(6 pages) Reg. No. : ....

Code No. : 20294 E **Sub. Code : AMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The complex form of CR equations is ——————

(a) $f_x = -if_y$ (b) $f_x = if_y$

(c) $f_y = -if_x$ (d) $f_x = f_y$

2. If $f(z)=z^2$, then the value of $v(x, y)$ is ——————

(a) x^2-y^2 (b) $2xy$

(c) xy (d) x^2+y^2

3. If C is the circle with centre a and radius r then $\int_C \frac{dz}{(z-a)^n} =$ —————— where $n \neq 1$ is ——————

(a) $2\pi i$ (b) $\frac{\pi i}{2}$

(c) 0 (d) πi

4. $\int_C (z-a)^n \, dz =$ —————— for every closed curve C where $n \geq 0$

(a) 0 (b) $2\pi i$

(c) πi (d) $\frac{\pi i}{2}$

5. Maclaurin's series for e^z is ——————

(a) $1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+\ldots+\frac{z^n}{n!}+\ldots$

(b) $1-\frac{z}{1!}+\frac{z^2}{2!}-\frac{z^3}{3!}+\ldots-\frac{z^n}{n!}+\ldots$

(c) $z-\frac{z^3}{3!}+\frac{z^4}{4!}\ldots$

(d) $1-z+z^2-z^3+\ldots+(-1)^n z^n+\ldots$

6. The residue of $\cot z$ at $z=0$ is

(a) 1 (b) 0

(c) i (d) $-i$

7. $\int_C \frac{dz}{2z+3}$ (where C is $|Z|=2$) = ——————

(a) $2\pi i$ (b) πi

(c) 0 (d) None of these

8. The poles of $f(z)=\frac{z^2}{(z-2)(z+3)}$ is

(a) 2,3 (b) -2, 3

(c) 2, -3 (d) -2, -3

9. The fixed point of the transformation $w=\frac{1}{z-2i}$ is ——————

(a) 0 (b) i

(c) $-i$ (d) $2i$

10. Which one of the following is not a bilinear transformation?

(a) $w=z$ (b) $w=\bar{z}$

(c) $w=1+z$ (d) $w=1-z$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $\lim_{z \to 2} \frac{z^2-4}{z-2} = 4$.

Or

(b) Verify Cauchy-Riemann equations for the function $f(z)=z^3$.

12. (a) State and prove Liouville's theorem.

Or

(b) Evaluate $\int_C \left(\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\right) dz$ where C is the circle $|z| = 3$.

13. (a) Expand $f(z)=\frac{z-1}{z+1}$ as a Taylor's series about the point $z=1$.

Or

(b) Find the residue of $\frac{1}{(z^2+a^2)^2}$ at $z=ai$.

14. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

Or

(b) Evaluate $\int_C \tan z\,dz$ where C is $|z|=2$.

15. (a) Find the image of the circle $|Z-3i|=3$ under the map $w=\frac{1}{z}$.

Or

(b) Find the bilinear transformation which maps the points $z=-1,1,\infty$ respectively on $w=-i,-1,i$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive C.R equations in polar coordinates.

Or

(b) If $f(z)$ is analytic prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

17. (a) State and prove Cauchy's integral formula.

Or

(b) Let f be analytic inside and on a simple closed curve C. Let z be any point inside C. Prove that $f'(z)=\frac{1}{2\pi i}\int_C \frac{f(\varepsilon)}{(\varepsilon-z)^2}d\varepsilon$.

18. (a) State and prove Cauchy's residue theorem.

Or

(b) Find the residue of $\frac{1}{z-\sin z}$ at its pole.

19. (a) Prove that $\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-a^2}}(-1<a<1)$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1+x^2}dx=\frac{\pi}{2e}$.

20. (a) Prove that any bilinear transformation preserves cross ratio.

Or

(b) Prove that a bilinear transformation preserves inverse points.