(7 pages) **Reg. No. :** ....

Code No. : 10418 E **Sub. Code : CMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

First Semester

Mathematics — Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The locus of the limiting position of the intersecting points of any two curves of a family of curves is called ———— of the family.

   (a) involute (b) evolute

   (c) curvature (d) envelope

2. The curvature of the circle is ———— of its radius.

   (a) half (b) square

   (c) reciprocal (d) square root

3. $\Gamma(n+1)$ = ————

   (a) $(n-1)\Gamma(n)$ (b) $(n+1)!$

   (c) $(n+1)\Gamma(n)$ (d) $n!$

4. $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ = ————

   (a) π (b) $\sqrt{\pi}$

   (c) $\frac{\pi}{2}$ (d) $\frac{\sqrt{\pi}}{2}$

5. $\int_0^4\int_0^4 dx\,dy$ = ————

   (a) 4 (b) 16

   (c) 12 (d) 8

6. $\int_0^a\int_0^a\int_0^a a^2\, dx\, dy\, dz =$ ————

(a) a^3 (b) a^4

(c) a^5 (d) a^2

7. If $f(a)$ and $f(b)$ have like signs, ———— roots of $f(x)=0$ lie between a and b.

(a) even number of

(b) no

(c) odd number of

(d) either (a) or (b)

8. The sum of roots of the equation $ax^4+4bx^3+6cx^2+4dx+e=0$ is ————

(a) $-4b$ (b) $\frac{4b}{a}$

(c) $\frac{-4b}{a}$ (d) $4b$

9. The coefficients of an odd degree reciprocal equation have all like signs. Then ———— is its root.

(a) -1 (b) 1

(c) ± 1 (d) 0

10. The number of positive roots of the equation $x^5-6x^2-4x+5=0$ is ————

(a) atleast two (b) atmost one

(c) atleast one (d) atmost two

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the radius of curvature at any point of the catenary $y = c\cosh\frac{x}{c}$ is equal to the length of the portion of the normal intercepted between the curve and the axis of x.

Or

(b) Develop an equation of a curve which forms the envelope of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{k^2-a^2}=1$ where 'a' is the parameter.

12. (a) Change the order of integration in $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xy\, dx\, dy$ and evaluate it.

Or

(b) Using Jacobians, evaluate $\iint_R (x-y)^4 e^{x+y}\, dx\, dy$ where R is the square with vertices (1, 0), (2, 1), (1, 2) and (0, 1).

13. (a) Evaluate :

(i) $\int_0^\infty e^{-x^2} dx$

(ii) $\int_0^{\frac{\pi}{2}} \sin^{10}\theta \, d\theta$.

Or

(b) Evaluate the integral $\iint x^p y^q \, dy \, dx$ over the triangle $x \geq 0$, $y \geq 0$, $x + y \leq 1$ in terms of Gamma functions.

14. (a) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.

Or

(b) If $a + b + c + d = 0$, show that
$$\frac{a^5 + b^5 + c^5 + d^5}{5} = \frac{a^2 + b^2 + c^2 + d^2}{2} \cdot \frac{a^3 + b^3 + c^3 + d^3}{3}.$$

15. (a) Increase by 7, the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

Or

(b) Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has atleast four imaginary roots.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Or

(b) (i) Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point (2, 1).

(ii) Prove that the radius of curvature at any point of cycloid $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$.

17. (a) Evaluate $\iiint xyz \, dx \, dy \, dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Use the substitution $x + y + z = u$, $y + z = uv$, $z = uvw$ to evaluate the integral $\iiint [xyz(1 - x - y - z)]^{\frac{1}{2}} \, dx \, dy \, dz$ taken over the tetrahedral volume enclosed by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

18. (a) Establish relation between Beta and Gamma functions.

Or

(b) Evaluate in terms of Gamma functions, the integral $\iiint x^{p} y^{q} z^{r}\, dx\, dy\, dz$ taken over the volume of the tetrahedron given by $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x + y + z \leq 1$.

19. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in Geometric progression and hence solve $27x^3 + 42x^2 - 28x - 8 = 0$, whose roots are in geometric progression.

Or

(b) Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

20. (a) Solve the equation :

$$6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0.$$

Or

(b) Solve the equation :
$x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing its second term.
