(6 pages) **Reg. No. :** ....

Code No. : 10089 E **Sub. Code : SEMA 6 B**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2023

Sixth Semester

Mathematics – Major Elective

FUZZY MATHEMATICS

(For those who joined in July 2017-2019)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following symbol is used for universal set?

   (a) α (b) a

   (c) A (d) X

2. A set whose members can be labelled by the positive integers is called a ————.

   (a) open set (b) closed set

   (c) countable set (d) uncountable set

3. Let $f : X \to Y$ be an arbitrary crisp function and $A \in \mathcal{F}(X)$. Then ————.

   (a) $A \subset f^{-1}(f(A))$ (b) $A \supset f^{-1}(f(A))$

   (c) $A \subseteq f^{-1}(f(A))$ (d) $A \supseteq f^{-1}(f(A))$

4. Let $A, B \in \mathcal{F}(X)$ and $\alpha, \beta \in [0,1]$. Then $\alpha \leq \beta \Rightarrow$

   (a) $\alpha_A \supseteq \beta_A$ (b) $\alpha_A \supseteq \beta +_A$

   (c) $\alpha +_A \supseteq \beta_A$ (d) $\alpha_A \subseteq \beta_A$

5. The standard fuzzy intersection is the only ———— τ-norm.

   (a) Archimedean

   (b) Strictly Archimedean

   (c) Idempotent

   (d) Involutive

6. $u(a,b) = \min(1, a+b)$ is known as ————.

   (a) Standard union (b) Algebraic sum

   (c) Bounded sum (d) Drastic union

7. If $A = [0, 1]$, $B = [1, 2]$ and $C = [-2, -1]$, then $A,(B+C) =$ ————.

   (a) [–1, 1] (b) [–2, 2]

   (c) [1, 1] (d) [2, 2]

8. To Qualify as a fuzzy number, a fuzzy set A on $\mathbb{R}$ must be ————.

(a) convex (b) not convex

(c) sub normal (d) normal

9. The set of vectors X that satisfy all given constraints is called a ————.

(a) Cost vector

(b) Feasible set

(c) Constraint matrix

(d) Right hand - side vector

10. Fuzzy decision making was introduced by

(a) Bellman (b) Blin

(c) Whinston (d) Datiz

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $S(A, B) = \frac{|A \cap B|}{|A|}$.

Or

(b) Prove that the absorption law.

12. (a) Let $A, B \in \mathcal{F}(X)$. Then prove that $\alpha \in [0, 1]$, $A = B$ if and only if $\alpha_A = \alpha_B$ and $A = B$ iff $\alpha +_A = \alpha +_B$.

Or

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then prove that for any $A \in \mathcal{F}(X)$, f fuzzfied by the extension principle satisfies the equation $f(A) = U_{\alpha \in [0,1]} f(\alpha +_A)$.

13. (a) If C is a continuous fuzzy complement, then prove that C has a unique equilibrium.

Or

(b) For all $a, b \in [0, 1]$, prove that max $(a, b) \le u(a, b) \le u_{\max}(a, b)$.

14. (a) Let Min and Max be binary operations on R defined by

$$Min(A, B)(z) = \sup_{z=\min(x,y)}[A(x), B(y)]$$

$$Max(A, B)(z) = \sup_{z=\max(x,y)}[A(x), B(y)]$$

respectively. Then prove that, for any $A, B, C \in R$,

$$Min[A, Max(B, C)] = Max[Min(A, B), Min(A, C)$$

Or

(b) Explain the arithmetic operations on intervals.

(a) Define fuzzy linear programming problem.

Or

(b) Solve the following fuzzy linear programming problem:

Max. $Z = 5x_1 + 4x_2$

Such that $\langle 4,2,1\rangle x_1 + \langle 5,3,1\rangle x_2 \le \langle 24,5,8\rangle$

$\langle 4,1,2\rangle x_1 + \langle 1,5,1\rangle x_2 \le \langle 12,6,3\rangle$

$x_1, x_2 \ge 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

(a) Prove that a fuzzy set A of $\mathbb{R}$ is convex if and only if $A(\lambda x_1 + (1-\lambda)x_2) \ge \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0, 1]$, where min denotes the minimum operator.

Or

(b) Define the following :

(i) α-cut

(ii) Strongly α-cut

(iii) Level set

(iv) Support of a fuzzy set A

(v) The height of A

(vi) Normal and subnormal fuzzy sets.

17. (a) State and prove first decomposition theorem.

Or

(b) For any $A \in \mathcal{F}(X)$, $A = U_{\alpha\Lambda(A)} \alpha_A$ where $\Lambda(A)$ is the set of A, α_A defined by $\alpha_A = \alpha \cdot \alpha_{A(x)}$ and U denotes standard fuzzy union $\Lambda(A) = \{\alpha / \alpha = A(x) \text{ for } x \in X\}$.

18. (a) State and prove first characterization theorem of fuzzy complement.

Or

(b) For all $a, b \in [0, 1]$, prove that $i_{\min}(a, b) \le i(a, b) \le \min(a, b)$ where $i_{\min}$ denotes the drastic intersection.

19. (a) State and prove characterization theorem for fuzzy number.

Or

(b) Explain the lattice of fuzzy numbers.

20. (a) Explain the individual decision making.

Or

(b) Explain the multiperson decision making.