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Reg. No. : ....

Code No. : 10421 E **Sub. Code : CMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics – Core

ABSTRACT ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the group $(z_8, \oplus)$, the order of 2 is ____________.

(a) 0 (b) 1

(c) 3 (d) 4

2. In the group $\{1, w, w^2\}$, $w^3 = 1$, under usual multiplication, the identity element is ____________.

(a) 0 (b) 1

(c) 2 (d) 3

3. The generator of the cyclic group $(z_8, \oplus)$ is ____________.

(a) 0 (b) 2

(c) 3 (d) 6

4. If H is a subgroup of G, then $a \in H \Rightarrow aH =$ ____________.

(a) H (b) a

(c) φ (d) G

5. In the quotient group G/N, N is ____________.

(a) a subgroup of G

(b) a cyclic sub group of G

(c) a normal sub group of G

(d) an abelian sub group of G

6. The product of two even permutation is ____________ permulation.

(a) Cycle (b) Odd

(c) Even (d) Odd and even

7. In the ring $(Z, +, \cdot)$, units are ____________.

(a) 0 (b) 1

(c) -1 (d) $1, -1$

8. In the ring $(Q, +, \cdot)$, identity element is ____________.

(a) 0 (b) 1

(c) 2 (d) 3

9. If $f : R \to R'$ be an ring isomorphism then $f(-a) =$ ____________ $\forall a \in R$.

(a) $-f(a)$ (b) $-a$

(c) $f(a)$ (d) a

10. If $f : R \to R'$ is $1-1$, then $Kref =$ ____________.

(a) R (b) R'

(c) $0'$ (d) $\{0\}$

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) In R^*, $a * b = \frac{ab}{2}$ then prove that $(R^*, *)$ is a group.

Or

(b) Let G be a group and $a \in G$. Let $H_a = \{x \mid x \in G \text{ and } ax = xa\}$. Prove that H_a is a subgroup of G.

12. (a) Prove that a subgroup of cyclic group is cyclic.

Or

(b) State and prove Euler's theorem.

13. (a) Show that if a group G has exactly one subgroup H of given order, then prove that H is a normal subgroup of G.

Or

(b) Let $f : G \to G'$ be an isomorphism. Let $a \in G$. Then prove that order of a is equal to the order of $f(a)$.

14. (a) Show that every field is an integral domain.

Or

(b) Prove that any finite integral domain is a field.

15. (a) Let R and R' be rings and $f : R \to R'$ be a homomorphism. Then S is an ideal of R, prove that $f(S)$ is an ideal of R'.

Or

(b) Let $f : R \to R'$ be a homomorphism. Prove that Kerf is an ideal of R'.

[P.T.O.]

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Let A and B be two subgroups of a group G. Then prove that AB is a subgroup of G iff $AB = BA$.

Or

(b) If H, K are two finite subgroups of a group G, then show that $|HK| = \frac{|H||K|}{|H \cap K|}$.

17. (a) Let H be a subgroup of a group G. Then prove that

(i) $aH = bH \Rightarrow a^{-1}b \in H$

(ii) $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$

(iii) $a \in bH \Rightarrow aH = bH$.

Or

(b) State and prove Lagrange's theorem.

18. (a) State and prove Cayley's theorem.

Or

(b) If a permutation $p \in s_n$ is a product of r transposition and also a product of s transpositions, then prove that either r and s are both even or both odd.

19. (a) Let R be a commutative ring with identity. Prove that a_n ideal M of R is maximal iff R/M is a field.

Or

(b) Prove that Z_n is an integral domain $\Leftrightarrow$ n is prime.

20. (a) State and prove fundamental theorem of homomorphism.

Or

(b) Prove that the only isomorphism $f : Q \rightarrow Q$ is the identity map.
