(8 pages) Reg. No. : .....

Code No. : 20419 E **Sub. Code : CMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Second Semester

Mathematics — Core

DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2021–2022 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The solution of the equation $p^2 - 9p + 18 = 0$ is

(a) $(x - 3y)(x - 6y) = 0$

(b) $(x - 3y + c)(x - 6y) = 0$

(c) $(3x - y)(x - 6y + c) = 0$

(d) $(3x - y + c)(6x - y + c) = 0$

2. The solution of the equation $\frac{d^2x}{dt^2} - n^2x = 0$ is

(a) $x = A\cos nt + B\sin nt$

(b) $t = A\cos nx + B\sin nx$

(c) $x = e^t A\cos nt$

(d) $x = e^t B\sin nt$

3. The complementary function of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 3y = x^2$ is

(a) $e^{\sqrt{3}x} + e^{-\sqrt{3}x}$

(b) $Ax^{\sqrt{3}} + Bx^{-\sqrt{3}}$

(c) $Ae^{\sqrt{3}x} + Be^{-\sqrt{3}x}$

(d) $(A + B)e^{\sqrt{3}z}$

4. The particular integral of $x^2D^2y = e\log^x$ is ——

(a) $x_e\log_x$ (b) $\frac{x}{2}\log x$

(c) $\frac{x^2}{2}\log x$ (d) $x^2\log x$

5. Angle between the two perpendicular lines is

(a) $\cos^{-1}(0)$ (b) $\sin^{-1}\left(\frac{\pi}{2}\right)$

(c) $\cos^{-1}\left(\frac{\pi}{2}\right)$ (d) $\sin^{-1}(0)$

6. Equation of the plane through the point (1, 1, 1) and parallel to $11x + 2y + 2z = 1$ is

(a) $11x + 2y + 2z = 1$

(b) $11x + 2y + 2z + 15 = 0$

(c) $2x + y + 11z = 2$

(d) $11x + 2y + 2z = -1$

7. Equation of the straight line joining the points (2, 5, 0) and (−1, 6, 3) is

(a) $\frac{x-3}{3} = \frac{y-5}{-1} = \frac{z-8}{5}$

(b) $\frac{x+1}{3} = \frac{y-6}{-1} = \frac{z-3}{-3}$

(c) $\frac{x+3}{1} = \frac{y+1}{11} = \frac{z-5}{3}$

(d) $\frac{x+1}{-3} = \frac{y-6}{1} = \frac{z-3}{-5}$

8. Direction cosines of the line joining the points (3, −5, 4) and (1, −8, −2) is

(a) $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

(b) $\left(\frac{4}{7}, \frac{-13}{7}, \frac{2}{7}\right)$

(c) $\left(\frac{1}{7}, \frac{1}{7}, \frac{1}{7}\right)$

(d) $\left(\frac{7}{2}, \frac{3}{7}, \frac{7}{6}\right)$

9. The plane section of the sphere is a

(a) circle (b) straight line

(c) parabola (d) plane

10. Centre and the radius of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ is

(a) (0, 2, 4) and 16

(b) (0, −1, 2) and −4

(c) (0, 1, 2) and 4

(d) (0, −2, −4), and 16

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve : $y - \log p - 3x = 0$.

Or

(b) Solve : $(D^2 + 5D + 4)y = x^2 + 7x + 9$.

12. (a) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$.

Or

(b) Solve : $x^2\frac{d^2y}{dx^2} + y = 3x^2$.

13. (a) Show that the points (4, 0, 1), (3, 2, −1), (5, 4, 0) and (6, 2, 2) are the vertices of a square.

Or

(b) Show that the projection of a finite straight line AB on another straight line CD is $AB\cos\theta$.

14. (a) Find the perpendicular distance from $A(-1,3,9)$ to the line $\frac{x-13}{5} = \frac{y+8}{-8} = \frac{z-31}{1}$.

Or

(b) Find the condition that two given straight lines should be coplanar.

15. (a) Find the equation of the sphere which has its centre at the point (6, −1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

Or

(b) Show that the spheres

$x^2 + y^2 + z^2 + 6x + 10y + 22z = 245$

$x^2 + y^2 + z^2 - 12x - 14y - 18z + 141 = 0$ touch each other. Find the point of contact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Reduce the equation $e^{3x}(p-1) + p^3e^{2x} = 0$ to Clairaut's form by suitable substitution and solve it.

Or

(b) Solve : $\frac{dx}{dt} + 2x - 3y = t$.

17. (a) Solve : $(D^2 + 1)y = x^2e^{2x} + x\cos x$.

Or

(b) Solve : $(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$.

18. (a) Find the length and equation of the shortest distance between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Or

(b) Find the equation of the plane parallel to the plane $x - 2y + 2z - 3 = 0$ and whose perpendicular distance from (1, 2, 3) is 1.

19. (a) Find the condition that two given straight lines in the symmetrical form should be coplanar.

Or

(b) If the volume of the tetrahedron with vertices $(a, 1, 2)$, (3, 0, 1), (4, 3, 6), (2, 3, 2) is 6 cubic units, find the value of a.

20. (a) Find the equation of then sphere through the circle $x^2 + y^2 + z^2 = 0$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

Or

(b) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lines on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

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