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Reg. No. :..

Code No. : 10067 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 – 2019)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. The real part of the function $f(z) = z\,\bar{z}$ is ————.

(a) x^2 (b) $x + y$

(c) $x^2 + y^2$ (d) xy

2. If $f(z) = u + iv$ is analytic and $f(z) \neq 0$, then $\nabla^2 \log|f(z)| =$ ————.

(a) $|f'(x)|^2$ (b) $\log|f'(x)|$

(c) 1 (d) 0

3. The length of the circle $z = a + re^{it}$, $0 \le t \le 2\pi$ is ————

(a) $2\pi r$ (b) πr^2

(c) 2π (d) $2r$

4. The value of $\int_C \frac{z\,dz}{z^2 - 9}$, $C : |z| = 2$ is ————.

(a) $4\pi i$ (b) πi

(c) 0 (d) $2\pi i$

5. The singular point of $f(z) = \tan z$ is ————.

(a) $\frac{\pi}{2} + n\pi,\ n \in z$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{2} + n\pi,\ n \in \mathbb{N}$ (d) 0

6. Residue of the function $f(z) = \frac{e^z}{z^2}$ at $z = 0$ is ————.

(a) $1/2$ (b) 1

(c) 0 (d) $\frac{1}{z}$

7. Where we evaluated the $\int_0^{2\pi} f(\cos\theta, \sin\theta)\,d\theta$, type of integrals?

(a) $|z| > r$ (b) $|z| < r$

(c) $|z| = r$ (d) $|z| = 1$

8. In the integral type $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)^2}\,dx$, the value of $f(z)$ is ————.

(a) $\frac{1}{(z^2 + a^2)^2}$ (b) $\frac{e^{iz}}{(z^2 + a^2)^2}$

(c) $\frac{\cos z}{(z^2 + a^2)^2}$ (d) $\frac{\cos z}{(z^2 + a^2)^3}$

9. The fixed point of the transformation $w = z + a$ is

(a) None (b) 1

(c) 0 (d) ∞

10. A bilinear transformation having ∞ as the only fixed point is ————.

(a) inversion

(b) translation

(c) contraction

(d) magnification

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove complex form of $C - R$ equations.

Or

(b) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are both hormonic, but $u + iv$ is not analytic.

12. (a) Evaluate $\int_C |z|^2\,dz$, where C is the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$.

Or

(b) State and prove Morera's theorem.

13. (a) Expand $f(z) = \sin z$ in a Taylor series about $z = \pi/4$.

Or

(b) Evaluate $\int_C \frac{z^2\,dz}{(z-2)(z+3)}$, where C is the circle $|z| = 4$.

14. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$.

Or

(b) Evaluate $\int_0^{\infty} \frac{dx}{x^2+1}$.

15. (a) Find the image of the circle $|z-3i|=3$ under the map $w=\frac{1}{2}$.

Or

(b) Show that the transformation $w=\frac{5-4z}{4z-2}$ maps the unit circle $|z|=1$ into a circle of radius unity and centre $-1/2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that $u=\log\sqrt{x^2+y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function $f(z)$.

Or

(b) If $f(z)$ is analytic, then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

17. (a) State and prove Cauchy's integral formula.

Or

(b) Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2}dz$, where C is $|z|=3$.

18. (a) Expand $\frac{1}{(z-1)(2-1)}$ as a power series in z in the regions

(i) $|z|<1$

(ii) $1<|z|<2$

(iii) $|z|>2$

Or

(b) State and prove Rouche's theorem.

19. (a) Prove that $\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}, (-1<a<1)$.

Or

(b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$.

20. (a) Find the bilinear transformation which maps the points $-1, 1, \infty$ respectively on $-i, -1, i$.

Or

(b) Find the image of the strip $2<x<3$ under $w=\frac{1}{z}$.