(7 pages) Reg. No. : ....

Code No. : 20420 E **Sub. Code : CMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Third Semester

Mathematics – Core

SEQUENCES AND SERIES

(For those who joined in July 2021 – 2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The l.u.b. of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$ is ————.

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) 3

2. $\lim_{n\to\infty} \frac{1}{2^n} =$ ————.

(a) 1 (b) 0

(c) 2 (d) ∞

3. $\lim_{n\to\infty} \frac{\sin n}{n} =$ ————.

(a) 1 (b) n

(c) 0 (d) ∞

4. $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} =$ ————.

(a) 0 (b) n

(c) $\frac{1}{n}$ (d) 1

5. The sequence $\left(\frac{1}{n}\right)$ is a

(a) Divergent sequence

(b) Oscillating sequence

(c) Cauchy sequence

(d) Not Cauchy sequence

6. Any Convergent sequence is ————.

(a) bounded (b) not bounded

(c) oscillates (d) none

7. The series $\sum \frac{1}{n \log n}$ ————.

(a) converges (b) diverges

(c) oscillates (d) bounded

8. The series $\sum \frac{x^n}{n^n}$ ————.

(a) converges (b) diverges

(c) converges to 1 (d) oscillates

9. The series $\sum \frac{(-1)^{n+1}(n+1)}{2^n}$ ————.

(a) diverges

(b) converges

(c) absolutely converges

(d) oscillates

10. The series $\sum (-1)^n \left(1+\frac{1}{n}\right)$ ————.

(a) oscillates (b) converges

(c) diverges (d) bounded

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing (a) or (b).

11. (a) Prove that $n^n(n+1)^{2n} > 4^n(n!)^3$ where $n \in N$.

Or

(b) Prove that $(1^m + 2^m + \ldots + n^m)2^m > n(n+1)^m$ for any rational number $m > 1$.

12. (a) Let $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$, show that (a_n) converges.

Or

(b) Show that if $|r| < 1$ then $(nr^n) \to 0$.

13. (a) Prove that $\frac{1}{n}((n+1)(n+2)\ldots(n+n))^{\frac{1}{n}} \to \frac{4}{e}$.

Or

(b) Show that $\lim_{n\to\infty} \frac{n!}{n^n} = 0$.

14. (a) Discuss the convergence of the series $\sum \frac{\sqrt{n+1}-\sqrt{n}}{nP}$.

Or

(b) Discuss the convergence of the series $\sum \frac{1^2+2^2+\ldots+n^2}{n^4+1}$.

15. (a) State and prove Abel's test.

Or

(b) State and prove Cauchy's root test.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the following :

(i) sequences in $\mathbb{R}$

(ii) bounded sequences

(iii) monotonic sequences

(iv) convergent sequences

Or

(b) If $a_1, a_2, \ldots, a_n$ are positive real numbers each $a_i < 1$, $a \le i \le n$ then prove that

(i) $(1+a_1)(1+a_2)\ldots(1+a_n) < \frac{1}{1-s}$ if $s<1$

(ii) $(1-a_1)(1-a_2)\ldots(1-a_n) < \frac{1}{1+s}$

17. (a) Show that

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{1!}+\ldots+\frac{1}{n!}\right) = e.$$

Or

(b) Show that $\lim_{n\to\infty}\frac{\log n}{nP} = 0$ if $P>0$.

18. (a) State and prove Cesaro's theorem.

Or

(b) Prove :

(i) any convergent sequence is a Cauchy sequence

(ii) any Cauchy sequence is a bounded sequence.

19. (a) State and prove Comparison test.

Or

(b) State and prove D'Alembert's ratio test.

20. (a) State and prove Leibnitz's test.

Or

(b) Discuss the convergence of the series $\sum\left(1+\frac{1}{2}+\ldots+\frac{1}{n}\right)\frac{\sin n\theta}{n}$.

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