(7 pages) **Reg. No. :** ....

Code No. : 20421 E **Sub. Code : CMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023

Fourth Semester

Mathematics — Core

ABSTRACT ALGEBRA

(For those who joined in July 2021-2022 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answers.

1. The identity element of the group $(Z_n, \oplus)$ is ________.

(a) 4 (b) n

(c) 0 (d) $-n$

2. In the group $(R^*, \cdot)$, the order of -1 is ________.

(a) 1 (b) 3

(c) 4 (d) 2

3. A generator of the cyclic group $(2Z, +)$ is ________.

(a) 0 (b) 2

(c) 4 (d) 6

4. If H is a subgroup of G, then $a \in H \Rightarrow aH =$ ________

(a) H (b) a

(c) φ (d) G

5. $p = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} p^{-1} =$

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$

6. If $f = R^* \to R^*$, by $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ then kerf = ________.

(a) R (b) Z

(c) N (d) R^+

7. In the ring $(R, +, \cdot)$, the identity element is ________.

(a) 0 (b) 3

(c) 2 (d) 1

8. In the ring R, $a \in R$ and $a^2 = a$ then $a + a =$ ________.

(a) 0 (b) a

(c) $2a$ (d) 1

9. If $f : R \to R'$ is an isomorphism, then $f(0) =$ ________.

(a) 1 (b) $0'$

(c) G (d) -1

10. $f : R \to R'$ is $1-1$ then kerf = ________

(a) R (b) R'

(c) $0'$ (d) $\{0\}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $G = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \Big/ x \in R^* \right\}$, then prove that G is a group under matrix multiplication.

Or

(b) Let G be a group and $a, b \in G$. Then prove that order of a = order of $b^{-1}ab$.

12. (a) If H is a subgroup of G, then prove that $a \in bH \Rightarrow aH = bH$.

Or

(b) State and prove Fermat's theorem.

13. (a) If H and K are normal subgroup of a group G, then prove that $H \cap K$ is a normal subgroup of G.

Or

(b) Let G be a group show that $f : G \to G$ given by $f(x) = x^{-1}$ is an isomorphism $\Leftrightarrow G$ is abelian.

[P.T.O.]

14. (a) Show that a ring R has no zero divisors iff cancellation law is valid in R.

Or

(b) Prove that the only ideal of field F are $\{0\}$ and F.

15. (a) Let $f : R \to R'$ be a homomorphism. Prove that kerf is an ideal of R'.

Or

(b) Let R, R' be rings and $f : R \to R'$ be a homomorphism. Then prove that, if S is a subring of R, then $f(s)$ is a subring of R'

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let G be the set of all real numbers except -1. Define $*$ on G by $a * b = a + b + ab$. Prove that $(G,*)$ is a group.

Or

(b) Prove that the union of two subgroups of a group G is a subgroup iff one is contained in the other.

17. (a) State and prove Lagrange's theorem.

Or

(b) Prove that a subgroup of cyclic group is cyclic.

18. (a) State and prove Cayley's theorem.

Or

(b) Prove that any finite cyclic group of order n is isomorphic to $(Z_n, \oplus)$.

19. (a) Let R be any commutative ring with identity. Let P be an ideal of R. Then prove that P is a prime ideal $\Leftrightarrow$ R/P is an integral domain.

Or

(b) Prove that the set F of all real numbers of the form $a + b\sqrt{2}$, $a, b \in Q$ is a field under the usual addition and multiplication of real numbers.

20. (a) State and prove fundamental theorem of homomorphism.

Or

(b) Let R and R' be rings and $f : R \to R'$ be an isomorphism. Then prove that

(i) R is commutative $\Rightarrow R'$ is commutative.

(ii) R is an integral domain $\Rightarrow R'$ is an integral domain.