(6 pages) Reg. No. : ....

Code No. : 10420 E **Sub. Code : CMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Third Semester

Mathematics — Core

SEQUENCES AND SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The g.l.b. of the sequence $1, \frac{1}{2}, \frac{1}{3}, \cdots \frac{1}{n}, \cdots$ is ———

   (a) 1

   (b) 0

   (c) ∞

   (d) $\frac{1}{2}$

2. $\lim\limits_{n \to \infty} \frac{1}{n^2} =$ ———

   (a) 0 (b) ∞

   (c) 1 (d) $\frac{1}{n}$

3. $\lim\limits_{n \to \infty} \left(n^{1/n}\right) =$ ———

   (a) 0 (b) ∞

   (c) 1 (d) $\frac{1}{n}$

4. $\lim\limits_{n \to \infty} \frac{3n-4}{2n+7} =$ ———

   (a) $\frac{3}{2}$ (b) $\frac{2}{3}$

   (c) $\frac{4}{7}$ (d) $\frac{-4}{7}$

5. The series $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$

   (a) converges to zero

   (b) diverges to ∞

   (c) converges to one

   (d) diverges to $-\infty$

6. $\sum \frac{1}{4n^2 - 1} =$ ———

   (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

   (c) -1 (d) 0

7. The series $\sum \frac{1}{(\log n)^n}$ is ———

   (a) divergent (b) diverges to ∞

   (c) convergent (d) none

8. The series $\Sigma \frac{x^n}{n}$ converges if ———

   (a) $x = 1$ (b) $x > 1$

   (c) $x = 0$ (d) $x < 1$

9. $\sum (-1)^n \left(1 + \frac{1}{n}\right)$ ———

   (a) is oscillating (b) diverges

   (c) converges (d) none

10. The series $\sum (-1)^n \sin\left(\frac{1}{n}\right)$ ———

    (a) diverges (b) converges

    (c) is constant (d) converges to $\frac{1}{n}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Cauchy - Schwarz inequality.

    Or

    (b) Define bounded sequence. Give an example.

12. (a) Show that $\left((-1)^n\right)$ is not a convergent sequence.

    Or

    (b) Show that if $(a_n) \to a$ and $K \in \mathbb{R}$ then $(Ka_n) \to Ka$.

13. (a) Show that $\lim\limits_{n \to \infty} \frac{n!}{n^n} = 0$.

    Or

    (b) Prove that any Cauchy sequence is a bounded sequence.

14. (a) Test the convergence of the series $\frac{1}{3}x + \frac{1}{3}\cdot\frac{2}{5}x^2 + \frac{1}{3}\cdot\frac{2}{5}\cdot\frac{3}{7}x^3 + \cdots$

    Or

    (b) Test the convergence of $\sum \frac{n^3 + a}{2^n + a}$.

[P.T.O.]

15. (a) Show that the series $\sum \frac{(-1)^{n+1}}{\log(n+1)}$ converges.

Or

(a) Prove that $\sum_{n=2}^{\infty}\left(\frac{\sin n}{\log n}\right)$ is convergent.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) For any two real numbers x and y, prove

(i) $|x+y| \leq |x| + |y|$

(ii) $|x-y| \geq \big||x| - |y|\big|$.

Or

(b) (i) If a,b,c are positive real numbers such that $a^2+b^2+c^2=27$ then show that $a^3+b^3+c^3 \geq 81$.

(ii) Define monotonic sequences. Give an for each example.

17. (a) (i) Prove that a sequence cannot converge to two different limits.

(ii) Prove if $(a_n) \to a$ and $a_n \geq 0$ for all n then $a \geq 0$.

Or

(b) Discuss the behaviour of the geometric sequence (r^n).

18. (a) State and prove Cauchy's first limit theorem.

Or

(b) State and prove comparison test.

19. (a) State and prove Kummer's test.

Or

(b) State and prove Gauss's test.

20. (a) State and prove Dirichlet's test.

Or

(b) State and prove Abel's test.
