(6 pages)

Reg. No. : ....

Code No. : 10295 E   Sub. Code : AMMA 62

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2020 only)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The Independence number of K_p is _____

    (a) 2   (b) p

    (c) $p-1$ (d) 1

2. $K_{m,n} =$ _____

    (a) $K_m + K_n$ (b) $K_m \cup K_n$

    (c) $\overline{K}_m + \overline{K}_n$ (d) $K_m \cap K_n$

3. Petersen graph is _____ graph

    (a) Eulerian

    (b) Hamiltonian

    (c) Eulerian, Non-Hamiltonian

    (d) Non Eulerian, Non-Hamiltonian

4. If e is a bridge of a graph G, then $w(G-e) =$ _____

    (a) $w(G)$ (b) $w(G)+1$

    (c) $w(G)-1$ (d) $w(G)-2$

5. (A) Every edge of a tree is a bridge

    (B) A block has no cut vertex

    (a) (A), (B) are correct

    (b) (A) is correct, (B) is wrong

    (c) (A) is wrong, (B) is correct

    (d) (A), (B) are wrong

6. The chromatic number of a tree is _____

    (a) 1   (b) 2

    (c) 4   (d) 0

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7. The number of faces in K_4 is _____

    (a) 4   (b) 2

    (c) 6   (d) 5

8. The thickness of K_9, $\theta(K_9) =$ _____

    (a) 1   (b) 2

    (c) 3   (d) 4

9. A digraph on p vertices is functional if the out degree of every vertex is _____

    (a) 1   (b) 2

    (c) p (d) $p-1$

10. (A) Strongly connected $\Rightarrow$ weakly connected

    (B) Weakly connected $\Rightarrow$ unilaterally connected

    (a) (A), (B) are correct

    (b) (A) is correct, (B) is wrong

    (c) (A) is wrong, (B) is correct

    (d) (A), (B) are wrong

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PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Show that sum of degrees of vertices of a graph is twice the number of edges.

    Or

    (b) If G is a (p, q) graph, show that $\delta \leq \dfrac{2q}{p} \leq \Delta$.

12. (a) If $\delta \geq K$ show that the graph G has a path of length K.

    Or

    (b) Show that a closed walk of odd length contains a cycle.

13. (a) Prove that every connected graph has a spanning tree.

    Or

    (b) Prove that G is a tree iff G is connected and every line of G is a bridge.

14. (a) Prove that K_5 is non-planar.

    Or

    (b) Prove that every planar graph G with $p \geq 3$ vertices has at least 3 points of degree less than 6.

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[P.T.O.]

15. (a) In a digraph D, show that the sum of indegrees of all the vertices equals sum of their out degrees, each being equal to the number of arcs.

Or

(b) If two diagraphs are isomorphic, prove that corresponding points have the same degree pair.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\dfrac{p^2}{4}\right]$.

Or

(b) (i) Prove that, every graph' is an intersection graph.

(ii) A (p, q) graph has t vertices of degree m and all other vertices are of degree n. Show that $(m-n)t + pn = 2q$.

17. (a) Prove that an edge x of a connected graph G is a bridge iff x is not on any cycle of G.

Or

(b) State and prove the necessary and sufficient condition for a partition of an even number to be graphical.

18. (a) Prove that a (p, q) graph G is a tree iff G is acyclic and $p = q+1$.

Or

(b) If G is a plane (p, q) graph in which every face is an n – cycle, prove that $q = \dfrac{n(p-2)}{n-2}$.

19. (a) State and prove Dirac's theorem.

Or

(b) Prove that every uniquely n – colourable graph is $(n-1)$ connected.

20. (a) Prove that a weak digraph D is Eulerian iff every vertex of D has equal indegree and out degree.

Or

(b) Find the chromatic polynomial of the graph with partition $(3, 3, 3, 3, 2)$.

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