(6 pages) **Reg. No. :** ..................................

**Code No. : 7378** **Sub. Code : ZMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Third Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2021-2022 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A closed trail whose origin and internal vertices are distinct is a ————
   (a) cycle (b) bipartite
   (c) empty (d) complete

2. The number of edges of $G$ incident with $v$ is called ———— of the vertex $v$.
   (a) regular (b) $K$-regular
   (c) degree (d) length

3. An acyclic graph is also called as a ————
   (a) tree (b) forest
   (c) path (d) cycle

4. A connected graph is a tree if and only if every edge is a ————
   (a) path (b) cycle
   (c) cut edge (d) same

5. A 3 regular graph with cut edges ———— have a perfect matching.
   (a) must (b) may
   (c) need not (d) none

6. A tree has atmost ———— perfect matching.
   (a) one (b) two
   (c) $\infty$ (d) 0

7. A ———— of a simple graph $G$ is subset $S$ of $V$ such that $G(S)$ is complete.
   (a) Tour (b) Euler
   (c) Clique (d) Independent

8. The value of $r(1, l) = r(K, 1) =$ ————
   (a) 0 (b) 1
   (c) 2 (d) $K$



9. For any graph the chromatic number $\chi \leq$ ————
   (a) $\Delta$ (b) $\Delta - 1$
   (c) $\Delta + 1$ (d) 0

10. Every critical graph is a ————
    (a) Euler (b) Cut vertex
    (c) Block (d) Cover

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $G$ be simple graph. Show that $\Sigma = \binom{v}{2}$ if and only if $G$ is complete.

    Or

    (b) Prove that in any graph, the number of vertices of odd degree is even.

12. (a) Prove that a connected graph is a tree if and only if every edge is a cut-edge.

    Or

    (b) Prove that a graph $G$ with $v \geq 3$ is 2 connected if and only if any two vertices of $G$ are connected by atleast two internally disjoint paths.



13. (a) Prove that a non empty connected graph is eulerian if and only if it has no vertices of odd degree.

    Or

    (b) Prove that a matching $M$ in $G$ is a maximum matching if and only if $G$ contains no $M$ - augmenting path.

14. (a) Prove that if $G$ is bipartite then $\chi' = \Delta$.

    Or

    (b) If $\delta > 0$ then prove that $\alpha' + \beta' = v$.

15. (a) Prove that in a critical path, no vertex cut is a clique.

    Or

    (b) Prove that if $G$ is $K$-critical, then $\delta \geq k - 1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Explain Dijktra's algorithm.

    Or

    (b) Prove that a graph is bipartite if and only if it contains no odd cycle.

17. (a) (i) Prove that a vertex $v$ of a tree is a cut vertex of $G$ if and only if $d(v) > 1$.

(ii) Prove that if $e$ is a link of $G$, $\tau(G) = \tau(G - e) + \tau(G.e)$.

Or

(b) Prove the relation $K \le K' \le \delta$.

18. (a) Prove that if $G$ is a simple graph with $\upsilon \ge 3$ and $\delta \ge \upsilon/2$ then $G$ is Hamiltonian.

Or

(b) Prove that if $G$ be a bipartite graph with bipartition $(X, Y)$ then $G$ contains a matching that saturates every vertex in $X$ if and only if $|N(S)| \ge |S|$ for all $S \subseteq X$.

19. (a) Prove that if $G$ is simple then either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

Or

(b) Prove that for any two integers $K \ge 2$ and $l \ge 2$ $r(K\,l) \le r(K, l-1) + r(K-1, l)$.



20. (a) If $G$ is a connected simple graph and is neither an odd cycle nor a complete graph then show that $\chi \le \Delta$.

Or

(b) If $G$ is a 4-chromatic then prove that $G$ contains a subdivision of $K_4$.

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