Reg. No. : .................................

Code No. : 10071 E    Sub. Code : SMMA 65/
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B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017-2020)

Time : Three hours           Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Choose the transcendental equation from the following ——————.
   (a) $x^3 - 1 = 0$      (b) $x^2 + x + 1 = 0$
   (c) $x = 1$            (d) $e^x - 1 = 0$

2. Regula-Falsi method is also known as ——————.
   (a) method of tangents
   (b) method of false position
   (c) back substitution
   (d) forward substitution

3. If $f(x) = x^2 + x + 1$, then taking $h = 1$, $\Delta f(x) =$ ——————.
   (a) $x + 2$           (b) $2(x-1)$
   (c) $2x$              (d) $2$

4. Choose the wrong statement from the following ——————.
   (a) $E = 1 + \Delta$       (b) $\nabla = E^{-1}$
   (c) $1 - E^{-1} = \nabla$  (d) $\delta = E^{1/2} - E^{-1/2}$

5. If $f(4) = 1$, $f(6) = 3$, then the interpolating polynomial is ——————.
   (a) $3x - 1$          (b) $x - 3$
   (c) $x - 3$           (d) $3x - 2$

6. Newton's backward interpolation formula is used when interpolation is required near the —————— of the table.
   (a) beginning         (b) middle
   (c) end               (d) at the average

7. By evaluating $\int_0^1 \frac{dx}{1+x^2}$ by numerical interpolation, we obtain an approximate value of ——————.
   (a) $\log_e^2$        (b) $\pi$
   (c) $\log_{10}^2$     (d) $e$

8. Error in Simpson's one third rule is of order _____.

   (a) $h^2$             (b) $h^4$

   (c) $h$              (d) $4h$

9. The order of the difference equation $y_{n+1} - 3y_n = 3^n$ is _____.

   (a) 2              (b) 0

   (c) 1              (d) 3

10. The order of the difference equation $y_{n+2} - 2y_{n+1} + y_n = 2^n$ is _____.

   (a) $n+1$          (b) 2

   (c) $n$             (d) $n+2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Using Newton-Raphson method, find correct to four decimal places, the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$.

     Or

   (b) Solve the equations $2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33$ by Gauss-elimination method.

12. (a) Prove that $(\Delta(f_i g_i) = f_i \Delta g_i + g_{i+1} \Delta f_i)$.

     Or

   (b) Evaluate $\Delta^{10}\left[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)\right]$ if the interval of differencing is 2.

13. (a) The following are data from the steam table :

| Temperature °C : | 140 | 150 | 160 | 170 | 180 |
|---|---|---|---|---|---|
| Pressure kgf/cm² : | 3.685 | 4.854 | 6.302 | 8.076 | 10.225 |

     Using Newton's formula, find the pressure of the steam for the temperature of 142°.

     Or

   (b) Using Gauss's backward formula, find the value of sales for the year 1966 given that

| Year : | 1931 | 1941 | 1951 | 1961 | 1971 | 1981 |
|---|---|---|---|---|---|---|
| Sales : | 12 | 15 | 20 | 27 | 39 | 52 |

14. (a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $x = 1.25$ from the following values of $x$ and $y$ :

| $x$ : | 1.00 | 1.05 | 1.10 | 1.15 |
|---|---|---|---|---|
| $y$ : | 1.00000 | 1.02470 | 1.04881 | 1.07238 |

| $x$ : | 1.20 | 1.25 | 1.30 |
|---|---|---|---|
| $y$ : | 1.09544 | 1.11803 | 1.14017 |

     Or

(b) Use the trapezoidal rule with $h = \frac{1}{4}$ to evaluate $\int_0^1 f(x)dx$ using the table below :

| $x$ : | 0.000 | 0.250 | 0.500 | 0.750 | 1.000 |
|---|---|---|---|---|---|
| $f(x)$ : | 0.79788 | 0.77334 | 0.70413 | 0.60227 | 0.48394 |

15. (a) Solve the difference equation.

$$y_{x+2} - 8y_{x+1} + 15y_x = 0$$

Or

(b) Solve $u_{x+2} - 6u_{x+1} + 9u_x = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Determine the root of $x^3 - 3 = 0$ correct to three decimal places, using the method of false position.

Or

(b) Solve by Gauss elimination procedure, the equations.

$3.15x - 1.96y + 3.85z = 12.95$,

$2.13x + 5.12y - 2.89z = -8.61$,

$5.92x + 3.05y + 2.15z = 6.88$.

17. (a) Represent the function

$x^4 - 12x^3 + 42x^2 - 30x + 9$ and its successive differences in factorial rotation in which the differencing interval $h = 1$.

Or

(b) Obtain the function whose first difference is

$x^3 + 3x^2 + 5x + 2$.

18. (a) Find log 337.5, by Laplace Everett formula, for the following data :

| $x$ : | 310 | 320 | 330 |
|---|---|---|---|
| $\log x$ : | 2.4913617 | 2.5051500 | 2.5185139 |
| $x$ : | 340 | 350 | 360 |
| $\log x$ : | 2.5314789 | 2.5440680 | 2.5563025 |

Or

(b) Prove Lagrange's interpolation formula in the form

$$f(x) = \sum_{r=0}^{n} \frac{\phi(x)f(x_r)}{(x - x_r)\phi'(x_r)}, \text{ where}$$

$$\phi(x) = \prod_{r=0}^{n}(x - x_r) \text{ and } \phi'(x) = \left[\frac{d}{dx}\phi(x)\right].$$

19. (a) Using the following data, find $f'(5)$.

| $x$ : | 0 | 2 | 3 | 4 | 7 | 9 |
|-------|---|----|----|-----|-----|-----|
| $f(x)$ : | 4 | 26 | 58 | 112 | 466 | 922 |

Or

(b) Dividing the range into 10 equal parts, find the approximate value of $\int_0^\pi \sin x \, dx$ by

(i) Trapezoidal rule (ii) Simpson's rule

0. (a) Solve the difference equation

$y_{n+3} - 3y_{n+1} + 2y_n = 0$ given $y_1 = 0, \; y_2 = 8$ and $y_3 = -2$.

Or

(b) Solve the equation

$y_{n+2} + y_{n+1} - 56y_n = 2^n(n^2 - 3)$.

———————