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Reg. No. : .....

Code No. : 10419 E **Sub. Code : CMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics — Core

DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The general solution of the differential equation $y - p(x+1) = p$ is

(a) $y = p(x+2)$ (b) $y = cx$

(c) $y = cx + 2c$ (d) $y = c(x+1)$

2. $\frac{1}{D^2 + a^2}\cos ax =$ _____.

(a) $\frac{x}{2a}\sin ax$ (b) $\frac{-x}{2a}\sin ax$

(c) $\frac{x}{2}\sin ax$ (d) $\frac{x}{a}\sin ax$

3. The differential equation with constant coefficients obtained from $x^2\frac{d^2y}{dx^2} + y = 3x^2$ by substituting $x = e^z$, $D = \frac{d}{dz}$ is

(a) $(D^2 - D + 1)y = 3x^2$ (b) $(D^2 + D - 1)y = 3z^2$

(c) $(D^2 - D + 1)y = 3z^2$ (d) $(D^2 - D + 1)y = 3e^{2z}$

4. The complementary function of $(x^2D^2 + xD + 1)y = \log x$ is

(a) $A + Bx$

(b) $A\cos(\log x) + B\sin(\log x)$

(c) $A + B$

(d) $(A + Bx)e^x$

5. The middle point of the line joining the points $(1, 2, 8)$ and $(1, 1, 3)$ is _____.

(a) $(1, 3, 11)$ (b) $\left(1, \frac{3}{2}, \frac{11}{2}\right)$

(c) $\left(1, \frac{2}{3}, \frac{11}{2}\right)$ (d) $\left(1, \frac{2}{3}, \frac{2}{11}\right)$

6. The angle between the planes $2x+4y-6z=1$ and $3x+6y-5z+4=0$ is _____.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) None of the above

7. A straight line is equally inclined to the three coordinate axes. Then that angle = _____.

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\cos^{-1}\left(\frac{1}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

8. On which plane does the line $\frac{x-4}{2}=\frac{y-2}{3}=\frac{z-3}{6}$ lie?

(a) $4x+3y+20z=5$ (b) $4x+2y+3z=2$

(c) $3x-4y+z=7$ (d) $2x-2y+z=1$

9. The radius of the sphere $2x^2+2y^2+2z^2-2x+2y-4z-5=0$ is _____.

(a) 2 (b) $\sqrt{\pi}$

(c) 1 (d) $\frac{\sqrt{\pi}}{2}$

10. The equation of the tangent plane of the sphere $x^2+y^2+z^2=9$ at (1, –2, 2) is

(a) $x-2y+2z+9=0$ (b) $x-2y+2z-9=0$

(c) $x+2y+2z+9=0$ (d) $x-2y-2z-9=0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $y=xp+x\left(1+p^2\right)^{\frac{1}{2}}$.

Or

(b) Sovle : $tdx=(t-2x)dt$

$tdy=(tx+ty+2x-t)dt$.

12. (a) Solve : $\left(D^3-3D^2+3D-1\right)y=x^2e^x$.

Or

(b) Solve : $x^2y''+3xy'+y=\frac{1}{(1-x)^2}$.

13. (a) Show that the points (2, 5, –4), (1, 4, –3), (4, 7, –6) and (5, 8, –7) are the vertices of a parallelogram.

Or

(b) Prove that the lines $\frac{x-3}{2}=\frac{y-2}{-5}=\frac{z-1}{3}$ and $\frac{x-1}{-4}=y+2=\frac{z-6}{2}$ are coplanar.

14. (a) Find the distance between the parallel planes $2x - 3y + 6z + 12 = 0$, $2x - 3y + 6z - 2 = 0$

Or

(b) Find the equation of the image of the line $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$.

15. (a) Find the equation of the sphere which has its center at the point (6, −1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

Or

(b) Show that the plane $2x + y - 2z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0$. Find the point of contact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve : $\frac{dx}{dt} + 2x - 3y = t$

$\frac{dy}{dt} - 3x + 2y = e^{2t}$.

Or

(b) Solve : $(px - y)(x + yp) = a^2 p$ (Take $x^2 = X$, $y^2 = Y$).

17. (a) Solve : $(D^2 - 2D + 4)y = e^x \cos x$.

Or

(b) Solve the differential equation $\frac{d^2y}{dx^2} + n^2 y = \cos nx$.

18. (a) Show that the lines whose direction cosines are related as $3l + 4m + 5n = 0$, $l^2 + m^2 - n^2 = 0$ are parallel.

Or

(b) A moving plane passes through a fixed point (α, β, γ) and intersects the coordinate axes at A, B, C. Show that the locus of centroid of the triangle ABC is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.

19. (a) Find the coordinates of the foot of the perpendicular drawn from the point (2, 3, 1) to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

Or

(b) Show that the lines $\frac{x-2}{1} = \frac{y-4}{+2} = \frac{z-5}{2}$ and $\frac{x-5}{+2} = y - 8 = \frac{z-7}{2}$ are coplanar. Find the point of intersection. Also, find the equation of the plane determined by the lines.

20. (a) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the center of the sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Or

(b) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9\left(x^2+y^2+z^2\right)=4k^2$.
