(6 pages)

Reg. No. : ....

Code No.: 7384   Sub. Code: ZMAM 42

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2023.

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2021-2022 onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A function which satisfies Laplace's equation $\Delta u = 0$ is said to be —————

    (a) harmonic   (b) Conjugate

    (c) compact   (d) morera

2. A series of the form $a_0 + a_1 x + a_2 x^2 + \ldots$ is called ————— series.

    (a) Sequence   (b) Laurent

    (c) Power   (d) Exponential

3. The points z and z^* are said to be ————— with respect to the circle c through z_1, z_2, z_3 if and only if $(z^* z_1 z_2 z_3) = \overline{(z\ z_1\ z_2\ z_3)}$

    (a) transitive   (b) symmetric

    (c) refluxive   (d) equal

4. The cross ratio is ————— under linear transformation.

    (a) Constant   (b) Same

    (c) Equal   (d) Invariant

5. The integral $\int_\gamma f\,dz$ with continuous f depends only the end points of γ if and only if f is the ————— of an analytic function in

    (a) Derivative   (b) Integrand

    (c) Zero   (d) Discontinuous

6. The length of a circle with equation $z = z(t) = a + \rho e^{it}, 0 \le t \le 2\pi$ is

    (a) 2π (b) $2\pi i$

    (c) $2\pi \rho$ (d) $\pi \rho$

7. If γ lies inside of a circle, then $n(\gamma, a) =$ ————— for all points a outside of the same circle.

    (a) 1   (b) 0

    (c) − 1   (d) 2

8. A function which is analytic and bounded in the whole plane must reduce to a —————.

    (a) 0   (b) − 1

    (c) constant   (d) Complex number

9. The ————— of $f(z)$ at an insolated singularity a is the unique number R such that $f(z) - R/(z-a)$ the derivative of a single valued analytic function in an annulus $0 < |z - a| < \delta$.

    (a) Pole   (b) Zero

    (c) Order   (d) Residue

10. A cycle γ is said to ————— the region Ω if and only if $n(r, a)$ is equal to 1 for all $a \in \Omega$ and either undefined or zero for $a \notin \Omega$.

    (a) Closed   (b) Open

    (c) Bound   (d) Compact

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Derive Taylor-Maclaurin series from power series.

    Or

    (b) Expand $\dfrac{2z + 3}{z + 1}$ in powers of $z - 1$.

12. (a) If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1 z_2 z_3 z_4)$.

    Or

    (b) Find the linear transformation which carries the circle $|z| = 2$ into $|z + 1| = 1$, the point $- 2$ into the origin and the origin into i.

13. (a) Prove that $\left| \int_r f\,dz \right| \le \int_r |f| \cdot |dz|$.

    Or

    (b) If $f(z)$ is analytic in an open disk Δ, then prove that $\int_r f(z)dz = 0$ for every closed curve γ in Δ.

14. (a) Define zeros and poles with examples.

Or

(b) If the piecewise differentiable closed curve γ does not pass through the point a then prove that $\int_\gamma \dfrac{dz}{z-a}$ is a multiple of $2\pi i$.

15. (a) State and prove argument principle.

Or

(b) Find the residue of the function $\dfrac{e^z}{(z-a)(z-b)}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Prove that if all zeros of a polynomial $P(z)$ lie in a half plane, then all zeros of the derivative $P'(z)$ lie in the same half plane.

Or

(b) Derive Cauchy-Riemann equations.

17. (a) Prove that the cross ratio $(z_1 z_2 z_3 z_4)$ is real if and only if the four points lie on a circle or on a straight line.

Or

(b) Prove the symmetric principle.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Prove that the line integral $\int_r p\,dx + q\,dy$ defined in Ω depends only on the end points of γ if and only if there exists a function $U(x,y)$ in Ω with the partial derivatives $\dfrac{\partial u}{\partial x} = p$, $\dfrac{\partial u}{\partial y} = q$.

19. (a) State and prove Taylor's theorem.

Or

(b) Derive Cauchy's integral formula.

20. (a) Compute $\int_0^\pi \dfrac{d\theta}{a + \cos\theta}$, $a > 1$ write about definite integral.

Or

(b) State and prove Cauchy residue theorem.

_____