(7 Pages)

Reg. No. : ....

Code No. : 10072 E **Sub. Code : SAMA 11/ AAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2017-2020)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The n^{th} degree equation $f(x)=0$ cannot have more than ———— roots.

(a) 4 (b) 6

(c) 7 (d) n

2. The equation $x^5+4x^4+3x^3+3x^2+4x+1=0$ is ———— equation.

(a) Quadratic (b) Biquadratic

(c) Reciprocal (d) None of these

3. How many imaginary roots will occur for the equation $x^6+3x^2-5x+1=0$?

(a) Atmost four (b) Exactly four

(c) Atleast four (d) None of these

4. Remove the fractional co-efficients from $x^3-\frac{1}{4}x^2+\frac{1}{3}x-1=0$ multiply the roots by

(a) 4 (b) 3

(c) -3 (d) 12

5. The characteristic equation of $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$ is

(a) $\lambda^2-2\lambda-1=0$ (b) $\lambda^2+2\lambda-1=0$

(c) $\lambda^2-2\lambda+1=0$ (d) $\lambda^2+2\lambda+1=0$

6. The eigen values of $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ are ————

(a) $-1, 2$ (b) $1, 2$

(c) $-1, -2$ (d) $1, -2$

7. General solution of $y = xp + ap^{-1}$ is $y =$ ________

(a) $cx + a$

(b) $2xc + a$

(c) $xc + ac^{-1}$

(d) $yx + ac^{-1}$

8. Partial differential equation from $z = ax + by + a^2$

(a) $z = px + py + a^2$ (b) $z = qx + py + a^2$

(c) $z = px + qy + a^2$ (d) None of these

9. $L[x^2] =$ ________

(a) $\frac{1}{S^2}$ (b) $\frac{2}{S^3}$

(c) $\frac{3}{S^4}$ (d) $\frac{4}{S^5}$

10. $L^{-1}\left[\frac{1}{S}\right] =$

(a) 1 (b) x

(c) e^{ax} (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve $x^4 + 2x^2 - 16x + 77 = 0$ given that one root is $-2 + i\sqrt{7}$.

Or

(b) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$ given that the roots are in Arithmetic progression.

12. (a) Transform the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$ whose roots are each diminished by 2.

Or

(b) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

13. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$.

Or

(b) Using Cayley-Hamilton theorem find the inverse of $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

14. (a) Eliminating the arbitrary functions f and g from $z = f(x+ay) + g(x-ay)$ form a partial differential equation.

Or

(b) From the partial differential equation by eliminating the arbitrary constants from $Z = axe^y + a^2e^{2y} + b$.

15. (a) (i) Prove that $L[\cosh ax] = \frac{s}{s^2 - a^2}$.

(ii) Prove that

$$L[f''(x)] = s^2 L[f(x)] - s f(0) - f'(0).$$

Or

(b) Find $L[\sin 2t \sin 3t]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in Arithmetic progression if $2p^3 - 9pq + 27r = 0$.

Or

(b) Solve $4x^4 - 20x^3 - 33x^2 - 20x + 4 = 0$.

17. (a) Find correct to two places of decimals the root of the equation $x^4 - 3x + 1$ that lies between 1 and 2 by Newton method.

Or

(b) Find by Horner's method, the positive root of $x^3 - 3x + 1 = 0$, lies between 1 and 2. Calculate it to three places of decimals.

18. (a) Find the eigen value and eigen vectors of $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

Or

(b) Verify Cayley-Hamilton theorem for $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

19. (a) Solve $y - 2px = f(xp^2)$.

Or

(b) Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$.

20. (a) Find $L^{-1}\left[\frac{s^2 - s + 2}{s(s-3)(s+2)}\right]$.

Or

(b) Find $L^{-1}\left[\log\frac{s^2 + 1}{s(s+1)}\right]$.
