Reg. No. : ....

Code No. : 10300 E **Sub. Code : AEMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Sixth Semester

Mathematics

Major Elective — FUZZY MATHEMATICS

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The law of absorption is ————

(a) $A \cup (B \cap C) = B$

(b) $A \cup B = B \cup A$

(c) $A \cup (A \cap B) = A$

(d) ϕ

2. The truth values of fuzzy sets are ————

(a) between 0 or 1 both exclusive

(b) between 0 and 1 both inclusive

(c) either 0 or 1

(d) 0.5

3. Second decomposition theorem states ————

(a) $A = \bigcup_{\alpha \in [0,1]} {}^{\alpha}A$ (b) $A = \bigcup_{\alpha \in [0,1]} {}^{\alpha+}A$

(c) $A = \bigcup_{\alpha \in \wedge(A)} {}^{\alpha}A$ (d) $A = \bigcup_{\alpha \in [0,1]} A$

4. Let A_{α_0} denotes the α – cut of a fuzzy set A at α_0. If $\alpha_1 < \alpha_2$ then

(a) $A_{\alpha_1} \supseteq A_{\alpha_2}$ (b) $A_{\alpha_1} \supset A_{\alpha_2}$

(c) $A_{\alpha_1} \subseteq A_{\alpha_2}$ (d) $A_{\alpha_1} \subset A_{\alpha_2}$

5. $u(a,b) = \min(1, a+b)$ is known as

(a) standard union

(b) algebraic sum

(c) bounded sum

(d) drastic sum

6. $i_w(a,b) =$ ————, $w > 0$.

(a) $1 - \min(1, ((1-a)^w + (1-b)^{1/w}))$

(b) $1 + \min(1, ((1-a)^w + (1-b)^{1/w}))$

(c) $1 - \min(1, ((1-a)^w - (1-b)^{1/w}))$

(d) $1 + \min(1, ((1-a)^w - (1-b)^{1/w}))$

7. A fuzzy number of a fuzzy set A on $\mathbb{R}$ must be

(a) subnormal (b) not convex

(c) convex (d) normal

8. Let A and B be two closed interval such that $A = [a_1, a_2]$ and $B = [b_1, b_2]$ then $B - A$ is

(a) $[b_1 - a_1, b_2 - a_2]$

(b) $[b_1 - a_2, b_2 - a_1]$

(c) $[b_2 - a_1, b_1 - a_2]$

(d) $[b_2 - a_2, b_1 - a_1]$

9. In a linear programming problem, the matrix $A = [a_{ij}], i \in N_m, j \in N_n$ is

(a) goal matrix

(b) consistent matrix

(c) cost matrix

(d) decision matrix

10. $F(x_i, x_j) =$ ————

(a) $\max[f(x_i,x_j)/f(x_j,x_i)]$

(b) $\min[1, f(x_i,x_j)/f(x_j,x_i)]$

(c) $\min[f(x_i,x_j)f(x_j,x_i)]$

(d) $\max[f(x_i,x_j), f(x_j,x_i)]$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) State fundamental properties of crisp set. Among them which are valid in fuzzy set.

Or

(b) Prove that $|A|+|B|=|A\cap B|+|A\cup B|$ is Fuzzy set.

2. (a) State and prove first decomposition theorem.

Or

(b) Let $f: X \to Y$ be an arbitrary crisp function. Then prove that for any $A \in F(X)$, f fuzzified by the extension principle satisfies the equation $f(A) = \bigcup_{\alpha\in[0,1]} f(\alpha + A)$.

13. (a) Let $<i,u,c>$ be a dual triple that satisfies the law of excluded middle and law of contradiction. Then, prove that $<i,u,c>$ does not satisfy the distributive laws.

Or

(b) Assume that a given fuzzy complement c has equilibrium e_c, and fuzzy complement has only one equilibrium point, then prove that $a \le c(a)$ iff $a \le e_c$ and $a \ge c(a)$ iff $a \ge e_c$.

14. (a) If $A(x) = \begin{cases} 0 & \text{if } x \le -1 \,\&\, x > 3 \\ \frac{x+1}{2} & \text{if } -1 < x \le 1 \\ \frac{3-x}{2} & \text{if } 1 < x \le 3 \end{cases}$,

$B(x) = \begin{cases} 0 & \text{if } x \le -1, x > 5 \\ \frac{x-1}{2} & \text{if } 1 < x \le 3 \\ \frac{5-x}{2} & \text{if } 3 < x \le 5 \end{cases}$ then find

(i) $\alpha_{(A+B)}$ (ii) $\alpha_{(A-B)}$ (iii) $\alpha_{(A\cdot B)}$ (iv) $\alpha_{(A/B)}$.

