(6 pages) **Reg. No. :** ....

Code No. : 10293 E **Sub. Code : AMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. If M is a discrete metric space then $B(a, 4)$ is

(a) $\{a\}$ (b) M

(c) ϕ (d) 4

2. Z is

(a) not open in R (b) open in R

(c) not closed (d) an interval

3. Which is not correct?

(a) $A \subset \overline{A}$ (b) $Int\,A \subset A$

(c) $Int\,A \subset \overline{A}$ (d) $\overline{A} \subset \text{int}\,A$

4. $D(Q) =$ ________.

(a) Q (b) ϕ

(c) R (d) Z

5. If $f : R \to R$ is continuous at a then $w(f, a)$ is

(a) 0 (b) a

(c) 1 (d) ∞

6. If $f : R \to R$ which is uniformly continuous?

(a) $f(x) = x^2$ (b) $f(x) = \sin x$

(c) $f(x) = \frac{1}{x}$ (d) none

7. Choose the correct statement

(a) R is connected

(b) Q is connected

(c) A subspace of a connected space is connected

(d) Union of two connected subsets of a metric space is connected

8. $\{(-n, n)/n \in N\}$ is an open cover for ________.

(a) R (b) N

(c) Z (d) Q

9. In $\mathbb{R}$, (5, 6) is a

(a) Closed Set (b) Compact

(c) Not a compact set (d) None

10. Any continuous function $f : [a, b] \to \mathbb{R}$ is

(a) onto (b) 1-1

(c) open (d) not onto

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions by choosing (a) or (b).

11. (a) Define discrete metric d. Prove that d is a metric on M.

Or

(b) Let (M, d) be a metric space. Let $A, B \subseteq M$. Then prove the following :

(i) $A \subseteq B \Rightarrow Int\,A \subseteq Int\,B$

(ii) $Int(A \cap B) = Int\,A \cap Int\,B$.

12. (a) Let (M, d) be a metric space. Prove that any convergent sequence in M is a Cauchy Sequence.

Or

(b) Prove that any discrete metric space is complete.

13. (a) Define Uniformly Continuous Function. Prove that the function $f : (0, 1) \to \mathbb{R}$ define by $f(x) = \frac{1}{x}$ is not uniformly continuous.

Or

(b) Prove : $f : \mathbb{R} \to \mathbb{R}$ is continuous at $a \in R$ $\Leftrightarrow w(f, a) = 0$.

14. (a) Prove that any continuous image of a connected set is connected.

Or

(b) Define a connected set. Prove that any discrete metric space M with more than one point is disconnected.

15. (a) Prove that any compact subset A of a metric space M is bounded.

Or

(b) Define totally bounded metric space. Prove that any compact metric space is totally bounded.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) For $p \geq 1$, let l_p denote the set of all sequences (x_n) such that $\sum_{n=1}^{\infty} |x_n|^p$ is convergent. Define $d(x, y) = \left\{\sum_{n=1}^{\infty} (x_n - y_n)^p\right\}^{\frac{1}{p}}$ where $x = (x_n)$ and $y = (y_n)$. Prove that d is a metric on l_p.

Or

(b) Let (M, d) be a metric space. Let x, y be two distinct points of M. Prove that there exists two disjoint open balls with centres x and y respectively.

17. (a) Prove that $\mathbb{R}^n$ with usual metric is complete.

Or

(b) State and prove Cantor's intersection theorem.

18. (a) If (M_1, d_1) and (M_2, d_2) are two metric spaces and $a \in M_1$ then prove that $f : M_1 \to M_2$ is continuous at a $\Leftrightarrow (x_n) \to a \Rightarrow (f(x_n)) \to f(a)$.

Or

(b) Let $f : [a, b] \to \mathbb{R}$ be a monotonic function. Prove that set of points of $[a, b]$ at which f is discontinuous is countable.

19. (a) Let A be a connected subset of M, $A \subseteq B \subseteq \overline{A}$. Show that $\overline{A}$ is connected.

Or

(b) Prove : A subspace of $\mathbb{R}$ is connected $\Leftrightarrow$ is is an interval.

20. (a) State and prove Heine Borel Theorem.

Or

(b) Prove : A metric space (M, d) is totally bounded $\Leftrightarrow$ every sequence in M has a Cauchy Subsequence.