(8 pages) **Reg. No. :** ....

Code No. : 10422 E **Sub. Code : CAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

First/Third Semester

Mathematics – Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. The least degree of an equation with real coefficients, two of whose roots are $1+i$ and $\sqrt{3}$ is
   (a) 2 (b) 3
   (c) 4 (d) 6

2. If 'a' is a root of $x^4 - 2x^3 + 6x^2 + 2x - 1 = 0$, then ________
   (a) $-a$ is also a root (b) $2a$ is also a root
   (c) $1/a$ is also a root (d) a^2 is also a root

3. One root of $x^3 - x - 3 = 0$ lies between ________
   (a) 0 and 1 (b) 1 and 2
   (c) 0 and –1 (d) –1 and –2

4. When the roots of the equation $3x^3 - 10x^2 + 9x + 2 = 0$ are multiplied by 3, the transformed equation is ________
   (a) $3x^3 - 100x^2 + 900x + 2000 = 0$
   (b) $27x^3 - 90x^2 + 27x + 2 = 0$
   (c) $3x^3 - 30x^2 + 81x + 54 = 0$
   (d) $x^3 - \frac{10}{3}x^2 + 3x + 2/3 = 0$

5. The characteristics equation of $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ is ________
   (a) $x^2 - 2x - 5 = 0$ (b) $x^2 + 2x + 5 - 0$
   (c) $-x^2 - 2x + 5 = 0$ (d) $-x^2 - 2x - 5 = 0$

6. The eigen values of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ are ________
   (a) $-1, 1$ (b) $-1, -1$
   (c) $1, -1$ (d) $1, 1$

7. The complementary function of $(D^3 - 3D^2 + 3D - 1)y = x^3$ is ______

(a) $e^x(a + bx + cx^2)$

(b) $e^{-x}(a\cos x + b\sin x + c)$

(c) $ae^x + be^{-x} + ce^{2x}$

(d) $e^{-x}(a + bx + cx^2)$

8. The partial differential equation by eliminating the arbitrary constants a and b form $z = axy + b$ is ______

(a) $px + qy = 0$ (b) $py + qx = 0$

(c) $px - qy = 0$ (d) $py - qx = 0$

9. $L(\sqrt{x}) =$ ______

(a) $\frac{\sqrt{\pi}}{2s^{3/2}}$ (b) $\frac{1}{s^2}$

(c) $\frac{1}{\sqrt{s}}$ (d) $\frac{\pi}{2s^{3/2}}$

10. $L^{-1}[F(s+a)] =$ ______

(a) $e^{ax}L^{-1}[F(s)]$ (b) $e^{-ax}L^{-1}[f(s)]$

(c) $e^{ax}L[f(s)]$ (d) $1/a\,F\left(\frac{s}{a}\right)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression.

Or

(b) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

12. (a) Increase the roots of equation $4x^5 - 2x^3 + 7x - 3 = 0$ by 2.

Or

(b) Apply Newton's method to obtain the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

13. (a) Find the eigen values of the matrices $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$.

Or

(b) Verify Cayley Hamilton's theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

14. (a) Solve

(i) $(D^2+D+1)^2 y = 0$

(ii) $(D^2+D+1)y = \sin 2x$

Or

(b) Focus on the method of eliminating constants a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and form a partial differential equation.

15. (a) Apply Laplace transform to the functions

(i) $t^2 + \cos 2t \cos t + \sin^2 2t$

(ii) e^{at}

Or

(b) Find the inverse Laplace transform of

(i) $\frac{1}{(s+3)^2+25}$ and

(ii) $\frac{s}{(s+2)^2}$

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ find

(i) $\sum\left(\frac{1}{\alpha}\right)$

(ii) $\sum\left(\frac{\alpha}{\beta}\right)$

(iii) $\sum\left(\frac{1}{\alpha\beta}\right)$

(iv) $\sum \alpha^2\beta$

(v) $\sum \alpha^3$

Or

(b) Find the roots of $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$

17. (a) Apply Horner's method to find the positive root of $x^3 - x - 3 = 0$ correct to two places of decimals.

Or

(b) Show that $x^3 + 3x - 1 = 0$ has only one real root and calculate it correct to two places of decimals.

18. (a) Using Cayley Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$.

Or

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

19. (a) Solve :

(i) $(D^3 - 3D^2 + 3D - 1)y = x^2 e^x$

(ii) $(D^2 + 5D + 6)y = x^2$

Or

(b) Solve: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$.

20. (a) Applying Laplace transformation, find

(i) $L(x^2 \cosh ax)$

(ii) $L\left(\frac{1 - \cos x}{x}\right)$

Or

(b) Applying inverse Laplace transformation, find

(i) $L^{-1}\left(\frac{1 + 2s}{(s+2)^2 (s-1)^2}\right)$

(ii) $L^{-1}\left(\frac{s^2 - s + 2}{s(s-3)(s+2)}\right)$
