(6 pages) **Reg. No. :** ____________

Code No. : 20423 E **Sub. Code : CMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2021-2022)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. In the discrete metric space M the diameter of $A = \{1, 5, 7, 9\}$ is

   (a) 0 (b) 1

   (c) 9 (d) 8

2. In R with usual matric the open ball $B(-1, 1)$ is

   (a) $[-2, 0]$ (b) $[-2, 0)$

   (c) $[-1, 1)$ (d) $(-2, 0)$

3. The incorrect statement is ————

   (a) $D(Z) = \phi$ (b) $D(Z) = Q$

   (c) $D(Q) = R$ (d) $D(Q \times Q) = R \times R$

4. The incorrect statement is

   (a) Q is second category

   (b) R is second category

   (c) l_2 is of second category

   (d) Any complete metric is of second category

5. If $f : R \to R$ is continuous then ————

   (a) f is 1 – 1

   (b) f is onto

   (c) f is uniformly continuous

   (d) none of the above

6. In $[0, 1]$ with usual metric the closure of $A = Q \cap (0, 1)$ ————

   (a) $(0, 1)$ (b) A

   (c) $[0, 1]$ (d) $[0, 1)$

7. Any connected subset of R containing more than one point is

   (a) bounded (b) finite

   (c) uncountable (d) countable

8. Which of the following is a disconnected subset of R^2

   (a) $\{(x, y) : x^2 + y^2 = 1\}$

   (b) $\{(x, y) : x = 0\}$

   (c) $\{(x, y) : x = 0 \text{ or } y = 0\}$

   (d) $\{(x, y) : x, y \in R\}$

9. Which of the following subset of R is both compact and connected?

   (a) R (b) $(0, 1)$

   (c) $[0, 100]$ (d) Q

10. In a discrete metric space, the only connected subsets are

    (a) finite sets

    (b) the whole set

    (c) singleton sets

    (d) all proper subsets

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) If d is a metric on M, prove that $\sqrt{d}$ is a metric on M.

    Or

    (b) Prove that in any metric space the union of any family of open sets is open.

12. (a) Prove that in any metric space the union of a finite number of closed sets is closed.

    Or

    (b) Prove that any discrete metric space is complete.

13. (a) Let (M, d) be a metric space. Let $a \in M$. Show that the function $f : M \to R$ defined by $f(x) = d(x, a)$ is continuous.

    Or

    (b) Prove that the functions $f : R \to R$ defined by $f(x) = \sin x$ is uniformly continuous on R.

14. (a) Prove that A metric space M is connected iff there does not exist a continuous function f from M onto the discrete metric space $\{0, 1\}$.

Or

(b) Let M be a metric space. Let A be a connected subset of M. If B is a subset of M such that $A \subseteq B \subseteq \overline{A}$, then B is connected.

15. (a) Prove that any compact subset A of a metric space (M, d) is closed.

Or

(b) Prove that any totally bounded metric space is separable.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Let (M, d) be a metric space. Define $\rho(x, y) = 2d(x, y)$. Then prove that ρ and p are equivalent metrics.

Or

(b) Prove that let M be a metric space and let M_1 be a subspaces of M. Let $A_1 \subseteq M_1$. Then A_1 is open in M_1 iff there exists an open set A in M such that $A_1 = A \cap M_1$.

17. (a) Prove that A subset A of a complete metric space M is complete iff A is closed.

Or

(b) State and prove Baire's category theorem.

18. (a) Prove that let (M_1, d_1) and (M_2, d_2) be two metric spaces. A functions $f : M_1 \to M_2$ is continuous iff $f^{-1}(F)$ is closed in M_1 whenever F is closed in M_2.

Or

(b) Prove that the metric spaces $(0, 1)$ and $(0, \infty)$ with usual metrics are homeomorphic.

19. (a) Prove that A subspace of R is connected iff it is an interval.

Or

(b) State and prove intermediate value theorem.

20. (a) State and prove Heine Borel theorem.

Or

(b) Prove that A metric space (M, d) is totally bounded iff every sequence in M has a cauchy subsequence.