Code No. : 10068 E   Sub. Code : SMMA 62

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics – Core

NUMBER THEORY

(For those who joined in July 2017-2019 only)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer : .

1. The value of $\binom{n}{n}$ is _____.

    (a)   0   (b) n
    (c)   1   (d) $n!$

2. The value of 1! is _____.

    (a)   1   (b)   0
    (c) n (d)   none

3. gcd (−5, 5) is _____.

    (a)   − 5   (b)   5

    (c)   1   (d)   25

4. For any positive integers a and b the value of gcd (a,b) lcm (a,b) is _____.

   (a) ab (b) $a+b$

   (c) a (d) b

5. Repunit R_n consisting of n consecutive _____.

   (a) 2's (b) 1's

   (c) 3's (d) 0's

6. The value of $\pi_{4,1}(89)$ is _____.

   (a) 13 (b) 10

   (c) 8 (d) 9

7. Which one of the following is wrong?

   (a) $-12 \equiv 2 \pmod{7}$ (b) $13 \equiv 6 \pmod{7}$

   (c) $91 \equiv 0 \pmod{7}$ (d) $82 \equiv 4 \pmod{7}$

8. If $4x \equiv 0 \pmod{12}$, the value of x is _____.

   (a) 2 (b) 3

   (c) 4 (d) 5

9. Which one is the smallest pseudo prime to the base 5?

   (a) 91 (b) 217

   (c) 247 (d) 341

10. If p is a prime, then $(p-1)! \equiv -1$ _____.

    (a) mod 1 (b) mod p

    (c) mod $p-1$ (d) none

    PART B — (5 × 5 = 25 marks)

    Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the first principle of finite induction.

    Or

    (b) State and prove Pascal's rule.

12. (a) State and prove Euclid's lemma.

    Or

    (b) Find the gcd (12378, 3054).

13. (a) If p is a prime and $p|ab$, then prove that $p|a$ or $p|b$.

    Or

    (b) Prove that there is an infinite number of primes.

14. (a) Solve $18x \equiv 30 \pmod{42}$.

    Or

    (b) If $ca \equiv cb \pmod{n}$ and $d = \gcd(c,n)$, prove that $a \equiv b \pmod{n/d}$.

15. (a) State and prove Wilson theorem.

Or

(b) Factor the number 12499 using Fermat's method.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Archimedean property.

Or

(b) State and prove binomial theorem.

17. (a) State and prove Division Algorithm.

Or

(b) Solve the linear diophantine equation $172x + 20y = 1000$.

18. (a) Prove that the number $\sqrt{2}$ is irrational.

Or

(b) If p_n is the nth prime number, then prove that $p_n \leq 2^{2^{n-1}}$.

19. (a) State and prove Chinese remainder theorem.

Or

(b) Solve $7x + 3y \equiv 10 \pmod{16}$

$2x + 5y \equiv 9 \pmod{16}$.

20. (a) State and prove Fermat's theorem.

Or

(b) Clarify the proof of Wilson theorem with $p = 13$.