(8 pages) Reg. No. : ....

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B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics — Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2021–2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The locus of the centre of curvature for a curve is called ———— of the curve.

   (a) involute (b) evolute

   (c) envelope (d) curvature

2. The curvature of a circle is the reciprocal of its ————

   (a) centre (b) arc

   (c) radius (d) diameter

3. The value of $\int_0^1\int_0^1 xy\,dx\,dy =$ ————

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$

   (c) 2 (d) 1

4. The value of $\int_1^c\int_1^b\int_1^a \frac{1}{xyz}\,dx\,dy\,dz =$ ————

   (a) $\log(abc)$ (b) $\log\left(\frac{1}{abc}\right)$

   (c) $\log a \log b \log c$ (d) $\frac{1}{\log(abc)}$

5. $\Gamma\left(\frac{1}{2}\right) =$ ————

   (a) π (b) $\sqrt{\pi}$

   (c) $\frac{\pi}{2}$ (d) $\frac{\sqrt{\pi}}{2}$

6. $\Gamma(m)\Gamma(n) =$ ————

   (a) $\beta(m,n)$

   (b) $\Gamma(m+n)$

   (c) $\beta(m,n)\,\Gamma(m-n)$

   (d) $\beta(m,n)\,\Gamma(m+n)$

7. The least degree of the equation which as $\sqrt{5}+\sqrt{2}$ as one of its roots is ————

   (a) 2 (b) 3

   (c) 4 (d) either (b) or (c)

8. The sum of roots of the equation $27x^3+54x^2-28x-8=0$ is ————

   (a) –2 (b) 2

   (c) 54 (d) –54

9. The standard form of reciprocal equations is of ————

   (a) even degree with unlike sign

   (b) odd degree with like sign

   (c) odd degree with unlike sign

   (d) even degree with like sign

10. The number of negative roots of the equation $x^7+8x^5-x+9=0$ is ————

    (a) atleast one (b) atmost one

    (c) atleast two (d) zero

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Show that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$.

    Or

    (b) Develop the $p-r$ equation of the cardiod $r=a(1-\cos\theta)$.

12. (a) By changing into polar co-ordinates, evaluate the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)\,dx\,dy$.

    Or

(b) Evaluate $\iint_R xy\,dx\,dy$ where R is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = a^2$; $x^2 - y^2 = b^2$ and the circles $x^2 + y^2 = c^2$; $x^2 + y^2 = d^2$ $(0 < a < b < c < d)$.

13. (a) Evaluate :

(i) $\int_0^{\pi/2} \sin^7 \theta \cos^5 \theta \, d\theta$

(ii) $\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta$

Or

(b) Prove that $\iiint \frac{dx\,dy\,dz}{\left(1 - x^2 - y^2 - z^2\right)^{1/2}} = \frac{\pi^2}{8}$, the integration extended to all positive values of the variables for which the expression is real.

14. (a) Solve the equation

$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.

Or

(b) Evaluate $\frac{1}{\alpha^5} + \frac{1}{\beta^5} = \frac{1}{\gamma^5}$ where α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.

15. (a) Find the roots of the equation :

$x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$

Or

(b) Determine completely the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Show that in the parabola $y^2 = 4ax$ at the point 't', $\rho = -2a\left(1 + t^2\right)^{3/2}$, $X = 2a + 3at^3$, $Y = -2at^3$. Deduce the equation of the evolutes.

Or

(b) Find the envelope of the circles drawn on the radius vector of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as diameter.

17. (a) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) By changing into polar co-ordinates evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx\,dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ $(b > a)$.

18. (a) Express $\int_0^1 x^m \left(1 - x^n\right)^p dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5 \left(1 - x^3\right)^{10} dx$.

Or

(b) Evaluate in terms of Gamma functions the integral $\iiint x^p y^q z^r dx\,dy\,dz$ taken over the volume of the tetrahedron given by $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x + y + z \leq 1$.

19. (a) If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.

Or

(b) Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

20. (a) Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation.

Or

(b) Solve the equation $x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing its second term.