(8 pages)

Reg. No. : ....

Code No. : 10066 E **Sub. Code : SMMA 54/ AMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fifth Semester

Mathematics – Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017 – 2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $F\{f(x)\} = \bar{f}(s)$, then $F\{e^{-iax} f(x)\} =$ ________.

   (a) $\bar{f}(s-a)$ (b) $\bar{f}(x-a)$

   (c) $\bar{f}(x+a)$ (d) $\bar{f}(s+a)$

2. $F_c\{f'(x)\} =$ ________.

   (a) $sF_c\{f(x)\} - f(0)$ (b) $sF_s\{f(x)\} - f(0)$

   (c) $F_c\{f(x)\} - f(0)$ (d) $F_s\{f(x)\} - f(0)$

3. $F\{f''(x)\} =$ ________.

   (a) $(is)^2 \bar{f}(s)$ (b) $(is)\bar{f}(s)$

   (c) $(is)^2 \bar{f}(x)$ (d) $(is)\bar{f}(x)$

4. $F_s\{f(x)\cos ax\} =$ ________.

   (a) $\frac{1}{2}\left[\bar{f}_c(s+a) + \bar{f}_c(s-a)\right]$

   (b) $\frac{1}{2}\left[\bar{f}_s(s+a) + \bar{f}_s(a-s)\right]$

   (c) $\frac{1}{2}\left[\bar{f}_s(s+a) + \bar{f}_s(s-a)\right]$

   (d) $\frac{1}{2}\left[\bar{f}_c(s-a) + \bar{f}_c(s+a)\right]$

5. $F_s\{f(x)\} = \bar{f}_s(n) =$ ________.

   (a) $\int_0^l f(x) \sin(n\pi x)\, dx$

   (b) $\int_0^l f(x) \sin(\frac{n\pi x}{l})\, dx$

   (c) $\int_0^l f(x) \cos(n\pi x)\, dx$

   (d) $\int_0^l f(x) \cos(\frac{n\pi x}{l})\, dx$

6. $F_s\{f'(x)\}=$ ____________.

(a) $\frac{n\pi}{l}\bar{f}_s(n)$ (b) $\frac{-n\pi}{l}\bar{f}_s(n)$

(c) $\frac{n\pi}{l}f_c(n)$ (d) $\frac{-n\pi}{l}f_s(n)$

7. $Z\{a^{n-1}\}=$ ____________, if $n \geq 1$.

(a) $\frac{1}{z-a}$ (b) $\frac{z}{z-a}$

(c) $\frac{1}{z+a}$ (d) $\frac{z}{z+a}$

8. $Z(t^2)=$ ____________.

(a) $\frac{T^2z(z-1)}{(z+1)^3}$ (b) $\frac{T^2z(z+1)}{(z+1)^3}$

(c) $\frac{T^2z(z+1)}{(z-1)^3}$ (d) $\frac{T^2z(z-1)}{(z-1)^3}$

9. $z^{-1}\left\{\frac{z}{z-a}\right\}=$ ____________.

(a) a^{n-1} (b) a^2

(c) $e^{1/z}$ (d) a^n

10. $z^{-1}\left\{e^{a/z}\right\}=$ ____________.

(a) a^{n-1} (b) $\frac{a^{n-1}}{(n-1)!}$

(c) $\frac{a^n}{n!}$ (d) $n!a^{n-1}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) $e^{-ax}-e^{-bx}=\frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{u\sin xu}{(u^2+a^2)(u^2+b^2)}du$ $(a,b>0)$.

Using Fourier integral formula, prove that $e^{-ax}-e^{-bx}=\frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{u\sin xu}{(u^2+a^2)(u^2+b^2)}du$ $(a,b>0)$.

Or

(b) Find the Fourier transform of $f(x)$, defined as $f(x)=\begin{cases}1, & for\ |x|<a \\ 0, & for\ |x|>a\end{cases}$. Hence find $f\left\{f(x)\left[1+\cos\frac{\pi x}{a}\right]\right\}$.

12. (a) Find the Fourier sine transform of $f(x)$ defined as $f(x)=\begin{cases}\sin x, & when\, 0<x<a\\ 0, & when\ x>a\end{cases}$.

Or

(b) Use transform methods to evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$.

13. (a) Find the finite Fourier sine and cosine transforms of $\left(\frac{x}{\pi}\right)$ in $(0,\pi)$.

Or

(b) Find the finite Fourier sine transform of $\cos ax$ in $(0,\pi)$.

14. (a) Find the z-transform of $f(n)=\frac{2n+3}{(n+1)(n+2)}$.

Or

(b) Find the z-transform of $\cos^3 t$.

15. (a) Find $z^{-1}\left\{\frac{1+2z^{-1}}{1-z^{-1}}\right\}$ by the long division method.

Or

(b) Find $z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$, by using Residue theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation $(D^2-4D+4)y=xe^{-x}$, $x>0$, given that $y(0)=0$ and $y'(0)=0$.

Or

(b) Solve the one-dimensional heat flow equation $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ for a rod with insulated sides extending from $-\infty$ to ∞ and with initial temperature distribution given by $u(x,0)=f(x)$.

17. (a) Find the Fourier sine and cosine transforms of x^{n-1}. Hence deduce that $\frac{1}{\sqrt{x}}$ is self-reciprocal under both the transforms. Also find $F\left\{\frac{1}{\sqrt{|x|}}\right\}$.

Or

(b) Find $F_c\left(e^{-a^2x^2}\right)$ and hence find $F_s\left(xe^{-a^2x^2}\right)$.

18. (a) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<10$, using finite Fourier transforms, given that $u(0,t)=0$, $u(10,4)=0$, for $t>0$ and $u(x,0)=10x-x^2$ for $0<x<10$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}, 0<x<l$, using finite Fourier transforms, given that $\frac{\partial u}{\partial x}(0,t)=0$, $\frac{\partial u}{\partial x}(l,t)=0$, for $t>0$ and $u(x,0)=kx$, for $0<x<l$.

19. (a) Find the z-transform of the following functions.

(i) $\cos wt$,

(ii) $\sin wt$,

(iii) $e^{-at}\cos bt$,

(iv) $e^{-at}\sin bt$.

Or

(b) Find the z-transform of $f(n)*g(n)$, where

(i) $f(n)=\left(\frac{1}{2}\right)^n$ and $g(n)=\cos n\pi$

(ii) $f(n)=\begin{cases}\left(\frac{1}{3}\right)^n, & for\ n\geq 0\\ \left(\frac{1}{2}\right)^{-n}, & for\ n<0\end{cases}$ and

$g(n)=\left(\frac{1}{2}\right)^n U(n)$.

20. (a) Find $z^{-1}\left\{\frac{z^2}{(z+2)(z^2+4)}\right\}$, by the method of residues.

Or

(b) Solve the equation $y_{n+2}-7y_{n+1}+12y_n=2^n$, given that $y_0=y_1=0$.