(6 pages) Reg. No. : _____

Code No. : 20295 E **Sub. Code : AMMA 62**

B Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Sixth Semester

Mathematics – Core

GRAPH THEORY

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The number of graph with four vertices $V(G) = \{v_1, v_2, v_3, v_4\}$ having deg $(v_1) = 3$, deg $(v_2) = 2$, deg $(v_3) = 2$, deg $(v_4) = 2$ is _____

(a) 1 (b) 2

(c) 3 (d) 0

2. The number of edges in $K3,4$ is _____

(a) 7 (b) 12

(c) 3 (d) 4

3. If G is a connected graph, then $\omega(G)$ = _____

(a) 0 (b) 1

(c) 2 (d) None of these

4. Which of the following partitions is graphical?

(a) (3,3,3,1) (b) (3,2,1,1,1)

(c) (7,6,5,4,3,2,1) (d) (6,6,5,4,3,3,1)

5. Which of the following is Eulerian?

(a) K_{10} (b) $K_{3,10}$

(c) K_{15} (d) $K_{3,3}$

6. Every Hamiltonian graph is _____ connected.

(a) 2 (b) 3

(c) 4 (d) None of these

7. Which of the following is a planar graph?

(a) K_7 (b) K_6

(c) K_5 (d) K_4

8. Chromatic number of $\overline{K_5}$ is

(a) 2 (b) 3

(c) 5 (d) 1

9. If $f(G,\lambda) = \lambda^4 + 19\lambda^3 - 23\lambda^2 - 10\lambda$, then the number of points in G is _____

(a) 4 (b) 3

(c) 7 (d) 10

10. What is the in-degree of 1 in the following digraph?

(a) 1 (b) 2

(c) 3 (d) 4

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that, in any graph G, then number of points of odd degree is even.

Or

(b) Prove that $\overline{\lceil (G) = \lceil (\overline{G})}$.

12. (a) Prove that every non-trivial connected graphs has at least two points which are not cut points.

Or

(b) G is a $K-$ connected graph, prove that $q \geq \frac{pk}{2}$.

13. (a) Explain Fleury's Algorithm.

Or

(b) Let G be a (p,q) in which $p = q + 1$, a cyclic graph, prove that G is a tree.

14. (a) If G is a plane connected (p,q) graph without triangles and $p \geq 3$, prove that $q \leq 2p - 4$.

Or

(b) For any graph G, prove that $\chi \leq \Delta + 1$.

15. (a) Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

Or

(b) Explain three components of a diagraph.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the maximum number of lines among all p points graphs with no triangles is $\left\lfloor \frac{p^2}{4} \right\rfloor$.

Or

(b) Let G be a (p,q) graph. Prove that $L(G)$ is a (q_1, q_2) graph where $q_2 = \frac{1}{2}\left(\sum_{i=1}^{p} d_i^2\right) - q$.

17. (a) Let G, be a (p_1, q_1) graph and G_2 a (p_2, q_2) graph prove that $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph.

Or

(b) Prove that A partition $p = (d_1, d_2, \ldots d_p)$ of on even number into p parts with $p-1 \geq, d_1 \geq d_2 \geq \ldots \geq d_p$ is graphical is iff the modified $-1, d_{d_1+2}, \ldots d_p)$ partition $p' = (d_2 - 1, d_3 - 1, \ldots d_{d_1+1}$ is graphical.

18. (a) State and prove Chavatal theorem.

Or

(b) Prove that the following statements are equivalent for a connected graph.

(i) G is Eulerian

(ii) Every point of G has even degree

(iii) The set of edges of G can be partitioned into cycles

19. (a) The following statements are equivalent for any graph G

(i) G is 2-colourable

(ii) G is bipartite

(iii) Every cycle of G has even length

Or

(b) Prove that $\chi'(K_n) = \begin{cases} n \text{ if n is odd } (n \neq 1) \\ n-1 \text{ if n is even} \end{cases}$

20. (a) Prove that the coefficient of $f(G,-1)$ alternate in lign.

Or

(b) Prove that a graph G with $n \geq 2$ points is a tree iff $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.
