(8 pages)

Reg. No. : ....

Code No. : 10073 E **Sub. Code : SAMA 21/ AAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second/Fourth Semester

Mathematics

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2017-2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $\bar{f} = (axy - z^3)\bar{i} + (a-2)x^2\bar{j} + (1-a)xz^2\bar{k}$ is irrotational, Then the values of 'a' is ______________.

   (a) -4 (b) 4

   (c) 2 (d) 0

2. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla.\bar{r} =$ ______________.

   (a) 1 (b) 0

   (c) 3 (d) $x^2 + y^2 + z^2$

3. The values of $\iint_R dx\,dy$ where D is the circular disc $x^2 + y^2 \leq 5$ is ______________.

   (a) 5π (b) $2\sqrt{5}\pi$

   (c) $\sqrt{5}\pi$ (d) 25π

4. The values of $\iiint_D dx\,dy\,dz$ where D is the region banded by the sphere $x^2 + y^2 + z^2 = 9$ is ______________.

   (a) 36π (b) $\frac{4\pi}{3}$

   (c) 324π (d) $\frac{3\pi}{4}$

5. If C is the straight line joining (0, 0, 0) and (1, 1, 1), then $\int_c \bar{r}.d\bar{r}$ ______________.

   (a) $\frac{1}{2}$ (b) 1

   (c) 2 (d) $\frac{3}{2}$

6. The value of $\int_0^1 \int_0^1 (x^2 + y^2)\,dx\,dy$ is ____________.

(a) $\frac{3}{2}$ (b) 12

(c) $\frac{1}{2}$ (d) 2

7. If S is any closed surface enclosing as volume V and $\bar{f} = ax\bar{i} + by\bar{j} + cz\bar{k}$ then $\iint_s \bar{f}.\bar{n}\,ds =$ ____________.

(a) 3V (b) $(a+b+c)V$

(c) $(a+b+c)^3 V^3$ (d) 0

8. If R is any closed region of the xy-plane bounded by a simple closed curve C, then $\int_c y\,dx + x\,dy$ is ____________.

(a) 1 (b) π

(c) 0 (d) 2π

9. If $f(x)$ is an odd function, ____________.

(a) $f(x) = f(2x)$ (b) $f(x) = f(-x)$

(c) $f(x) = f(-2x)$ (d) $f(x) = -f(-x)$

10. $\int_0^{\pi} \sin mx \sin x\,dx$ ____________ (if $m = n$)

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{3}$ (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\bar{r}$ is the position vector of any point $P(x, y, z)$, prove that grad $r^n = nr^{n-2}\bar{r}$.

Or

(b) Prove that $div(r^n \bar{r}) = (n+3)r^n$. Deduce that $r^n\bar{r}$ is solenoidal iff $n = -3$.

12. (a) Evaluate $\int_0^1 \int_x^1 (x^2 + y^2)\,dy\,dx$.

Or

(b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x + 2y)\,dx\,dy$.

13. (a) Evaluate $\int_c \bar{f}.d\bar{r}$ where $\bar{f} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ and the bounded by $y = 0, y = b, x = 0, x = a$.

Or

(b) Evaluate $\iint_s \bar{f}.\bar{n}\, ds$ where $\bar{f} = (x + y^2)\bar{i} - 2x\bar{j} + 2yz\bar{k}$ and S is the surface of the plane $2x + y + 2z = 6$.

14. (a) Use Green's Thm in a plane to evaluate $\int_c (2x - y)dx + (x + y)dy$ where C is the boundary of the circle $x^2 + y^2 = a^2$ in the xy plane.

Or

(b) Using Stoke's theorem, evaluate $\int_c (\sin x - y)dx = \cos x dy$ where C is the boundary of the triangle whose vertices are (0, 0), $(\pi/2, 0), (\pi/2, 1)$.

15. (a) Find the half range sine series of $f(x) = a$ in $(0, l)$.

Or

(b) Find the half range cosine series of $f(x) = (\pi - x)^2$ in $(0, \pi)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that grade $(\bar{f}.\bar{g}) = \bar{f} \times curl\bar{g} + \bar{g} \times curl\bar{f} + (\bar{f}.\nabla)\bar{g} + (\bar{g}.\nabla)\bar{f}$

Or

(b) If $r = a\cos wt + b\sin wt$ where a, b are constant vectors and w is a constant, prove that $\bar{r} \times \frac{dr}{dt} = w(\bar{a} \times \bar{b})$ and $\frac{d^2r}{dt^2} + w^2\bar{r} = 0$.

17. (a) Find the area of the region D bounded by the parabola $y = x^2$ and $x = y^2$.

Or

(b) Evaluate $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyz\, dx\, dy\, dz$.

18. (a) Evaluate $\iint_S \bar{f}\cdot\bar{n}\,dS$ where $\bar{f}=(x^3-yz)\bar{i}-2x^2y\bar{j}+2\bar{k}$ and S is the surface of the cube bounded by $x=0$, $y=0$, $z=0$, $x=a$, $y=a$ and $z=a$.

Or

(b) Evaluate $\iint_S \bar{f}\cdot\bar{n}\,dS$ where $\bar{f}=4x\bar{i}-2y^2\bar{j}+z^2\bar{k}$ and S is the surface bounding the region $x^2+y^2=4$, $z=0$ and $z=3$.

19. (a) Verify Stoke's theorem for $\bar{f}=(x^2-y^2)\bar{i}+2xy\bar{j}$ in the rectangular region $x=0$, $y=0$, $x=a$, $y=b$.

Or

(b) Verify Green's theorem for $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ where C is the boundary of the region defined by the lines $x=0$, $y=0$ and $x+y=1$.

20. (a) Find the half range cosine series for the function $f(x)=x^2$ in $0\le x\le\pi$ and hence find the sum of the series $1-\frac{1}{2^2}+\frac{1}{3^2}-\ldots$.

Or

(b) Prove that the function $f(x)=x$ can be expanded in a series of cosines in $0<x<\pi$ as $x=\pi/2-\frac{4}{\pi}\left(\cos x/1^2+\frac{\cos 3x}{3^2}+\ldots\right)$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$.

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