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Reg. No. : ........................................

**Code No. : 10426 E** **Sub. Code : CSMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics

Skill Based Subject — VECTOR CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $\phi$ is a scalar function of $u$ and $\vec{a}$ is a constant vector then $\frac{d}{du}(\phi\vec{a}) =$ ________.

   (a) $\vec{a}\frac{d\phi}{du}$ (b) $\phi\frac{d\vec{a}}{du}$

   (c) $\phi$ (d) $\vec{a}$

2. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $|\vec{r}| = r$ then $\nabla r$ is

(a) 0

(b) $r$

(c) $\vec{r}$

(d) $\frac{\vec{r}}{r}$

3. If $\phi$ is such that $\nabla^2 \phi = 0$ then $\phi$ satisfies ———— equation.

(a) Gauss

(b) Laplace

(c) Stokes

(d) None

4. $\nabla \cdot (\nabla \times \vec{A}) =$ ————

(a) 1

(b) $\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(c) $\nabla^2 \vec{A}$

(d) 0

5. If $\vec{r} = x\vec{i} + y\vec{j} + 2\vec{k}$ then $\nabla \times \vec{r} =$ ————

(a) 1

(b) 2

(c) 3

(d) 0

6. If $\vec{f} = 2x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla \circ \vec{f} =$ ————

(a) 0

(b) 2

(c) 1

(d) 4

7. $\vec{f} = x^2\vec{i} - xy\vec{j}$ and $C$ is the line joining the points (0, 0) and (1, 1) then $\int_C \vec{f} \circ d\vec{r} =$ ________

(a) $\frac{7}{10}$ (b) $\frac{10}{7}$

(c) 1 (d) 0

8. The necessary and sufficient condition for $\int_a^b \vec{f} \circ d\vec{r}$ to be independent of the path is

(a) $\vec{f} = \nabla \phi$ (b) $\nabla \circ \vec{f} = 0$

(c) $\vec{f} = \frac{\nabla \phi}{|\nabla \phi|}$ (d) $\nabla \times \vec{f} = 0$

9. $\text{div}\vec{f}$ is ________.

(a) $\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$ (b) $\frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{f}}{\partial y} + \frac{\partial f}{\partial z}$

(c) $\frac{\partial f_1}{\partial x}\vec{i} + \frac{\partial f_2}{\partial y}\vec{j} + \frac{\partial f_3}{\partial z}\vec{k}$ (d) None of these

10. $\iiint_V \nabla \circ \vec{f} dV =$ ________

(a) $\iint_S \vec{f} \circ \vec{n} dS$ (b) $\int_S \vec{f} \circ \vec{n} dS$

(c) $\int_S \vec{f} \circ d\vec{r}$ (d) $\iint_S \nabla \circ \vec{f} dS$

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\vec{A}, \vec{B}, \vec{C}$ are functions of the scalar variable $u$, then derive an expression for $\frac{d}{du}\left[\vec{A}\,\vec{B}\,\vec{C}\right]$.

Or

(b) If $\vec{A}, \vec{B}, \vec{C}$ are functions of the scalar variable $u$, derive an expression for $\frac{d}{du}\left(\vec{A} \times C\vec{B} \times \vec{C}\right)$.

12. (a) Prove that $\nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi \nabla \phi - \phi \nabla \psi}{\psi^2}$.

Or

(b) If $\vec{F} = (x + 2y + az)\,\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational find the values of $a, b, c$.

13. (a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $P(1, -2, -1)$ in the direction of $PQ$ where $Q$ is $(3, -3, -2)$.

Or

(b) If $\vec{u}, \vec{v}$ are vector point functions then prove that $\nabla \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot curl\, \vec{u} - \vec{u} \cdot curl \vec{v}$.

14. (a) Evaluate $\int_C \phi\, ds$ where $C$ is the curve $x = t$, $y = t^2$, $z = (1-t)$ and $\phi = x^2 y(1+z)$ from $t = 0$ to $t = 1$.

Or

(b) If $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 1$, evaluate $\iint_S \left(x\vec{i} + 2y\vec{j} + 3z\vec{k}\right) \cdot d\vec{S}$.

15. (a) Evaluate $\iiint_V \nabla \cdot \vec{F} dV$, where

$\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and $V$ is the volume enclosed by the cube $0 \le x, y, z \le 1$.

Or

(b) Evaluate $\int_C \left(e^x dx + zy\, dy - dz\right)$ by Stoke's theorem where $C$ is the curve $x^2 + y^2 = 4$, $z = 2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove the following :

$$\nabla(\vec{u} \cdot \vec{v}) = (\vec{v} \cdot \nabla)\vec{u} + (\vec{u} \cdot \nabla)\vec{v} + \vec{v} \times curl\, \vec{u} + \vec{u} \times curl\, \vec{v}$$

Or

(b) Prove that $\nabla^2 f(r) = \dfrac{\partial^2 f}{\partial r^2} + \dfrac{z}{r}\dfrac{\partial f}{\partial r}$.

17. (a) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point (1, –2, –1) in the direction of vector $2\vec{i} - \vec{j} - 2\vec{k}$.

Or

(b) (i) If $\nabla\phi = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ then find $\phi$.

(ii) Prove that $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\nabla\times\vec{u}) - \vec{u}\cdot(\nabla\times\vec{v})$

18. (a) Evaluate $\int_C \vec{f}\circ d\vec{r}$ where $\vec{f} = xy\vec{i} + (x^2 + y^2)\vec{j}$ and $C$ is the rectangle in the $xy$-plane bounded by the lines $y = 2$, $x = 4$, $y = 10$, $x = 1$.

Or

(b) Evaluate $\iiint xyz\, dx\, dy\, dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

19. (a) Verify Green's theorem for $\int_C (x - 2y)\, dx + x dy$ where $C$ is the circle $x^2 + y^2 = 1$.

Or

(b) Evaluate $\iint_S \vec{f} \cdot \vec{n} dS$ where $\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and $S$ is the surface of the plane $2x + y + 2z = 6$ in the first octant.

20. (a) Verify Gauss divergence theorem for the vector function $\vec{F} = 2xz\vec{i} + yz\vec{j} + z^2\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Verify Stoke's theorem when $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ where $S$ is the surface of the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

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