(7 pages) **Reg. No. :** ....

Code No. : 10424 E **Sub. Code : CAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second/Fourth Semester

Mathematic — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$, then $\nabla \cdot \overline{r} =$ __________.

   (a) 1 (b) 0

   (c) 3 (d) $x^2 + y^2 + z^2$

2. A vector function $\overline{f}$ is called solenoidal if __________.

   (a) $div\overline{f} = 0$ (b) $grad\overline{f} = 0$

   (c) $div\overline{f} = \overline{0}$ (d) $curl\overline{f} = 0$

3. $\int_0^1\int_0^1 dydx =$ __________.

   (a) 2 (b) 1

   (c) 0.5 (d) None of the above

4. $\int_0^a\int_0^b\int_0^c dxdydz =$ __________.

   (a) $a+b+c$ (b) $a^3+b^3+c^3$

   (c) abc (d) $(abc)^3$

5. If C is the straight line joining (0, 0, 0) and (1, 1, 1) then $\int_c \overline{r}\cdot d\overline{r}$ is __________.

   (a) $\frac{1}{2}$ (b) 1

   (c) $\frac{3}{2}$ (d) 2

6. Value of $\int(xdy - ydx)$ around the circle $x^2 + y^2 = 1$ is __________.

   (a) 0 (b) $\frac{\pi}{2}$

   (c) π (d) 2π

7. Value of $\iint_S \hat{n} \times \overline{F} ds$ is ________.

(a) $\iiint_V div \overline{F} dV$ (b) $\iiint_V Curl \overline{F} dV$

(c) $\iiint_V \overline{F} dV$ (d) zero

8. If S is the sphere $x^2 + y^2 + z^2 = 1$ the value of $\iint_S \overline{r} \cdot \hat{n} ds$ is ________.

(a) $\frac{4}{3}\pi$ (b) 3π

(c) 4π (d) 2π

9. If $f(x)$ is an odd function then $\int_{-a}^{a} f(x) dx$ = ________.

(a) $2\int_0^a f(x) dx$ (b) 0

(c) $\frac{2}{\pi}\int_0^a f(x) dx$ (d) None of these

10. What is the period of the periodic function $\sin nx$?

(a) π (b) 2π

(c) $2n\pi$ (d) $\frac{2\pi}{n}$

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\nabla\phi = yz\overline{i} + zx\overline{j} + xy\overline{k}$ find $\phi(x, y, z)$.

Or

(b) Prove that $\overline{f} = (x^2 - yz)\overline{i} + (y - zx)\overline{j} + (z^2 - xy)\overline{k}$ is irrotational.

12. (a) Evaluate the following integral $\int_0^1\int_0^2 (x+2) dy dx$.

Or

(b) Evaluate the integral $\int_0^1 dx \int_0^2 dy \int_1^2 x^2 yz dz$.

13. (a) If $\overline{f} = (2y+3)\overline{i} + xz\overline{j} + (yz - x)\overline{k}$ evaluate $\int_C \overline{f} \cdot d\overline{r}$ along the path C the straight line joining (0, 0, 0) and (2, 1, 1).

Or

(b) Evaluate $\iint_S \overline{F} \cdot \hat{n} ds$ where $\overline{F} = (x + 2y^2)\overline{i} - 2x\overline{j} + 2yz\overline{k}$ where S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

14. (a) Calculate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the square by lines $x = 0$, $x = 1$, $y = 0$, $y = 1$.

Or

(b) Using Stoke's theorem calculate $\int_C (yzdx + zxdy + xydz)$ where C is the any closed curve.

15. (a) Find the Fourier series for the function $f(x) = x^2$ where $-\pi \le x \le \pi$.

Or

(b) Find the Fourier sine series for the function $f(x) = \pi - x$ in the interval $(0, \pi)$.

SECTION C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Find the unit normal at (6, 4, 3) to $xy + yz + zx = 54$.

Or

(b) Prove that $\overline{F} = 3y^4z^2\overline{i} + 4x^3z^2\overline{j} - 3x^2y^2\overline{k}$ is solenoidal.

17. (a) Estimate $\int_1^2 \int_2^3 \frac{dxdy}{xy}$.

Or

(b) Estimate $I = \int_0^{\log a} \int_0^{x} \int_0^{x+y} e^{x+y+z} dzdydx$.

18. (a) If $\overline{f} = (3x^2 + 6y)\overline{i} - 14yz\overline{j} + 20xz^2\overline{k}$ determine $\int_C \overline{f} \cdot d\overline{r}$ from (0, 0, 0) to (1, 1, 1) along the curve $x = t$, $y = t^2$, $z = t^3$.

Or

(b) Determine $\iint \overline{f} \cdot \hat{n} ds$ where $\overline{f} = (x^3 - yz)\overline{i} - 2x^2y\overline{j} + 2\overline{k}$ and S is the surface of the cube bounded by $x = 0$, $y = 0$, $z = 0$, $x = a$, $y = a$ and $z = a$.

19. (a) Verify Gauss divergence theorem for $\overline{f} = y\overline{i} + x\overline{j} + z^2\overline{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$, $z = 0$ and $z = k$.

Or

(b) Verify Stoke's theorem for $\overline{f} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\overline{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

20. (a) If $f(x) = -x$ in $-\pi < x < 0$

x in $0 \leq x < \pi$

express $f(x)$ as Fourier Series in the interval $-\pi$ to π. Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + ...$

Or

(b) Formulate a cosine-series in the range 0 to π for

$$f(x) = x \quad \left(0 < x < \frac{\pi}{2}\right)$$

$$= \pi - x \quad \left(\frac{\pi}{2} < x < \pi\right)$$