}{(s+2)^2}\right)$.

Or

(b) (i) Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$.

(ii) Find $L^{-1}\left(\frac{1}{s(s+2)^2}\right)$.

19. (a) Solve : $y''+2y'+5y = 3e^{-t}\sin t$ given that $y(0)=0$, $y'(0)=3$.

Or

(b) Solve by using Laplace transform :

$\frac{dx}{dt}+y = \sin t$, $\frac{dy}{dt}+x = \cos t$ given $x(0)=2$, $y(0)=0$.

20. (a) Show that $x^2 = \frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2}$ in $-\pi < x < \pi$. Deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots = \frac{\pi^2}{12}$.

Or

(b) Expand $y = \cos zx$ as a series of sines in $(0, \pi)$.

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