(8 pages) Reg. No. : ....

Code No. : 10427 E Sub. Code : CSMA 41

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics — Skill Based Subject

TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. 1 radian = ——— degree.

   (a) 52.79 (b) 57.29

   (c) 59.27 (d) 59.29

2. If $\tan\theta = \frac{1}{15}$ then $\theta \simeq$ ———

   (a) 3°47' (b) 3°41'

   (c) 3°49' (d) 3°94'

3. The value of $\tanh^{-1} x$ is ———

   (a) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$

   (c) $\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$ (d) $\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$

4. The value of i^i = ———

   (a) $e^{\frac{(4n-1)\pi}{2}}$ (b) $e^{\frac{-(4n+1)\pi}{2}}$

   (c) $e^{\frac{-(4n-1)\pi}{2}}$ (d) $e^{(4n-1)\pi}$

5. $L[f'(x)]$ = ———

   (a) $f(0) - SL[f(x)]$

   (b) $SL[f(x)] - f(0)$

   (c) $f'(0) - SL[f(x)]$

   (d) $S^2L[f(x)] - Sf(0) - f'(0)$

6. $L^{-1}[F(s+a)]$ = ———

   (a) $e^{-ax}L^{-1}[f(s)]$ (b) $e^{ax}L^{-1}[f(s)]$

   (c) $e^{ax}L[f(s)]$ (d) $\frac{1}{a}f\left(\frac{s}{a}\right)$

7. $L[xy''] =$ ———

(a) $\frac{d}{dS}\left(S^2L(y) - Sy(0) - y'(0)\right)$

(b) $\frac{d}{dS}\left(S^2L(y) - Sy'(0) - y(0)\right)$

(c) $-\frac{d}{dS}\left(S^2L(y) - Sy(0) - y'(0)\right)$

(d) $-\frac{d}{dS}\left(S^2L(y) - Sy'(0) - y(0)\right)$

8. $L(xy') =$ ———

(a) $-\frac{d}{dS}[SL(y) - y(0)]$

(b) $\frac{d}{dS}[SL(y) - y(0)]$

(c) $\frac{d}{dS}[SL(y) - y'(0)]$

(d) $-\frac{d}{dS}[SL(y) - y'(0)]$

9. The Fourier co-efficient a_0 for $f(x) = x^2$ in $(-\pi, \pi)$ is ———

(a) 0 (b) $\frac{2\pi^3}{3}$

(c) $\frac{2\pi^2}{3}$ (d) $\frac{\pi^3}{3}$

10. For any integer n, the value of $\cos n\pi$ is ———

(a) 1 (b) 0

(c) -1 (d) $(-1)^n$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that

$$2^5 \cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10.$$

Or

(b) Evaluate $\lim_{x \to \frac{\pi}{2}} \left[\frac{\sin x + \cos 2x}{\cos^2 x}\right]$.

12. (a) If $\cos(x+iy) = r(\cos\alpha + i\sin\alpha)$. Then prove that $y = \frac{1}{2}\log\left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$.

Or

(b) Sum to infinity the series $1 + \frac{c^2\cos 2\theta}{2!} + \frac{c^4\cos 4\theta}{4!} + \cdots + \infty$.

13. (a) Find $L\left[\frac{1-\cos x}{x}\right]$.

Or

(b) Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

14. (a) Using Laplace transform, solve $y'+3y = e^{-2x}$ given $y(0) = 4$.

Or

(b) Solve $(D^2+5D+6)y = e^{-x}$ given that $y(0) = 0$ and $y'(0) = 0$, using Laplace transform.

15. (a) If $f(x) = \begin{cases} -\pi/4 & if \quad -\pi < x < 0 \\ \pi/4 & if \quad 0 < x < \pi \end{cases}$, then find the Fourier series of $f(x)$.

Or

(b) Find the Fourier constant b_1, for the function $x\sin x$ in the half range $0 < x < \pi$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that $\frac{\cos 9\theta}{\cos\theta} = 256\cos^8\theta - 576\cos^6\theta + 432\cos^4\theta - 120\cos^2\theta + 9$.

Or

(b) Prove that

$$\cos^5\theta\sin^4\theta = \left(\frac{1}{2}\right)^8 [\cos 9\theta + \cos 7\theta - 4\cos 5\theta - 4\cos 3\theta + 6\cos\theta].$$

17. (a) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that

(i) $\theta = \frac{1}{2}n\pi + \frac{\pi}{4}$

(ii) $\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$.

Or

(b) Sum the series to infinity

$$x\sin\theta + \frac{x^3}{3}\sin 3\theta + \frac{x^5}{5}\sin 5\theta + \cdots + \infty.$$

18. (a) Find

(i) $L\left[t^2 + \cos 2t \cos t + \sin^2 2t\right]$

(ii) $L\left[xe^{-x}\cos x\right]$.

Or

(b) Find

(i) $L^{-1}\left[\frac{s}{(s+2)^2}\right]$

(ii) $L^{1}\left[\frac{s^2}{(s-1)^2}\right]$.

19. (a) Solve $y''-4y'+4y = x$ given that $y(0) = 0$ and $y'(0) = 1$ using Laplace transform.

Or

(b) Using Laplace transform, solve the equation $xy''-(2x+1)y'+(x+1)y = 0$ given that $y(0) = 0$.

20. (a) Find the Fourier series of $f(x) = |\sin x|$ in $(-\pi, \pi)$ of periodicity 2π.

Or

(b) Prove that the function $f(x) = x$ can be expressed in a series of cosine in $0 \le x \le \pi$ as $x = \frac{\pi}{2} - \frac{\pi}{4}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right]$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{8}$.