(6 pages) Reg. No. : ....

Code No.: 7384 **Sub. Code: ZMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2023.

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2021-2022 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A function which satisfies Laplace's equation $\Delta u = 0$ is said to be ——————

   (a) harmonic (b) Conjugate

   (c) compact (d) morera

2. A series of the form $a_0 + a_1x + a_2x^2 + \ldots$ is called —————— series.

   (a) Sequence (b) Laurent

   (c) Power (d) Exponential

3. The points z and z^* are said to be —————— with respect to the circle c through z_1, z_2, z_3 if and only if $(z^* z_1 z_2 z_3) = \overline{(z\, z_1\, z_2\, z_3)}$

   (a) transitive (b) symmetric

   (c) refluxive (d) equal

4. The cross ratio is —————— under linear transformation.

   (a) Constant (b) Same

   (c) Equal (d) Invariant

5. The integral $\int_\gamma f\,dz$ with continuous f depends only the end points of γ if and only if f is the —————— of an analytic function in

   (a) Derivative (b) Integrand

   (c) Zero (d) Discontinuous

6. The length of a circle with equation $z = z(t) = a + \rho e^{it}, 0 \le t \le 2\pi$ is

   (a) 2π (b) $2\pi i$

   (c) $2\pi\rho$ (d) $\pi\rho$

7. If γ lies inside of a circle, then $n(\gamma, a) =$ —————— for all points a outside of the same circle.

   (a) 1 (b) 0

   (c) – 1 (d) 2

8. A function which is analytic and bounded in the whole plane must reduce to a ——————.

   (a) 0 (b) – 1

   (c) constant (d) Complex number

9. The —————— of $f(z)$ at an insolated singularity a is the unique number R such that $f(z) - R/(z-a)$ the derivative of a single valued analytic function in an annulus $0 < |z-a| < \delta$.

   (a) Pole (b) Zero

   (c) Order (d) Residue

10. A cycle γ is said to —————— the region Ω if and only if $n(r, a)$ is equal to 1 for all $a \in \Omega$ and either undefined or zero for $a \notin \Omega$.

    (a) Closed (b) Open

    (c) Bound (d) Compact

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Derive Taylor-Maclaurin series from power series.

    Or

    (b) Expand $\frac{2z+3}{z+1}$ in powers of $z-1$.

12. (a) If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1\, z_2\, z_3\, z_4)$.

    Or

    (b) Find the linear transformation which carries the circle $|z| = 2$ into $|z+1| = 1$, the point – 2 into the origin and the origin into i.

13. (a) Prove that $\left|\int_r f\,dz\right| \le \int_r |f| \cdot |dz|$.

    Or

    (b) If $f(z)$ is analytic in an open disk Δ, then prove that $\int_\gamma f(z)dz = 0$ for every closed curve γ in Δ.

14. (a) Define zeros and poles with examples.

Or

(b) If the piecewise differentiable closed curve γ does not pass through the point a then prove that $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

15. (a) State and prove argument principle.

Or

(b) Find the residue of the function $\frac{e^z}{(z-a)(z-b)}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Prove that if all zeros of a polynomial $P(z)$ lie in a half plane, then all zeros of the derivative $P'(z)$ lie in the same half plane.

Or

(b) Derive Cauchy-Riemann equations.

17. (a) Prove that the cross ratio $(z_1\, z_2\, z_3\, z_4)$ is real if and only if the four points lie on a circle or on a straight line.

Or

(b) Prove the symmetric principle.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Prove that the line integral $\int_{r} p\,dx + q\,dy$ defined in Ω depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial u}{\partial x} = p, \frac{\partial u}{\partial y} = q$.

19. (a) State and prove Taylor's theorem.

Or

(b) Derive Cauchy's integral formula.

20. (a) Compute $\int_{0}^{\pi} \frac{d\theta}{a+\cos\theta}$, $a > 1$ write about definite integral.

Or

(b) State and prove Cauchy residue theorem.