

(6 pages)

Reg. No. :

Code No. : 10069 E Sub. Code : SMMA 63

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2023.

Sixth Semester

Mathematics – Core

GRAPH THEORY

(For those who joined in July 2017-2019)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In a (p, q) complete graph $q =$ _____.
- (a) $\frac{p(p-1)}{2}$
 (b) $p(p-1)$
 (c) $\frac{p-1}{2}$
 (d) $p-1$

2. If G_1 is a (p_1, q_1) graph and G_2 is a (p_2, q_2) graph, then $G_1 \cup G_2$ is a _____ graph.
- (a) $(p_1 p_2, q_1 q_2)$
 (b) $(p_1 + p_2, q_1 q_2)$
 (c) $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$
 (d) $(p_1 + p_2, q_1 + q_2)$
3. Which one of the following is a graphic sequence?
- (a) (7, 6, 5, 4, 3, 2, 1, 1)
 (b) (7, 6, 5, 4, 3, 2, 1)
 (c) (7, 6, 5, 4, 3, 2, 2, 2, 1)
 (d) (7, 6, 5, 4, 3, 1, 0, 0)
4. In any graph _____.
- (a) $\lambda \leq k \leq \delta$
 (b) $\delta \leq \lambda \leq k$
 (c) $k \leq \lambda \leq \delta$
 (d) $k \leq \delta \leq \lambda$
5. Which one of the following is an Eulerian graph?
- (a) k_2
 (b) k_3
 (c) k_4
 (d) k_6

Page 2 Code No. : 10069 E

6. If G is a (p, q) tree, $q =$ _____.
- (a) $p-1$
 (b) $p+1$
 (c) p
 (d) $\frac{p(p-1)}{2}$
7. The largest complete plane graph is _____.
- (a) k_3
 (b) k_4
 (c) k_5
 (d) k_6
8. Chromatic number of $\overline{K}_p =$ _____.
- (a) 1
 (b) p
 (c) $p-1$
 (d) 0
9. $f(k_2, \lambda)$
- (a) λ
 (b) λ^2
 (c) $\lambda(\lambda-1)$
 (d) $\lambda^2(\lambda-1)$
10. In any digraph, _____.
- (a) $\Sigma d^+(v) = p$
 (b) $\Sigma d^-(v) = p$
 (c) $\Sigma d^+(v) = 2q$
 (d) $\Sigma d^-(v) = q$

Page 3 Code No. : 10069 E

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\delta \leq \frac{2q}{p} \leq \Delta$.
- Or
- (b) Prove that in any graph, the number of vertices of odd degree is even.
12. (a) Prove that a graph with p points and $\delta \geq \frac{p-1}{2}$ is connected.
- Or
- (b) Prove that a connected graph with at least two points has at least two points which are not cut points.
13. (a) Write Fluery's algorithm.
- Or
- (b) Draw all trees with 6 vertices.
14. (a) If G is a plane graph in which every face is an n -cycle, prove that $q = \frac{n(p-2)}{n-2}$.
- Or
- (b) If G is k -critical, prove that $\delta \geq k-1$.

Page 4 Code No. : 10069 E
[P.T.O.]

15. (a) Find the chromatic polynomial of C_4 .

Or

(b) Prove that every point of an eulerian weak diagraph has equal indegree and outdegree.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the maximum number of lines among all p point graphs with no triangles

is $\left\lfloor \frac{p^2}{4} \right\rfloor$.

Or

(b) (i) Prove that any self complementary graph has $4n$ or $4n + 1$ vertices

(ii) Prove that $\Gamma(G) = \Gamma(\overline{G})$.

17. (a) Prove that a graph with atleast two points is bipartite iff all its cycles are of even length.

Or

(b) Prove that any closed walk of odd length contains a cycle.

Page 5 Code No. : 10069 E

18. (a) Prove that $C(G)$ is well defined.

Or

(b) Prove that every tree has a centre with one point or two adjacent points.

19. (a) State and prove Euler's theorem.

Or

(b) Prove that $\chi'(k_n) = n$ if n is odd and $n \neq 1$.

20. (a) For any (p, q) graph G , prove that $f(G, \lambda)$ is a p degree polynomial with constant term zero.

Or

(b) If every edge of a connected graph G is contained in atleast one cycle, prove that the edges of G can be oriented so that the resulting digraph is strongly connected.

Page 6 Code No. : 10069 E