Or

(b) Prove that $\min[A, MA\times(B,C)] = \max[\min(A,B), \min(A,c)]$.

15. (a) Explain the method of solving the fuzzy linear programming problem defined by $\max \sum_{j=1}^{n} c_j x_j$ such that $\sum_{j=1}^{n} A_{ij} x_j \leq B_i (i \in N_m)$, $x_j \geq 0 (j \in N_n)$ where B_i and A_{ij} are fuzzy numbers.

Or

(b) Solve the following by graphical method

$\min z = x_1 - 2x_2$

subject to $3x_1 - x_2 \geq 1, 2x_1 + x_2 \leq 6$,

$0 \leq x_2 \leq 2, x_1 \geq 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Prove that : A fuzzy set A on $\mathbb{R}$ is convex iff $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$, where min denotes the minimum operator.

Or

(b) (i) Prove that the law of contradiction and the law of excluded middle are violated for fuzzy sets.

(ii) Prove that the law of absorption is valid.

17. (a) Let $A, B \in F(X)$. Then prove that following properties hold for $\alpha, \beta \in [0,1]$

(i) ${}^{\alpha+}A \subseteq {}^{\alpha}A$

(ii) $\alpha \leq \beta$ implies ${}^{\alpha}A \supseteq {}^{\beta}A$ and ${}^{\alpha+}A \supseteq {}^{\beta+}A$

(iii) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ and ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

(iv) ${}^{\alpha+}(A \cap B) = {}^{\alpha+}A \cap {}^{\alpha+}B$ and ${}^{\alpha+}(A \cup B) = {}^{\alpha+}A \cup {}^{\alpha+}B$

(v) ${}^{\alpha}\overline{A} = {}^{(1-\alpha)+}\overline{A}$.

Or

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in F(X)$ and all $\alpha \in [0,1]$ prove that the following properties of fuzzified by the extension principle hold :

(i) ${}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$

(ii) ${}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$.

18. (a) State and prove second characterization theorem of fuzzy complements.

Or

(b) Let i be a t-norm and let $g:[0,1]\to[0,1]$ be a function such that g is strictly increasing and continuous in (0, 1) and $g(0)=0$, $g(1)=1$. Then prove that the following function i^g defined by $i^g(a,b)=g^{(-1)}(i(g(a),g(b)))$ for all $a,b\in[0,1]$, where $g^{(-1)}$ denotes the pseudo-inverse of i^g is also a t - norm.

19. (a) Let $*\in\{+,-,\cdot,/\}$ and let A,B denote continuous fuzzy numbers. Then prove that the fuzzy set $A*B$ defined by $(A*B)(z)=\sup_{z=x*y}\min[A(x),B(y)]$ $Z\in\mathbb{R}$ is a continuous fuzzy number.

Or

(b) Let MIN and MAX be binary operations on $\mathbb{R}$ defined by

$$MIN(A,B)(z)=\sup_{z=\min(x,y)}\min[A(x),B(y)]$$

$$MAX(A,B)(z)=\sup_{z=\max(x,y)}\min[A(x),B(y)],$$ for all $Z\in\mathbb{R}$. Then for any $A,B,C\in\mathbb{R}$, prove that

(i) $MIN(A,MAX(A\mid B))=A$

(ii) $MIN(A,MIN(B,C))=$
$MIN(MIN(A,B),C)$.

20. (a) Explain linear programming problem.

Or

(b) Solve the following fuzzy linear programming problem $\max z=\cdot4x_1+\cdot3x_2$ subject to $x_1+x_2\le B_1$, $2x_1+x_2\le B_2$, $x_1,x_2\ge 0$, where B_1 is defined by

$$B_1(x) = \begin{cases} 1 & \text{when} \quad x \le 400 \\ \dfrac{500 - x}{100} & \text{when} \quad 400 < x \le 500 \\ 0 & \text{when} \quad 500 < x \end{cases} \quad \text{and}$$

B_2 is defined.

$$\text{by } B_2(x) = \begin{cases} 1 & \text{when} \quad x \le 500 \\ \dfrac{600 - x}{100} & \text{when} \quad 500 < x \le 600 \\ 0 & \text{when} \quad 600 < x. \end{cases}$$