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M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Third Semester

Mathematics — Core

TOPOLOGY — I

(For those who joined in July 2021–2022 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If x is any set the collection of all subset of x is a topology it is called ————
   (a) Discrete topology (b) Non discrete
   (c) Trivial (d) Comparable

2. If Y is a sub space of X then a set U is ———— in Y.
   (a) Closed (b) Subset
   (c) Open (d) None

3. Let $f: X \to Y$ be a bijection. If both f and the inverense function are continuous then f is called
   (a) Imbedding (b) Topological space
   (c) Continuous (d) Homeomorphism

4. If A is a subspace of X, the inclusion function $j: A \to Y$ is continuous is ————
   (a) composites (b) constant
   (c) inclusion (d) none

5. The metric $\overline{d}$ is called ———— of d.
   (a) Metric space (b) Bounded space
   (c) Standard bounded (d) None

6. An uncountable product of R with itself is ————
   (a) Metrizable (b) Not metrizable
   (c) Topology (d) Isometric

7. The image of a connected space under a connected map is ————
   (a) Connected (b) Compact
   (c) Non compact (d) Non connected

8. The image of a compact space under a continuous map is ————
   (a) Connected (b) Compact
   (c) Non compact (d) None

9. If very infinite subset of X has a limit then x is called
   (a) Limit point compact (b) Limit point
   (c) Compact (d) Continuous

10. Let X be metrizable then x is
    (a) Countable (b) Continuous
    (c) Limit point (d) None of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set X with Y.

    Or

    (b) The topologies of $\mathbb{R}_\ell$ and $\mathbb{R}_K$ are strictly finer than the standard topology on $\mathbb{R}$ but are no comparable with one another – Prove.

12. (a) State and prove pasting lemma.

    Or

    (b) If $\mathcal{B}$ is a basis for the topology of X and C is a basis for the topology of Y the collection $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $C \in C\}$ is basic topology of $X \times Y$ - prove.

13. (a) Let d and d' be two metrics on the set X; let τ and τ' be the topologies they reduce respectively. Then prove that τ' is finer than τ if and only if for each x in X and each $\varepsilon > 0$ there exist $\delta > 0$ such that $B_{d'}(x, \delta) \subset B_d(x, \epsilon)$.

    Or

    (b) Let $f: X \to Y$; let X and Y be metrizable with metrics d_X and d_Y respectively. Then continuity of f is equivalent to the requirement that given $x \in X$ and given $\epsilon > 0$ there exist $\delta > 0$ such that $d_X(x, y) \Rightarrow d_Y(f(x), f(y)) < \varepsilon$ - prove.

14. (a) Prove that every closed subspace of a compact space is compact.

    Or

    (b) Let $f: X \to Y$ be a bijective continuous function. If X is compact and Y is hausdorff then prove that f homeomorphism.

[P.T.O]

15. (a) Let X be locally compact Hausdroff space. Let A be a subspace of X. If A is closed in X or open in X then prove that A is locally compact.

Or

(b) Compactness implies limit point compactness but not conversely – Prove.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X and Y.

Or

(b) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets in X such that for each open set U of X and each x in U there is a element C of $\mathcal{C}$ such that $x \in C \subset U$. Then prove that $\mathcal{C}$ is a basis for the topology of X.

17. (a) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ is given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$. Where $f_\alpha : \prod X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Then prove that the function f is continuous if and only if each function f_α is continuous.

Or

(b) Let X and Y are topological spaces. Let $f : X \to Y$. Then prove that the following are equivalent.

(i) f is continuous

(ii) for every subset A of X one has $f(\overline{A}) \subset \overline{f(A)}$

(iii) for every closed set B of Y the set $f^{-1}(B)$ is closed in X

(iv) for each $x \in X$ and each neighborhood V of $f(x)$ there is a neighborhood U of x such that $f(U) \subset V$. If the condition in (d) holds for the point x of X we say that f is continuous at the point x.

18. (a) The topologies of R^n induced by the Euclidean metric d and the aquare metric ρ are same as the product topology on R^n – Prove.

Or

(b) State and prove uniform limit theorem.

19. (a) Prove that the product of finitely many compact space is compact.

Or

(b) Every compact subspace of Hausdorff space is closed – Prove.

20. (a) Let X be a metrizable space. Then prove that the following are equivalent

(i) X is compact

(ii) X is limit point compact

(iii) X is sequentially compact.

Or

(b) Prove that a space X is locally compact Hausdorff space if and only if it has a unique one point compactification.

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