

(6 pages)

Reg. No. : \_\_\_\_\_

Code No. : 20295 E Sub. Code : AMMA 62

B Sc. (CBCS) DEGREE EXAMINATION,  
NOVEMBER 2023.

Sixth Semester

Mathematics - Core

GRAPH THEORY

(For those who joined in July 2020 only)

Time : Three hours

Maximum : 75 marks

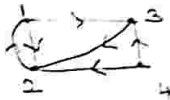
PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

- The number of graph with four vertices  $V(G) = \{v_1, v_2, v_3, v_4\}$  having  $\deg(v_1) = 3$ ,  $\deg(v_2) = 2$ ,  $\deg(v_3) = 2$ ,  $\deg(v_4) = 2$  is \_\_\_\_\_  
(a) 1 (b) 2  
(c) 3 (d) 0
- The number of edges in  $K_{3,4}$  is \_\_\_\_\_  
(a) 7 (b) 12  
(c) 3 (d) 4

- If  $f(G, \lambda) = \lambda^4 + 19\lambda^3 - 23\lambda^2 - 10\lambda$ , then the number of points in  $G$  is \_\_\_\_\_  
(a) 4 (b) 3  
(c) 7 (d) 10
- What is the in-degree of 1 in the following digraph?



- (a) 1 (b) 2  
(c) 3 (d) 4

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

- (a) Prove that, in any graph  $G$ , then number of points of odd degree is even.

Or

- (b) Prove that  $\overline{\overline{G}} = \overline{G}$ .

- (a) Prove that every non-trivial connected graphs has at least two points which are not cut points.

Or

- (b)  $G$  is a  $K$ -connected graph, prove that  $q \geq \frac{pk}{2}$ .

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- If  $G$  is a connected graph, then  $\omega(G) =$  \_\_\_\_\_  
(a) 0 (b) 1  
(c) 2 (d) None of these
- Which of the following partitions is graphical?  
(a) (3,3,3,1) (b) (3,2,1,1,1)  
(c) (7,6,5,4,3,2,1) (d) (6,6,5,4,3,3,1)
- Which of the following is Eulerian?  
(a)  $K_{10}$  (b)  $K_{4,10}$   
(c)  $K_{15}$  (d)  $K_{3,3}$
- Every Hamiltonian graph is \_\_\_\_\_ connected.  
(a) 2 (b) 3  
(c) 4 (d) None of these
- Which of the following is a planar graph?  
(a)  $K_7$  (b)  $K_6$   
(c)  $K_5$  (d)  $K_4$
- Chromatic number of  $\overline{K_4}$  is  
(a) 2 (b) 3  
(c) 5 (d) 1

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- (a) Explain Fleury's Algorithm.  
Or  
(b) Let  $G$  be a  $(p, q)$  in which  $p = q + 1$ , a cyclic graph, prove that  $G$  is a tree.
- (a) If  $G$  is a plane connected  $(p, q)$  graph without triangles and  $p \geq 3$ , prove that  $q \leq 2p - 4$ .

Or

- (b) For any graph  $G$ , prove that  $\chi \leq \Delta + 1$ .
- (a) Prove that  $\lambda^4 - 3\lambda^3 + 3\lambda^2$  cannot be the chromatic polynomial of any graph.

Or

- (b) Explain three components of a digraph.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

- (a) Prove that the maximum number of lines among all  $p$  points graphs with no triangles

$$\text{is } \left\lfloor \frac{p^2}{4} \right\rfloor.$$

Or

- (b) Let  $G$  be a  $(p, q)$  graph. Prove that  $L(G)$  is

$$\text{a } (q_1, q_2) \text{ graph where } q_2 = \frac{1}{2} \left( \sum_{i=1}^q d_i^2 \right) - q.$$

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[P.T.O.]

17. (a) Let  $G_1$  be a  $(p_1, q_1)$  graph and  $G_2$  a  $(p_2, q_2)$  graph prove that  $G_1 \times G_2$  is a  $(p_1 p_2, q_1 p_2 + q_2 p_1)$  graph.

Or

(b) Prove that A partition  $p = (d_1, d_2, \dots, d_p)$  of an even number into  $p$  parts with  $p-1 \geq d_1 \geq d_2 \geq \dots \geq d_p$  is graphical is iff the modified  $(-1, d_{d_1+2}, \dots, d_p)$  partition  $p' = (d_2 - 1, d_3 - 1, \dots, d_{d_1+1})$  is graphical.

18. (a) State and prove Chavatal theorem.

Or

(b) Prove that the following statements are equivalent for a connected graph.

- (i)  $G$  is Eulerian
- (ii) Every point of  $G$  has even degree
- (iii) The set of edges of  $G$  can be partitioned into cycles

19. (a) The following statements are equivalent for any graph  $G$

- (i)  $G$  is 2-colourable
- (ii)  $G$  is bipartite
- (iii) Every cycle of  $G$  has even length

Or

(b) Prove that  $\chi'(K_n) = \begin{cases} n & \text{if } n \text{ is odd } (n \neq 1) \\ n-1 & \text{if } n \text{ is even} \end{cases}$

20. (a) Prove that the coefficient of  $f(G, -1)$  alternate in sign.

Or

(b) Prove that a graph  $G$  with  $n \geq 2$  points is a tree iff  $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$ .

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